Summary Measures of Equalising Income Mobility Based on 'Three Is of Mobility' Curves

John Creedy and Norman Gemmell

WORKING PAPER 07/2022
March 2022

Working Papers in Public Finance

VICTORIA UNIVERSITY OF WELLINGTON
TE HERENGA WAKA

Chair in Public Finance
Wellington School of Business and Government

The Working Papers in Public Finance series is published by the Victoria Business School to disseminate initial research on public finance topics, from economists, accountants, finance, law and tax specialists, to a wider audience. Any opinions and views expressed in these papers are those of the author(s). They should not be attributed to Victoria University of Wellington or the sponsors of the Chair in Public Finance.

Further enquiries to:
The Administrator
Chair in Public Finance
Victoria University of Wellington
PO Box 600
Wellington 6041
New Zealand

Phone: +64-4-463-9656
Email: cpf-info@vuw.ac.nz

Papers in the series can be downloaded from the following website:
https://www.wgtn.ac.nz/sacl/centres-and-chairs/cpf/publications/working-papers

Summary Measures of Equalising Income Mobility Based on 'Three Is of Mobility' Curves

John Creedy and Norman Gemmell*

Abstract

This paper extends the 'Three Is of Mobility (TIM) Curve' framework, developed by Creedy and Gemmell (2019) to produce summary measures of equalising mobility between two periods, based on areas within the diagram. Two concepts of equalising mobility are considered. The first involves equalisation of incomes in the second period, achieved by a compression of incomes and no re-ranking. The second concept involves maximum redistribution in terms of the inequality of incomes measured over the two periods combined. This involves differential income growth and maximum re-ranking, whereby second-period incomes are 'swapped': the richest person becomes the poorest, and so on. The measures are illustrated using a large sample of taxpayers' incomes in New Zealand.

*Victoria School of Business and Government, Victoria University of Wellington, New Zealand. This paper is part of a project on 'Measuring Income Inequality, Poverty, and Mobility in New Zealand', funded by an Endeavour Research Grant from the Ministry of Business, Innovation and Employment (MBIE) and awarded to the Chair in Public Finance at Victoria University of Wellington. The Confidential anonymized individual taxpayer income data used in this paper were provided to the authors by the New Zealand Inland Revenue Department (IRD) and are not publicly available. We are grateful to Sean Comber for extracting the IRD data.

1 Introduction

The value of diagrams to summarise income distribution characteristics is exemplified by the famous Lorenz curve, which has become a standard device to illustrate the nature of cross-sectional income distributions. With individual observations arranged in ascending order, the Lorenz curve plots (within a box of unit height and base) the cumulative proportion of total income against the corresponding cumulative proportion of individuals. This provides much more information 'at a glance', about relative income inequality, than either the density function or the distribution function alone, and can quickly allow qualitative comparisons between different periods or population groups. It gives rise to the famous Gini inequality summary measure, in terms of the area contained between the Lorenz curve and the diagonal line of equality, expressed as a proportion of the area represented by maximum inequality.

A challenge arises in the context of income mobility, where the same individuals are observed in, say, two different years and where the 'basic data' are in the form of a joint distribution. Where it is required to illustrate the main characteristic of mobility in a simple diagram, Creedy and Gemmell (2019) proposed a convenient curve that reflects several important features of differences in income growth rates, conditional on initial income.[1] With individuals ranked in ascending order of initial income, the curve plots the cumulative proportional income change *per capita* (not, as in other growth curves, per head of the cumulated sub-group), against the corresponding proportion of individuals.[2] This diagram enables three characteristics of mobility – incidence, intensity and inequality – to be clearly illustrated: it is referred to as a 'Three Is of Mobility', or TIM, curve, following the terminology adopted by Jenkins and Lambert (1997) in the context of cross-sectional

[1] Associated diagrams in the context of mobility into and out of poverty are presented in Creedy and Gemmell (2018). For an introduction to the various curves, see Creedy and Gemmell (2022). For practical purposes, as seen below, the income growth rates can be approximated by log-changes.

[2] For example, Van Kerm (2009) and Jenkins and Van Kerm (2016) introduce an 'income growth profiles' (IGP), and a cumulative version (CIGP) in which income growth is calculated per head of the cumulated group.

poverty. The 'end value' of the curve is the average proportional growth rate, and dividing all values by this gives a 'normalised' TIM curve, denoted nTIM. The TIM curve concept is described briefly in Section 2.

If all initial incomes are subject to the same proportional growth rate the TIM curve is a straight line from the origin, at a slope given by the average growth rate. If there is a systematic tendency for mobility to be equalising, over the whole range of incomes, the TIM curve is concave. Comparisons among different periods or population groups can easily be made using nTIM curves. If one (normalised) TIM curve lies above a second curve, it can be said that the first displays unequivocally more equalising mobility, in terms of relative income changes. However, it may be desired to provide a quantitative measure of the extent to which such mobility differs between two curves. The value of a scalar summary measure increases in situations where nTIM curves intersect, such that one curve displays more equalising mobility than the other over only a range of the distribution. In the context of inequality comparisons using Lorenz curves, a similar need for a scalar summary measure arises where Lorenz curves intersect or where quantitative comparisons are needed to supplement qualitative comparisons between distributions.

The present paper is concerned with the question of whether an overall summary measure of the equalising extent of mobility can be defined, as with the Gini inequality measure and the Lorenz curve in the cross-sectional case, in terms of the area contained by the nTIM curve and the straight line of equal proportional growth? This requires a statement of what is meant by 'equalising mobility' as a benchmark against which to compare the nTIM curve. Two concepts are considered. One is in terms of equalising cross-sectional incomes in the second period: in this context the re-ranking of individuals is considered to 'frustrate' redistribution, since only a compression of incomes is required (with all individuals moving to the mean). The second concept is in terms of inequality based on a longer accounting period, and here a 'maximum re-ranking' standard is relevant, in which individuals 'swap' positions and incomes in the second period, whereby the richest becomes the poorest, and so on.[3]

[3] Measures of positional changes within the distribution are examined in detail in Creedy

First the basic TIM curve is defined briefly in Section 2: a more formal statement is given in the Appendix. Section 3 proposes summary measures of equalising and disequalising mobility, in terms of only differential income growth and second-period inequality. Section 4 considers a longer accounting period and introduces the role of rank-order changes. Illustrations using data from New Zealand's Inland Revenue department are provided in Section 5. Conclusions are in Section 6.

2 The TIM Curve

Jenkins and Lambert (1997) demonstrated that three important dimensions of cross-sectional poverty can be summarised by the following curve. Let x_i denote individual i's income, for $i = 1, ..., n$. For a specified poverty line, x_p, poverty gaps are defined by $g(x_i) = 0$ for $x_i > x_p$ and $g(x_i) = x_p - x_i$ for $x_i < x_p$. With incomes arranged in ascending order, plot $\frac{1}{n}\sum_{i=1}^{k} g(x_i)$ against $\frac{k}{n}$, for $k = 1, ..., n$. That is, the total cumulative poverty gap per capita is plotted against the associated proportion of people. The curve conveniently displays the *incidence* of poverty (the headcount poverty measure), its *intensity* (the income gap, $x_p - x_i$), and its *inequality* (the dispersion of incomes below x_p). They therefore named the curve the 'three Is of poverty', or TIP, curve. The slope at any point is equal to the average poverty gap. A flattening of the curve therefore shows the extent to which the average poverty gap falls as income rises towards x_p. Thus, inequality among the poor is reflected in the curvature of the TIP curve. The curve becomes horizontal beyond x_p. Poverty is unambiguously higher where a TIP curve lies wholly above and to the left of an alternative TIP curve.

The TIP curve relates to poverty within a specified period of time over which income is measured. However, it is possible to define a related curve in the context of income growth between two periods. Creedy and Gemmell (2019) define the 'three Is of mobility', or TIM, curve as follows. Define the logarithm of income, $y_i = \log x_i$, for individuals $i = 1, ..., n$. Hence $y_{i,t} - y_{i,t-1}$ is (approximately) person i's proportional change in income from period $t-1$

and Gemmell (2019).

to t. With log incomes ranked in ascending order, plot $\frac{1}{n}\sum_{i=1}^{k}(y_{i,t}-y_{i,t-1})$ against $h=\frac{k}{n}$, for $k=1,...,n$. Thus the TIM curve plots the cumulative proportional income change per capita against the corresponding proportion of individuals, h. A more formal statement of the TIM curve is given in the Appendix.

A TIM curve allows focus on the mobility of a particular group of low-income individuals: those with incomes below $x(h)$, for the proportion, h, of the population. In this framework h captures the *incidence* of the particular group of concern. Similarly, the *intensity* and *inequality* dimensions of mobility in terms of income growth are reflected in the shape of the TIM curve, by analogy with the TIP curve.

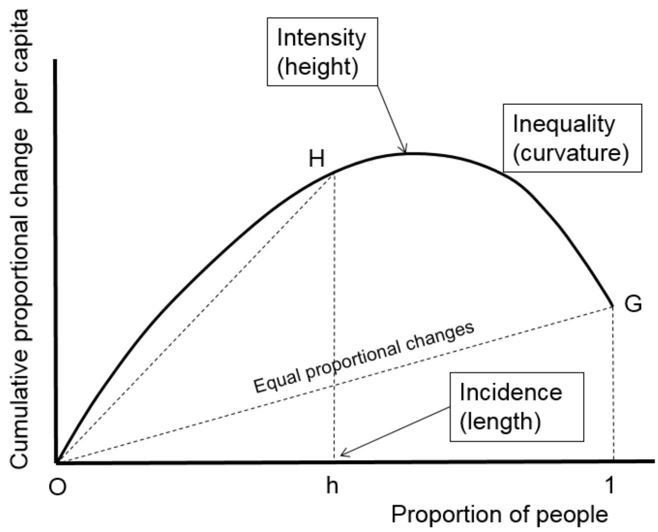

Figure 1: A TIM Curve

A hypothetical example of a TIM curve is shown in Figure 1, with $h=k/n$ on the horizontal axis. This reflects a situation in which relatively lower-income individuals receive proportional income increases which are greater than that of average (geometric mean) income. Hence the TIM curve, OHG, lies wholly above the straight line OG. The TIM curve in practice is not necessarily smooth and concave over the whole range, as shown in Figure

1. There can be income reductions as well as increases and, depending on how they are spread across the initial income distribution, and the curve can be more or less 'jagged' depending on the extent of non-systematic income-related changes.

If all incomes increase by the same proportion, the TIM curve is the straight line OG. The height, G, indicates the average growth rate of the population as a whole, with the height, H, indicating the average growth rate for those below $x(h)$. Furthermore, inequality is reflected in the degree of curvature. For example, the curvature of the arc OH relative to the straight line OH indicates that lower income individuals have higher (more unequal) growth than those individuals to the left of, but closer to, h.

Suppose interest is focussed on those below the h^{th} percentile, indicated in Figure 1. There is less 'inequality of mobility' within the group below h, shown by the fact that the TIM curve from O to H is closer to a straight line than the complete curve OHG.[4] The TIM curve also shows that the income growth of those below h is larger than that of the population as a whole. The average growth rate among the poor (the intensity of their growth) is given by the height H.

If it is preferred to assess mobility from *relative* income growth rates, some normalisation of the TIM curves is required. For example, comparing the income mobility experienced across different periods, the arithmetic mean income growth rate, g, is likely to vary across periods, such that the height of point G in Figure 1 differs. This can make comparisons of the degree of inequality of mobility across periods difficult. In this case equivalent normalised TIM curves, or nTIM curves, can be obtained where each TIM is normalised by the average growth rate for each period. With normalisation, $M_{h,t}$ reaches a value of 1 at $h = 1$, though $M_{h,t}$ values can exceed 1 at lower values of h, as illustrated in Figure 1. This normalisation allows the degree of concavity or convexity of each TIM curve to be directly compared.

[4] There is a potential ambiguity in the use of the term 'inequality' here since the TIP curve refers to a cross-sectional distribution whereas the TIM curve refers to income changes.

3 A Summary Measure

In the case of the Lorenz curve, the definitions of complete equality and inequality (whereby only one person has a positive income while all others are zero) are straightforward to define and envisage. In the former case, the Lorenz curve corresponds to the diagonal line of equality, and in the latter case it corresponds (for a large enough population) to the base and right hand side of the unit-square box. Any actual curve must lie between these two extremes, and the arrangement of incomes in ascending order ensures that the Lorenz curve is convex. In the context of mobility as differential growth, attempting to define the extremes is not so straightforward. However, one extreme case, that of a relative inequality-preserving mobility process, is simple. It is illustrated in Figure 1, where the normalised TIM curve is a straight line corresponding to the diagonal shown.

Consider one extreme form of inequality-reducing mobility, where income changes are either zero or positive. If only the poorest person has an income increase, while all other incomes remain unchanged, the normalised TIM curve is simply a horizontal line, after following the vertical axis up to 1. But this is an arbitrary case. Another possibility is the set of proportional income changes which produce equal incomes in the second period, equal to the actual average in that period. In terms of incomes, $x_{2,i}$ and $x_{1,i}$, for person $i = 1, ..., n$ in periods 2 and 1, for growth rates, g_i, and second-period arithmetic mean income, $\bar{x}_2$, this requires $x_{2,i} = x_{1,i}(1 + g_i) = \bar{x}_2$, or (for strictly positive initial incomes):[5]

$$g_i = \left(\frac{\bar{x}_2}{x_{1,i}}\right) - 1 \tag{1}$$

However, these g_is produce a TIM curve having an average growth rate that differs from the actual average growth rate, which means that the normalised version adjusts the cumulative growth rates per capita by a different amount from that used to obtain the actual TIM curve.[6] Suppose it is required to

[5] If changes are expressed as log-income-changes, then $g_i = \log \bar{x} - \log x_{1,i}$. This uses the approximation $\log(1 + g_i) = g_i$.

[6] This essentially arises because of the basic property that the average of ratios is not the same as the ratio of averages.

have equal incomes, $\widetilde{x}$, in the second period *and* an average growth rate equal to the actual rate, g. Using logarithmic changes to approximate proportional changes:

$$g = \frac{1}{n} \sum_{i=1}^{n} (\log x_{2,i} - \log x_{1,i}) \tag{2}$$

The $\widetilde{x}$ and associated g_i must now satisfy:

$$g_i = \log \widetilde{x} - \log x_{1,i} \tag{3}$$

and:

$$\frac{1}{n} \sum_{i=1}^{n} g_i = g \tag{4}$$

Substituting (3) into (4) and equating with (2) gives:

$$\log \widetilde{x} = \frac{1}{n} \sum_{i=1}^{n} \log x_{2,i} \tag{5}$$

Furthermore, substitution into (3) gives the set of growth rates needed:

$$g_i = \left(\frac{1}{n} \sum_{i=1}^{n} \log x_{2,i} \right) - \log x_{1,i} \tag{6}$$

Figure 2 illustrates two normalised TIM curves for a given initial income distribution in period 1. The solid line is the actual nTIM curve, and the higher dashed line is the hypothetical curve which would arise from the application of proportional income changes according to equation (6). The question is whether a useful measure of the degree of systematic equalising mobility can be obtained in this diagram. Clearly, the area B (between the nTIM curve and the diagonal line of equal proportional changes) alone does not provide an appropriate measure, since the scope for equalising differential income growth depends on the initial income distribution. The same dynamic process (in terms, say, of the relationship between the proportional growth rate and initial income) gives a smaller area for a relatively more equal distribution than for a more unequal distribution (for the same overall income growth rate). The maximum area is $A+B$, where A is the area between the actual nTIM and the hypothetical 'fully equalising-mobility' nTIM. Consider a 'degree of equalising mobility' measure, M_E, defined as:

$$M_E = \frac{B}{A + B} \tag{7}$$

The maximum value this can take is 1 while the minimum is 0 (when relative incomes do not change). The example shown is one in which there is systematic second-period-equalising mobility, in that the nTIM curve is everywhere above the diagonal nTIM of equal proportional changes, which is the dominant case in empirical applications.

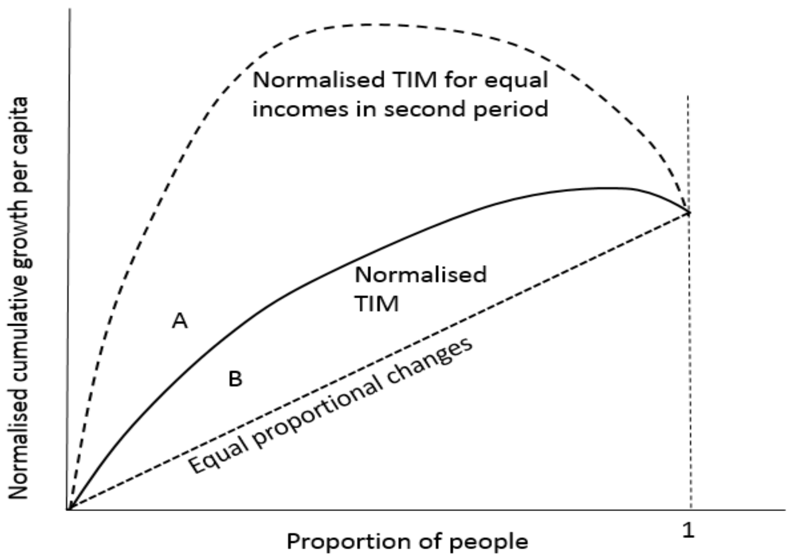

Figure 2: Actual nTIM Curve and nTIM corresponding to Equal Second-Period Incomes and an Average Growth Rate Equal to the Actual Rate

However, there may be cases where there are sufficiently large disequalising changes, along with other equalising changes in other ranges of the distribution, so that the normalised TIM curve moves below the diagonal ('equal-proportional change') line for an initial part of its length.

It is important to recognise that the equalising TIM or nTIM curve is not uniquely defined, as it depends on the form of the initial distribution. A more equal distribution in period 1 produces a lower and less-concave curve. Nevertheless, the fact that a measure of equalising mobility depends on the initial distribution is not really surprising. For example, it corresponds to the fact that, in a slightly different context (though one involving movement from one distribution to another), income tax progressivity measures depend on the initial or pre-tax income distribution.

The characteristics of the two curves, the nTIM and the associated 'equalising nTIM' curve that generates complete equality in the second period, are illustrated in Figure 3 for a simple hypothetical example of a population consisting of just 7 individuals. Suppose their incomes in the first period, arranged in ascending order, are 100, 200, 500, 1000, 1500, 3000, 4500. The corresponding incomes in the second period, following a mobility process, are 180, 280, 680, 1200, 1600, 3400, 4600. The normalised TIM curve, the solid line in the diagram, clearly demonstrates that the process is equalising. However, the curve is still some distance from the curve associated with a fully equalising process.

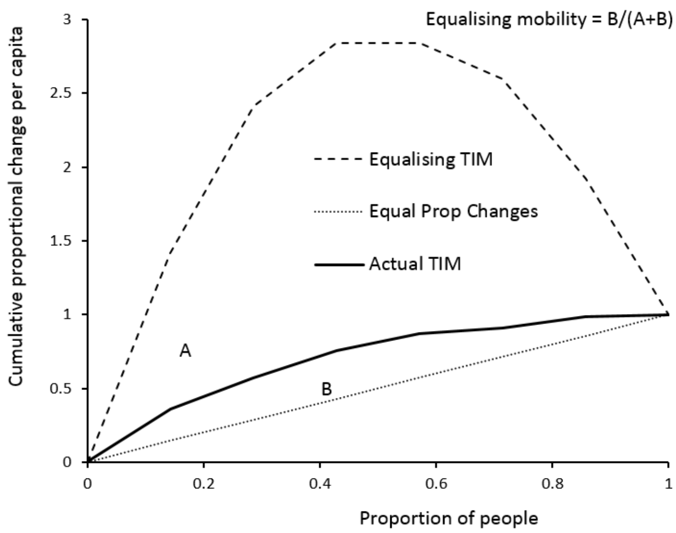

Figure 3: Hypothetical Example of Normalised TIM Curve and Associated Extreme Equalising Mobility

In general, denote the proportions of people and cumulative growth per capita by p_j and q_j respectively, for $j = 1, ..., n$, with $p_1 = q_1 = 0$ and $p_n = q_n = 1$. The areas may be found using the standard trapezoidal rule for the area, I, beneath the piecewise-linear curve:

$$I = \frac{1}{2} \sum_{j=2}^{n} (p_j - p_{j-1})(q_j + q_{j-1}) \tag{8}$$

Remembering that 0.5 – the area below the diagonal line of equal proportional changes – must be subtracted to obtain the relevant areas between the nTIM curve and the diagonal, the area B for this hypothetical case is found to be 0.2076 and the area $A+B$ is 1.576. Hence the degree of equalisating mobility, $M_E = 0.132$.

Although systematic inequality-increasing mobility has not been observed in practice, the concept of extreme inequality-increasing mobility is somewhat different. The actual nTIM curve arising from a systematically disequalising process would of course be consistently below the diagonal equal-proportional-changes line. One substantial inequality-increasing case arises if only the richest person has a positive income increase, and all other incomes remain constant. The nTIM curve follows the bold line around the base and right-hand side of the unit box. But of course this is not the extreme case, as it could be taken further by allowing for negative income changes. Thus, an extreme inequality-increasing case could be defined as a mobility pattern that reduces all but the maximum income to zero, and transfers all income to the initially richest person. In this case, all $g_i = -1$, except for the richest person. The corresponding nTIM curve is then uniquely defined. It is a downward sloping '45 degree' straight line from the origin until reaching $h = 1$, when it becomes vertical with a height of $+1$.

This concept is shown in Figure 4. The area of the triangle indicated by CDE represents the area representation of the furthest distance from the line of equal proportional changes. This area is equal to 1 (half the base, of 2, multiplied by the height of 1). The area between the actual nTIM curve and the line of equal changes is the area, A. Hence in this case, a measure of disequalising mobility, M_D, is simply given by the area A, so that $M_D = A$. If

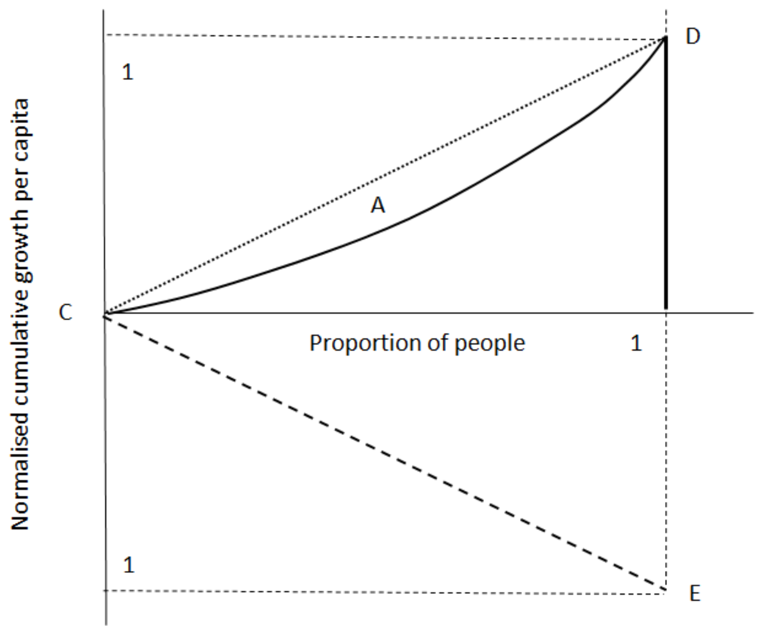

Figure 4: Maximum Inequality-Increasing Income Mobility

there are disequalising and equalising ranges so that the nTIM curve crosses the diagonal line one or more times, a measure of disequalising mobility may be obtained by taking $M_E - M_D$.

4 Positional Changes and the TIM curve

In the previous section, extreme inequality-reducing mobility was considered in terms of the hypothetical pattern of changes, g_i, giving rise to equal incomes in the second period, and with the same average growth rate as actual incomes. With incomes arranged in ascending order, complete equality can be achieved by a compression of incomes towards the arithmetic mean income. That is, equality can be achieved without any changes in the rank-order of individuals in the income distribution. Changes in the relative positions, or re-ranking, actually 'frustrate' an otherwise systematic equalising mobility process. Again there is a corresponding analogy with tax progressivity analyses. A progressive tax structure may, for various reasons, introduce some rank-order changes: for example, it can arise if the tax system allows individual-specific changes to the tax function, depending on the non-income characteristics of individuals (such as the number of dependents and associated tax deductions).

However, it is possible to consider a process of equalising mobility in which an extreme type of re-ranking takes place along with differential income growth. If equality is viewed not in terms of second-period incomes but in terms of a longer accounting period – incomes measured over the two periods – a further type of maximum equalising mobility can be defined. An extreme form of longer-accounting-period equalisation can be achieved by a process involving a complete reordering (maximum re-ranking) of period 2's incomes. Hence, the poorest in period 2 changes place with the richest, the second-poorest person changes place with the second-richest person, and so on.

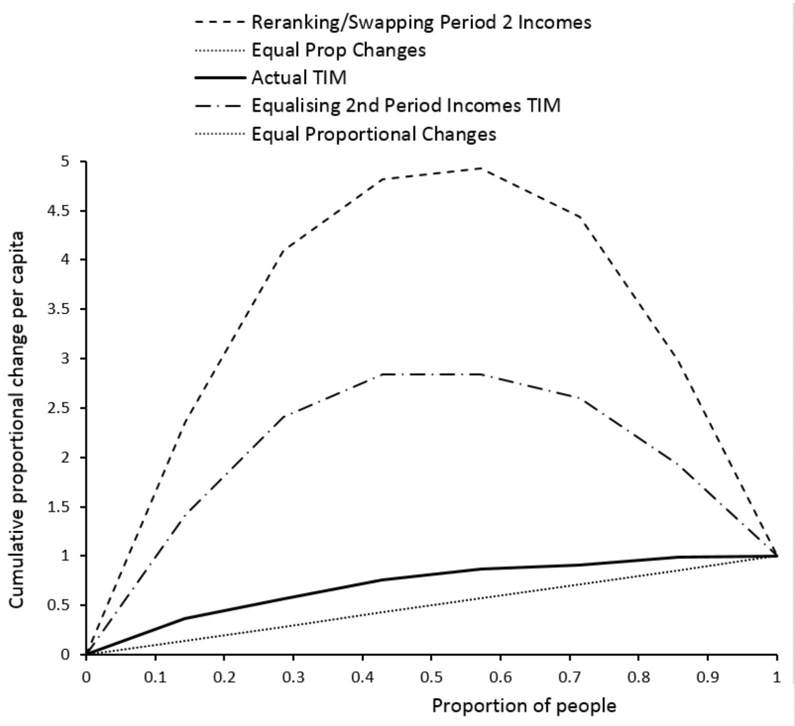

Figure 5: Hypothetical Example With Maximum Reranking and Period 2 Income Swapping

This complete reversal of ranks, combined with a 'swapping of incomes', with the richest person simply replacing the poorest, and so on across the distribution, produces no change in annual relative income inequality in period 2. Furthermore, the combination of 'changing places' and 'swapping incomes' in period 2 does not produces complete equality, when incomes are measured over two periods, because of the differential income growth from period 1 to period 2. However, it may be regarded as providing an alternative basis with which to compare the actual normalised TIM curve: it combines maximum re-ranking with a reasonable view of a maximum equalising-growth distribution of income changes.

Consider again the simple numerical example above, with just seven individuals. Figure 5 shows, in addition to the 'equalising-period-2 income' nTIM, a 'maximum re-ranking combined with income-swapping' curve. It is clear from the figure that this new TIM curve represents a distribution of income growth changes that, combined with the greatest degree of re-ranking that arises from swapping period 2 income (from the top to the bottom of the distribution, and so on), produces a greater extent of equalisation of incomes. This is because in this benchmark case, longer-period incomes become significantly more equal, rather than simply equalising period 2's incomes. This suggests that an additional 'extreme' measure of equalising changes can be obtained by comparing the area underneath the actual nTIM with that below this new hypothetical nTIM curve. This is illustrated in the following section using a practical example.

5 Illustrative Examples

This section illustrates the nTIM curves and associated measures, using Inland Revenue Department (IRD) data from a 2 per cent random sample of New Zealand personal income taxpayers. A confidential dataset was obtained from IRD, giving incomes of a constant group of individuals in 2006 and 2010. To avoid the exercise being contaminated by taxpayers with very low incomes, such as small part-time earnings of children, or small capital incomes of non-earners, individuals with annual incomes less than \$1,000 were omitted from the sample. This yielded a usable sample of 32,970 individuals.

Figure 6 shows, as a solid line, the normalised TIM curve for income changes between 2006 and 2010. This lies above the straight line of equal proportional changes over the whole of its length, demonstrating a substantial amount of equalising mobility over the period. The dashed line in Figure 6 is the normalised TIM curve corresponding to the hypothetical case of equalisation of incomes in the year 2010. This completely smooth curve arises from a compression or squeezing of the distribution towards its mean, and avoids any re-ranking. Although the actual nTIM curve looks quite smooth, this is an artifact of the scale of the diagram: when magnified it is a jagged edge, reflecting the existence of many individuals experiencing relative income reductions in the lower-income ranges and relative income increases in the higher ranges despite the systematic equalising tendency over the whole range. The combined effect is to produce many rank-order changes and to maintain approximate stability in the inequality of incomes in 2006 and 2010: the variance of logarithms in fact increases very slightly from 0.711 to 0.738.[7]

Using equation (7), the extent of equalising mobility, M_E, as defined above, is found to be 0.325. However, care must be taken in interpreting this value, as discussed further below. For example, it does not mean that, over the five-year period, differential income growth has moved the initial (2006) income distribution a third of the way towards equality.

Figure 7 adds to Figure 6 the hypothetical nTIM that arises from the maximum re-ranking, that is 'reverse ranking' with income swapping, of period 2 (2010) incomes. This diagram also indicates a number of relevant areas, reflecting the distance between curves using an area measure of distance. The figure also replaces percentiles on the horizontal axis with the equivalent population proportions, to facilitate measurement comparisons. The relevant areas are given in Table 1.

[7] These values necessarily apply to the constant large sample of individuals who were present in both years. In the broader cross-sectional distributions, there are exits and entrants into the population of taxpayers, which also combine to stabilise the distribution of income in any year.

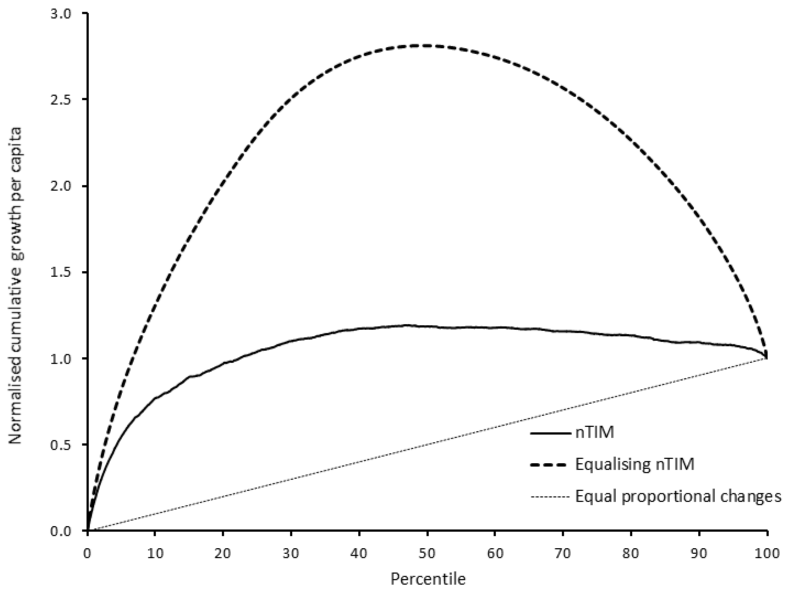

Figure 6: Normalised and Equalising TIM curves: NZ Income Taxpayers 2006 to 2010

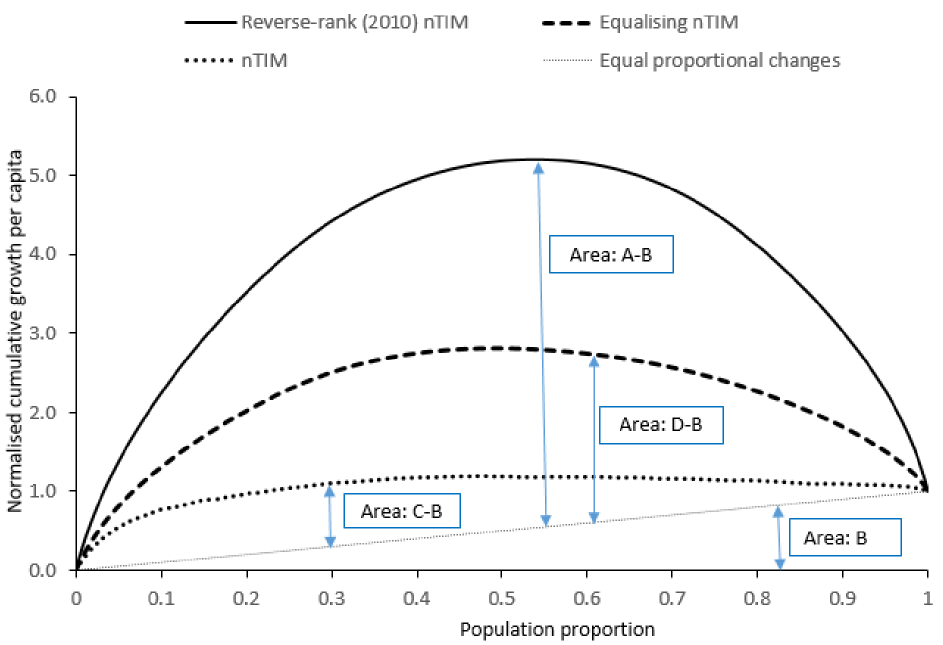

Figure 7: Alternative New Zealand nTIM Curves

Figure 7 and the table show, firstly, that area B, under the line of equal proportional changes, is equal to 0.5, being half of the box with sides of unit length. This provides a convenient benchmark against which other areas may be compared. Secondly, it can be seen in Table 1 that the area, C, beneath the nTIM curve is 1.04. Hence, the area between the nTIM and the line of equal proportional changes, labelled $C - B$, is equal to 0.54. Following the same approach, the area between the 'income swapping' nTIM and the line of equal proportional changes, area $A - B$, is equal to 1.66. Furthermore, the area between the 'equalising nTIM' and the line of equal proportional changes, area $D - B$, is equal to 3.35.

Table 1: Areas Under nTIM Curves in Figure 7

Area under curve:	Area label in Figure 7	Value	Value (net of EPC)
Equal prop. changes (EPC)	B	0.50	-
nTIM	C	1.04	0.54
Equalising nTIM	D	2.16	1.66
Reverse-rank (RR) nTIM	A	3.85	3.35
Ratios:			
(i) $\frac{\text{nTIM}}{\text{RR nTIM} - \text{EPC}}$	$\frac{C-B}{A-B}$	$\frac{0.54}{3.35}$	0.161
(ii) $\frac{\text{nTIM}}{\text{Equalising nTIM} - \text{EPC}}$	$\frac{C-B}{D-B}$	$\frac{0.54}{1.66}$	0.325

Table 1 includes two ratios, both measuring the area beneath the nTIM curve (net of the area B) relative to (i) the (net) area beneath the 'income swapping/reverse rank' nTIM curve; and (ii) the area beneath the 'equalising (second period income) nTIM' curve. It can be seen that ratio (i) is equal to 0.161 (16.1%), while ratio (ii) is equal to 0.325 (32.5%). These values can be interpreted as follows. The extent to which observed income mobility between 2006 and 2010 is equalising, measured by the area $C-B$ in Figure 7, represents about 16 per cent of the maximum that would be achieved if actual 2010 incomes had instead be reallocated such that there was a complete re-ranking of all individuals in the sample.

It is also the case that the observed mobility represents around 32 per cent of that which would achieve complete income equality in 2010 purely via income compression towards the mean (that is, with no rank changes). As noted earlier, this is the value of the measure M_E in equation (7). However, since the actual nTIM curve captures a mixture of 'pure' income compression between 2006 and 2010, and a substantial degree of re-ranking of individuals' incomes, comparing the two areas ($D - B$ and $C - B$) in ratio (ii) is not straightforward. Ratio (i), on the other hand, provides a readily interpretable comparison of the progressivity of actual mobility with the extreme progressive case of income swapping (a given total) to achieve a complete re-ranking of individuals' incomes, and 'extreme equalising' changes when viewed from a longer-accounting-period perspective.

6 Conclusions

This paper has extended the 'Three Is of Mobility (TIM) Curve' framework, initially developed by Creedy and Gemmell (2019). The TIM curve, obtained by plotting the cumulative growth *per capita* against the corresponding proportion of people (ranked in ascending-income order), provides a convenient illustration of systematic equalising tendencies in differential income growth. While such visual qualitative comparisons (between time periods or geographical regions or demographic groups) are useful, in some applications it is desirable to have quantitative summary measures of equalising mobility between two periods. The present paper has shown that measures can be based, as in the famous Lorenz curve used to depict cross-sectional inequality, on areas within the diagram. These are area measures of the 'distance' from the TIM curve to two alternative curves which depict, in different senses, hypothetical extreme equalising mobility cases.

The first case involves the equalisation of incomes in the second period, such that all second-period incomes are equal to the actual average, and the average growth rate is equal to the actual average growth over the relevant period. This involves a compression of incomes and no re-ranking. If second-period equality is treated as 'extreme equalisation', then any re-ranking of incomes (generated by non-systematic changes) can be regarded as 'frustrating' redistribution.

The second concept involves a different concept of maximum redistribution, defined in terms of the inequality of incomes measured over the two periods combined. This hypothetical extreme involves a combination of differential income growth with maximum re-ranking, whereby second-period incomes are 'swapped': the richest person becomes the poorest, and so on. In this case, maximum re-ranking is viewed as a fundamental component of equalising change.

The measures were illustrated using a large sample of taxpayers' incomes in New Zealand, obtained from confidential unit-record files. It is suggested that these measures of equalising mobility can usefully augment the visual information provided by the TIM curve concept.

Appendix: A Formal Statement of the TIM Curve

The TIM curve can be specified more formally as follows, ignoring i subscripts for convenience. Suppose incomes are described by a continuous distribution where $H(x_t)$ and $F(y_t)$ denote respectively the distribution functions of income and log-income at time t, with population size, n. For incomes ranked in ascending order, the TIM curve plots the cumulative proportional income changes, $y_t - y_{t-1}$, per capita, denoted $M_{h,t}$, against the corresponding proportion of people, h, where:

$$h = F(y_{h,t-1}) \tag{9}$$

Thus $y_{h,t-1} = F^{-1}(h)$ is log-income corresponding to the h^{th} percentile, and the TIM curve plots $M_{h,t}$, given by:

$$M_{h,t} = \int_0^{y_{h,t-1}} (y_t - y_{t-1})\, dF(y_{t-1}) \tag{10}$$

against h.

Let μ_t denote the arithmetic mean of log-income (that is, the logarithm of the geometric mean, G_t, of income, x_t. Equation (10) can be written as:

$$M_{h,t} = \int_0^{y_{h,t-1}} \left\{(y_t - \mu_t) - \left(y_{t-1} - \mu_{t-1}\right)\right\} dF(y_{t-1}) + \left(\mu_t - \mu_{t-1}\right) F(y_{h,t-1}) \tag{11}$$

The term, $y_t - \mu_t$ is equal to $\log(x_t/G_t)$. Hence $(y_t - \mu_t) - \left(y_{t-1} - \mu_{t-1}\right)$ is the proportional change in *relative* income. Thus, $M_{h,t}$ consists of the cumulative proportional change in income *relative* to the geometric mean, *plus* a component that depends only on the proportional change in geometric mean income.

Let g denote the proportional change in geometric mean, $\mu_t - \mu_{t-1}$, and suppose the proportional change in relative income depends on income in $t-1$, so that $(y_t - \mu_t) - \left(y_{t-1} - \mu_{t-1}\right)$ can be written as the function, $g^*(y_{t-1})$.[8]

[8] Here, the letter, g, represents a growth rate, contrasting with its use earlier where it represented a poverty gap.

Then (11) can be expressed as:

$$M_{h,t} = \int_0^{y_{h,t-1}} g^* (y_{t-1})\, dF (y_{t-1}) + gh \tag{12}$$

If all individuals receive exactly the same relative income change, then relative positions are unchanged and $g^* (y_{t-1}) = 0$ for all y_{t-1}. Hence, $M_{h,t}$ plotted against h is simply a straight line through the origin with a slope of g. This means that the extent to which it is systematically equalising or disequalising – in relation to inequality in the second period – over any range of the income distribution can be seen immediately by the extent to which the TIM curve deviates from a straight line, which in turn depends on the properties of $g^* (y_{t-1})$.

References

[1] Creedy, J., and Gemmell, N. (2018) Income dynamics, pro-poor mobility and poverty persistence curves. *Economic Record*, 94, 306, 316-328.

[2] Creedy, J. and Gemmell, N. (2019) Illustrating income mobility: new measures. *Oxford Economic Papers*, 71, 733-755.

[3] Creedy, J. and Gemmell, N. (2022) Illustrating income mobility and poverty persistence. *Australian Economic Review*, forthcoming.

[4] Jenkins, S.P. and Lambert, P.J. (1997) Three "I"s of poverty curves, with an analysis of UK poverty trends. *Oxford Economic Papers*, 49, 317-27.

[5] Jenkins, S.P. and Van Kerm, P. (2016) Assessing individual income growth. *Economica*, 83, 679-703.

[6] Van Kerm, P. (2009) Income mobility profiles. *Economics Letters*, 102, 93-95.

About the Authors

John Creedy is Professor of Public Finance at Wellington School of Business and Government, Victoria University of Wellington, New Zealand.
Email: john.creedy@vuw.ac.nz

Norman Gemmell is the Chair of Public Finance at Wellington School of Business and Government, Victoria University of Wellington, New Zealand.
Email: norman.gemmell@vuw.ac.nz