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Abstract

This paper reports new estimates of simple regression models of income dynamics, using a special New Zealand anonymous dataset compiled from Inland Revenue data. The pattern of relative income changes, despite being subject to considerable complexity, can be described succinctly using a simply autoregressive stochastic process in which Galtonian regression towards the mean is combined with serial correlation in the stochastic term. The parameters of the model have convenient interpretations and can be easily estimated using limited longitudinal data. There is substantial regression towards the mean combined with negative serial correlation: these imply that relatively high income individuals have, on average, lower proportional increases in income from one year to the next compared with those with lower incomes, and those with a large increase in one year are more likely to experience a decrease the following year. Both these effects, though combined with a stochastic component that on its own would tend to increase inequality over time, are sufficient to ensure that inequality falls as the accounting period over which incomes are measured increases, and there is no systematic tendency for annual inequality to rise.

*We are grateful to the NZ Inland Revenue Department for making the anonymous longitudinal data available.

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1 Introduction

This paper reports new estimates of simple regression models of income dynamics, using a special New Zealand anonymous dataset compiled from Inland Revenue data. Despite the considerable complexity of the underlying process of income change from year to year for individuals, useful summary information can be obtained using simple specifications involving variations of the Galtonian model of 'regression towards the mean', based on a small number of regression parameters. In addition to providing succinct descriptive information which can usefully supplement cross-sectional inequality measures, such models are useful when it is required to include income dynamics in larger economic models. For example, when estimating taxpayers' responses to changes in marginal tax rates, using the concept of the elasticity of taxable income (with respect to changes in the net-of-tax rate) it is useful to be able to capture non-tax related changes in incomes, so that only those changes arising from behavioural responses to tax rates can be isolated.[1] Other contexts include the construction of life-cycle similation models designed to examine, for example, superannuation schemes or tax incidence over the lifetime. In addition, descriptive information about mobility is important when examining inequality. Attitudes towards changing cross-sectional inequality may well be influenced by changes in the characteristics of relative income changes, which influence inequality when measured over longer accounting periods.

The use of tax administration data means that very little information – other than taxable income – is available for each individual. Hence the analysis cannot consider a range of demographic or other variables (such as education, occupation and location) which may be expected to be relevant. Nevertheless, it is found that a substantial proportion (over three quarters) of the variation in individuals' earnings in any year can be 'explained' by

[1] See, for example, Carey *et al.*(2015).

incomes in the preceeding two or three years. Earlier estimates of income dynamics in New Zealand also made use of Inland Revenue Data.[2] It is therefore useful to compare the present estimates with previous results.

Section 2 presents results for the basic Galtonian model – itself an extension of the famous Gibrat stochastic model – combined with first-order serial correlation, which may be described as the 'workhorse' of simple dynamic models.[3] Section 3 extends the model to examine whether a second-order process is appropriate. The effect of mobility on inequality measured over different accounting periods is considered in Section 4. Brief conclusions are in Section 5.

2 The Galton Process with First-Order Serial Correlation

This section presents the basic Galton model and reports estimates for New Zealand.

2.1 Specification

Let y_t denote an individual's income in period t, and if m_t is geometric mean income, then $z_t = \log(y_t/m_t)$ is the logarithm of the ratio of the individual's income to geometric mean (the 'relative income position'). A simple, but convenient, statistical description of the process of relative income change is

[2] The first published regression estimates for NZ were obtained by Creedy (1996). However, an earlier little known analysis, of longitudinal data obtained from the Personal Incomes Survey for eight consecutive years (1980 to 1987), is contained in Smith and Templeton (1990). They provided a number of tables of transition matrices between income quintiles. Transition matrices were also reported by Hyslop (2000) using IRD data for 1994 to 1997. Crawford (2009) used the Linked Income Supplement of the Household Labour Force Survey to examine transition matrices for hourly earnings over 1997 to 2004. The same data source was used by Ballantyne *et al.* (2003) to examine child poverty dynamics. Transition matrices, obtained using SoFIE data from 2002 to 2010, were reported by Carter *et al.* (2014).

[3] The seminal works are Gibrat (1931) and Galton (1889).

the Galtonial model:[4]

$$z_t = \beta z_{t-1} + u_t \tag{1}$$

where β reflects 'regression towards the mean' and u_t is a random term. The Gibrat process is the special case where $\beta = 1$.

The basic Galton model can be extended by adding a first-order autoregressive process to the u_t, whereby:[5]

$$u_t = \rho u_{t-1} + \varepsilon_t \tag{2}$$

and the ε_t are independently distributed. Combining the two equations gives:

$$z_t = (\beta + \rho)\, z_{t-1} - \rho\beta z_{t-2} + \varepsilon_t \tag{3}$$

Rewrite this as:

$$z_t = A z_{t-1} + B z_{t-2} + \varepsilon_t \tag{4}$$

The parameters, A and B can be estimated using income data for a constant sample of individuals over three consecutive years. However, it can be seen that β and ρ cannot be identified. Writing $\beta = -B/\rho$ and substituting into $A = \beta + \rho$ gives ρ as the root of the quadratic $B + A\rho - \rho^2 = 0$. But from empirical values, this generally has two real roots, one of which is negative and the other is positive and close to unity. If the alternative route is taken of substituting $\rho = -B/\beta$ into $A = \beta + \rho$, exactly the same quadratic is obtained in terms of β. The problem is essentially that a model which has a high degree of positive autocorrelation in the u_t and a huge degree of regression towards the mean is observationally equivalent to one in which there is a small amount of negative serial correlation in the u_t and a 'reasonable' amount of regression. The contrast suggests that it is reasonable to impose the *a priori* assumption that the positive root is interpreted as β.

[4] On the Gibrat process and resulting equilibrium distributions, see Aitchison and Brown (1954, 1957) and Brown (1976).

[5] The Galton process was used to examine early UK longitudinal data by Hart (1976a, b). Serial correlation was added by Creedy (1974). See also Creedy and Hart (1979).

Consider rewriting (3) as:

$$z_t - z_{t-1} = (\beta + \rho - \rho\beta - 1)\, z_{t-1} + \rho\beta\, (z_{t-1} - z_{t-2}) + \varepsilon_t \tag{5}$$

Hence, the proportional change in relative income from $t-1$ to t can be said to depend on (the logarithm of) the individual's relative income position in $t-1$ and the previous proportional change from $t-2$ to $t-1$. Hence, for a given previous proportional change, if $y_{t-1} < m_{t-1}$ and $\beta + \rho\,(1-\beta) < 1$ the proportional change is positive. The process is thus 'equalising' if $\beta + \rho\,(1-\beta) < 1$. In addition, if the previous income change is positive, the next proportional change is more likely to be negative, depending of course on the relative income position at $t-1$.[6]

Figure 1 shows the systematic changes implied by (5), that is, ignoring the stochastic term, ε, for the combination of $\beta = 0.9$ and $\rho = -0.15$. The profiles show the proportional change in y/m from period $t-1$ to t (measured on vertical axis), for variations in the ratio of y/m in period $t-1$, for alternative assumptions about the proportional change $\Delta z = z_{t-2} - z_{t-1}$. For ease of interpretation, it is assumed that median income remains unchanged. Hence if the previous income change is zero and income in $t-1$ is equal to the median, the proportional change in income from $t-1$ to t is zero, that is, the same as the change in the median.

2.2 Estimates for New Zealand

The data used here are IRD panel microdata compiled from individuals' tax return information and/or employer records. It includes detailed information on taxable income and its component sources (such as, wages and salaries, taxable benefits, dividends and so on). Selection into the sample is through the final two digits of an individual's IRD number. This gives a random

[6] Although the same interpretation follows for the alternative *a priori* assumption that β is negative and ρ is positive and close to one, it makes no sense in terms of the basic process described by (1).

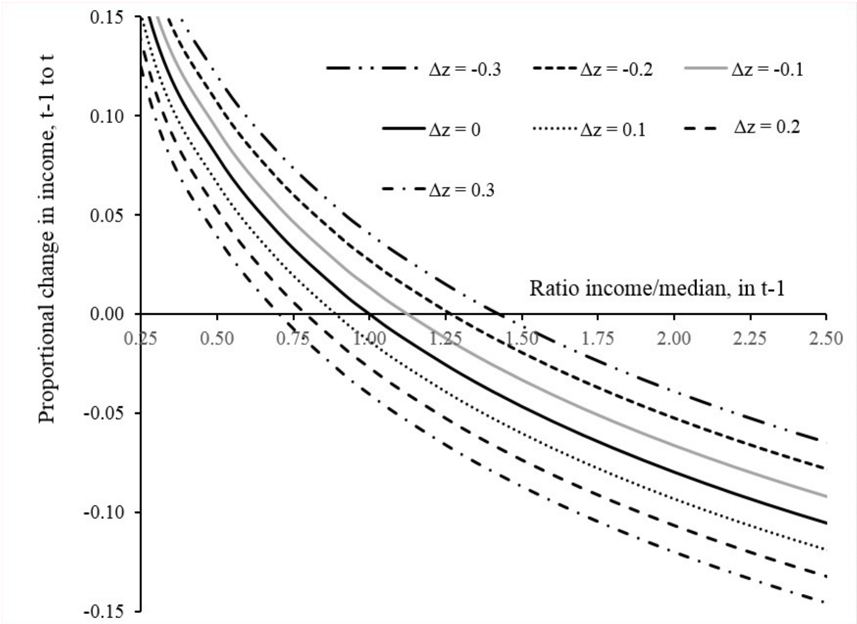

Figure 1: Proportional Income Changes Depending on Initial Income and Previous Change

sample with a sampling rate of 2%. To be included, an individual must either have PAYE earnings (which include taxable welfare benefits) or be a tax return filer. The sample does not capture individuals with zero income or people receiving only interest, dividends or PIE Portfolio Investment Entities (PIE) income that are fully taxed at source, unless they filed a tax return.

Nineteen consecutive years of data from 1994 to 2012 (year ending 31 March, the end of the tax year) have been used and the sampling method has remained unchanged for the full period. The New Zealand tax system taxes individuals on their personal (not family) income so there is no information available on family or household composition or indeed household income. Similarly, as the dataset is compiled from tax returns, no information exists on an individual's educational status, ethnicity, occupation and so on.

The IRD microdata contains large numbers of individuals who earned extremely low incomes. For example, the data include some students or

Table 1: First-Order Serial Correlation; Regressions

	Coefficient on:		t-Values		
t	z_{t-1}	z_{t-2}	(z_{t-1})	(z_{t-2})	R^2
1997	0.71745	0.17389	143.74	35.50	0.739
1998	0.71196	0.17514	148.69	36.86	0.746
1999	0.72383	0.18702	143.62	37.92	0.747
2000	0.72436	0.19016	140.77	36.59	0.735
2001	0.68618	0.16356	146.47	34.34	0.731
2002	0.73517	0.15291	162.73	35.64	0.758
2003	0.73048	0.15409	154.53	33.18	0.753
2004	0.74517	0.14780	161.58	32.93	0.756
2005	0.73519	0.16505	160.14	36.71	0.756
2006	0.74942	0.14747	161.93	32.42	0.750
2007	0.72184	0.16972	159.60	38.13	0.749
2008	0.72471	0.15334	158.20	34.37	0.735
2009	0.71772	0.15952	157.00	36.07	0.727
2010	0.71828	0.17215	160.17	39.52	0.732
2011	0.71705	0.17034	168.04	40.52	0.747
2012	0.73784	0.16637	167.97	39.07	0.752

others who engage in minimal part time work to supplement their incomes, those with small amounts of interest on savings deposits, or election day workers who work for a single day once every three years. Individuals were included in the following regressions if they recorded taxable that satisfies minimum and maximum criteria for all of the relevant years. The maximum income allowable was pegged at $500,000 in 2012. Previous years' values were then adjusted for inflation using New Zealand Consumer Price Index time series, giving a maximum criteria in 1994 of $325,300. The minimum requirement was set at one quarter of the gross rate of superannuation in each year.

Table 1 reports a series of regression results from applying ordinary least squares to equation (3), for all working age individuals, for whom information was available over the three years. For example, where $t = 1997$ (the first row of the table), this means that three consecutive years, 1995 $(t-2)$, 1996

Table 2: First-Order Serial Correlation: Parameters

t	β	ρ
1997	0.9088	-0.1913
1998	0.9054	-0.1934
1999	0.9258	-0.2020
2000	0.9290	-0.2047
2001	0.8734	-0.1873
2002	0.9043	-0.1691
2003	0.9014	-0.1709
2004	0.9079	-0.1628
2005	0.9155	-0.1803
2006	0.9112	-0.1618
2007	0.9086	-0.1868
2008	0.8959	-0.1712
2009	0.8958	-0.1781
2010	0.9079	-0.1896
2011	0.9052	-0.1882
2012	0.9189	-0.1811

$(t-1)$ and 1997 (t) were used. Table 2 converts the coefficients into the corresponding values of β and ρ. These results show a relatively stable value of β over the period, with a slight increase in the value of ρ (that is, a slight reduction in the absolute value). The results may be compared with earlier regression results for NZ obtained by Creedy (1996), who used information about taxable incomes in 1991, 1992 and 1993, for males and females in a range of age groups. Although there were slight differences among age groups, and the regression term, β, was a little lower for females compared with males, the similarities in the overall results are perhaps surprising, given the calendar time differences involved.

3 An Extension: Second-order Serial Correlation

Suppose the Galtonial process of regression towards the geometric mean, in (1), is combined with a second-order autoregressive form for the u_t, as follows:

$$u_t = \rho u_{t-1} + \gamma u_{t-2} + \varepsilon_t \tag{6}$$

Then lagging (1) and multiplying by the corresponding terms in (6) gives:

$$\rho z_{t-1} - \rho\beta z_{t-2} = \rho u_{t-1} \tag{7}$$

and:

$$\gamma z_{t-2} - \gamma\beta z_{t-3} = \gamma u_{t-2} \tag{8}$$

Hence substitution produces:

$$z_t = (\beta + \rho)\, z_{t-1} + (\gamma - \rho\beta)\, z_{t-2} - \gamma\beta z_{t-3} + \varepsilon_t \tag{9}$$

Given information about the incomes of a constant sample of individuals in four consecutive years, it is thus possible to estimate the parameters A, B and C, where:

$$z_t = A z_{t-1} + B z_{t-2} + C z_{t-3} + \varepsilon_t \tag{10}$$

with:

$$A = \beta + \rho \tag{11}$$

$$B = \gamma - \rho\beta \tag{12}$$

$$C = -\gamma\beta \tag{13}$$

As before, the question arises of whether the basic parameters, β, γ and ρ can be recovered from the estimates of A, B and C. Writing $\beta = A - \rho$ and $\gamma = -C/\left(A - \rho\right)$, substitution into the (12) gives, $-C/\left(A-\rho\right) - \rho\left(A-\rho\right) = B$, and hence the following cubic in ρ:

$$(BA + C) + \left(A^2 - B\right)\rho - 2A\rho^2 + \rho^3 = 0 \tag{14}$$

In practice, this cubic produces only one real root, which is negative. Having solved for ρ, (11) can be used to get β and (12) can be used to get γ.

The fact that only one real root is produced in practice makes it easier to solve (14), as follows. Consider the cubic:

$$a_0x^3 + a_1x^2 + a_2x + a_3 = 0 \tag{15}$$

and let:

$$\alpha = \left(a_2 - \frac{a_1^2}{3a_0}\right)/a_0 \tag{16}$$

and:

$$\varphi = \left(a_3 + \frac{2a_1^3}{27a_0^2} - \frac{a_2a_1}{3a_0}\right)/a_0 \tag{17}$$

If $d = -4\alpha^3 - 27\varphi^2$ is negative, there is only one real root. Letting $s = a_1/3a_0$ and $u = \left\{-\frac{\varphi}{2} + \left(\frac{-d}{108}\right)^{0.5}\right\}^{1/3}$ with $v = \left\{-\frac{\varphi}{2} - \left(\frac{-d}{108}\right)^{0.5}\right\}^{1/3}$, the root is given by $u + v - s$.

As with the first-order serial correlation case, it is possible to rearrange the expression in (9), so that the proportional change in relative income from $t-1$ to t is a function of the (logarithm of) relative income in $t-1$, z_{t-1}, and the two preceeding proportional changes, $z_{t-1} - z_{t-2}$, and $z_{t-2} - z_{t-3}$. Equation (9) can be rewritten as:

$$\begin{aligned} z_t - z_{t-1} &= \{\beta + (1-\beta)(\gamma+\rho) - 1\} z_{t-1} \\ &\quad + \{\beta(\gamma+\rho) - \gamma\}(z_{t-1} - z_{t-2}) \\ &\quad + \gamma\beta(z_{t-2} - z_{t-3}) + \varepsilon_t \end{aligned} \tag{18}$$

For example, consider the case above where, by assumption, income relative to the median is 0.5, and $z_{t-1} - z_{t-2} = 0.10$. For $\beta = 0.9$ and $\rho = -0.15$ it can be found that the proportional change in relative income (excluding the stochastic term) is 0.066. If, now, second-order serial correlation is introduced with other parameters unchanged and $\gamma = -0.05$, the proportional change becomes 0.077, approximately one percentage point higher than in the first-order example.

This approach, particularly as the single root of the above cubic equation turns out in practice to be a small negative number, appears at first sight to avoid the previous difficulty raised by the inability to identify the parameters β and ρ. However, it can be seen that the coefficient on z_{t-1} is unchanged if values of ρ and β are interchanged, although the coefficients on the past changes are affected. Furthermore, the process turns out to be observationally equivalent to a slight modification, in which the specification has a second-order Galton process with first-order serial correlation. Despite this complexity, it is argued that the above specification is the most appropriate interpretation: for further discussion, see the appendix.

Table 3 reports the regression results for (10).[7] The associated parameter values describing the dynamics are given in Table 4.

Table 3: Second-Order Serial Correlation: Regressions

	Coefficient on:			t-Values			
t	z_{t-1}	z_{t-2}	z_{t-3}	(z_{t-1})	(z_{t-2})	(z_{t-3})	R^2
1997	0.6804	0.1453	0.0847	125.38	22.32	16.23	0.748
1998	0.6792	0.1320	0.0943	131.02	21.00	18.75	0.755
1999	0.6973	0.1559	0.0759	127.71	24.40	14.91	0.756
2000	0.6907	0.1451	0.1015	124.22	21.24	18.58	0.745
2001	0.6537	0.1279	0.0869	129.69	20.30	16.60	0.740
2002	0.7125	0.1271	0.0662	147.02	22.66	14.33	0.772
2003	0.6961	0.1072	0.0997	135.59	17.90	22.45	0.766
2004	0.7164	0.1163	0.0797	142.09	19.18	16.75	0.768
2005	0.6959	0.1424	0.0834	138.72	23.76	18.08	0.770
2006	0.7146	0.1219	0.0809	140.25	20.04	17.10	0.762
2007	0.6871	0.1530	0.0737	138.54	25.36	15.66	0.762
2008	0.6910	0.1236	0.0851	137.15	20.68	18.07	0.748
2009	0.6809	0.1260	0.0901	135.48	20.91	19.22	0.738
2010	0.6906	0.1296	0.0907	140.20	22.00	19.90	0.743
2011	0.6852	0.1293	0.0935	148.52	23.28	21.33	0.756
2012	0.7035	0.1347	0.0849	146.63	24.16	19.47	0.760

[7] In this case, individuals were included if their income exceeded $5,000 in each year.

Table 4: Second-Order Serial Correlation: Parameter Estimates

t	β	ρ	γ
1997	0.9333	-0.2529	-0.0908
1998	0.9184	-0.2393	-0.0878
1999	0.9262	-0.2289	-0.0561
2000	0.9230	-0.2323	-0.0692
2001	0.8661	-0.2124	-0.0560
2002	0.8972	-0.1847	-0.0387
2003	0.8887	-0.1926	-0.0639
2004	0.8955	-0.1791	-0.0441
2005	0.8889	-0.1929	-0.0291
2006	0.8910	-0.1764	-0.0353
2007	0.8737	-0.1866	-0.0100
2008	0.8670	-0.1760	-0.0290
2009	0.8585	-0.1777	-0.0266
2010	0.8667	-0.1760	-0.0229
2011	0.8603	-0.1751	-0.0213
2012	0.8731	-0.1696	-0.0134

Generally, the introduction of an additional lag produces a slightly lower value of β, implying a greater degree of regression towards the (geometric) mean, and slightly higher ρ. The results from the two specifications can be compared in Figure 2. The value of β obtained from both specifications is relatively stable over the period, except for a dip in 2001. It is possible that this arose from the introduction of a higher top marginal income tax rate in that year. The announcement in advance of the tax change allowed individuals to shift some of their income between periods.

4 Inequality and the Accounting Period

An obvious implication of relative income mobility is that the inequality of the annual cross-sectional distribution is not generally equal to the inequality of income in a cohort, when income is measured over a longer accounting period. However, the complexity of the relationship between single-period,

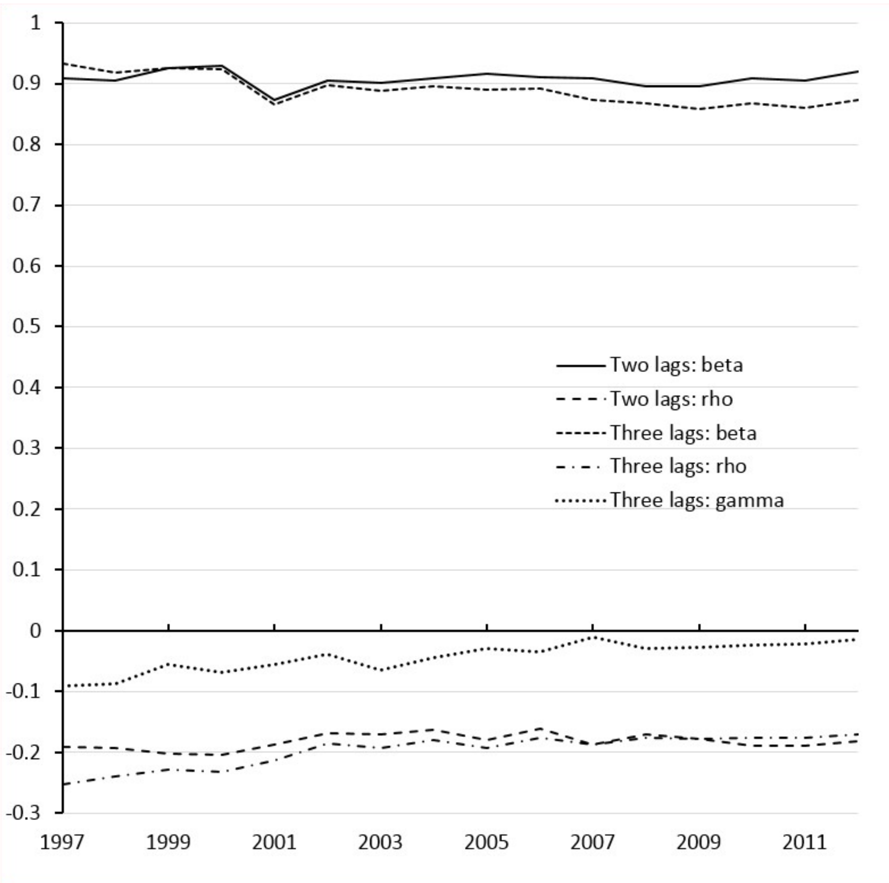

Figure 2: Mobility Estimates: First- and Second-Order Serial Correlation

cross-sectional, and longer-period income inequality means that unambiguous general statements about relative orders of magnitude of inequality measures cannot be made.[8] However, if annual inequality among a cohort of individuals is not increasing substantially, inequality tends to decrease, at a decreasing rate, as the accounting period is lengthened. The extent of the reduction depends on, among other things, the degree of Galtonian regression towards the mean and the nature of the random component of income change.

Examples of this reduction are shown in Figures 3, for Atkinson inequality measures with inequality aversion parameters, ε, of 0.2 and 0.9. This shows two starting years, 1994, and 2001, with the accounting period extended from one to eight and nineteen years respectively. Both the eight and nineteen year profiles reveal similar downward trends with the rate of decline in longer-period inequality higher for the higher degree of inequality aversion. For example, for the lower degree of aversion, inequality reaches 75 per cent of its 2001 annual value (not shown in the diagram) after six years of cumulation. However, for the higher value of aversion of $\varepsilon = 0.9$, longer-period inequality reaches 75 per cent of its 2001 value after only 3 years, and 50 per cent after twelve years.

The top panel of the figure also suggests that, starting in 1994, there is a temporary rise in inequality seven years later, in 2001. This is associated with the 2000-2001 tax reform which raised the top marginal income tax rate from 33 to 39 per cent, and is known to have led to some higher incomes being diverted from the personal tax code (for example, via use of trusts and incorporation).[9] It is noteworthy that this '2001 effect' is not evident for the higher inequality aversion profile ($\varepsilon = 0.9$) in the lower panel of Figure 3. This higher inequality aversion assumption gives a much smaller weight to higher incomes.

Figure 4 shows the equivalent profile for the Gini coefficient as the ac-

[8] On inequality and the time period, see, for example, Creedy (1985, 1997).

[9] See Carey et al. (1995) for details.

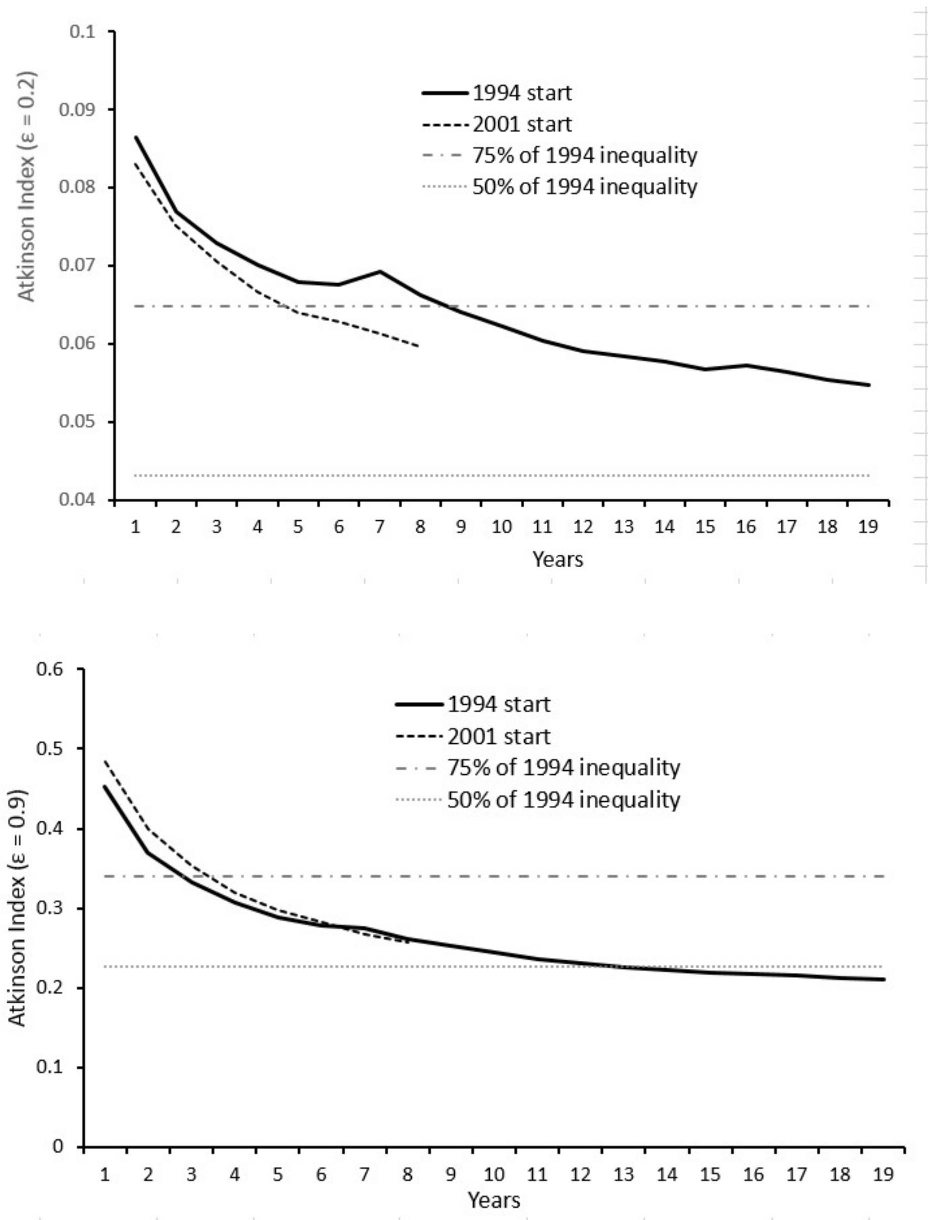

Figure 3: Atkinson Inequality Measure as the Accounting Period is Extended

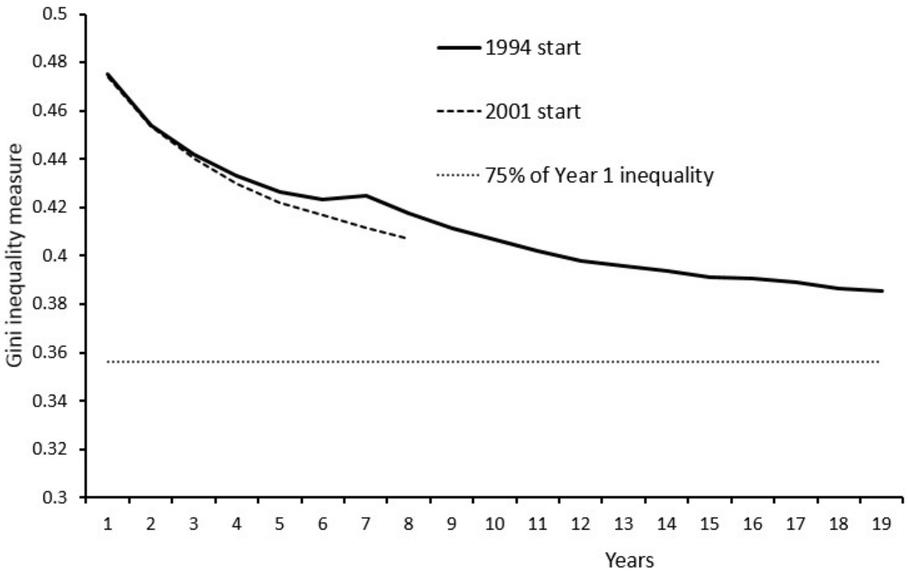

Figure 4: The Gini Coefficient as the Accounting Period is Extended

counting period is extended in the same manner as in Figure 3. While the overall decline as the accounting period is increased is somewhat less than for either Atkinson measure, otherwise the patterns are very similar: after nineteen years the Gini remains above 75 per cent of its starting value in 1994. The Gini coefficient also displays the '2001 effect' associated with the tax reform, which is not observed with a 2001 start year.

5 Conclusions

In debates on income inequality, considerable emphasis is usually placed on changes in a measure of annual incomes over time. Yet, an important characteristic of individual incomes is that they are not constant: indeed they are subject to considerable variability over time. The pattern of relative income changes, despite being subject to considerable complexity, can be described succinctly using a simply autoregressive stochastic process in which Galtonian regression towards the mean is combined with serial correlation in the stochastic term. The parameters of the model have convenient interpretations and can be easily estimated using limited longitudinal data. Using longitudinal data for New Zealand individuals, it was found that there is substantial regression towards the mean combined with negative serial correlation: these imply that relatively high income individuals have, on average, lower proportional increases in income from one year to the next compared with those with lower incomes, and those with a large increase in one year are more likely to experience a decrease the following year. Both these effects, though combined with a stochastic component that on its own would tend to increase inequality over time, are sufficient to ensure that inequality falls as the accounting period over which incomes are measured increases, and there is no systematic tendency for annual inequality to rise. Despite the simplicity of the dynamic process specified, it is nevertheless capable of explaining about 75 per cent of the variation in annual income.

Appendix: An Alternative Process

Suppose, instead of the specification in Section 3, that z is thought to be determined by a second-order process rather than the simple Galton process considered earlier:

$$z_t = \beta z_{t-1} + \gamma z_{t-2} + u_t \tag{A.1}$$

If this process is combined with the first-order form for u_t in (2), then by lagging (A.1) and multiplying by ρ:

$$\rho z_{t-1} - \rho\beta z_{t-2} - \rho\gamma z_{t-3} = \rho u_{t-1} \tag{A.2}$$

and substituting in (A.1) gives:

$$z_t = (\beta + \rho)\, z_{t-1} + (\gamma - \rho\beta)\, z_{t-2} - \rho\gamma z_{t-3} + \varepsilon_t \tag{A.3}$$

Given estimates of the coefficients A, B and C, now obtained from the regression:

$$z_t = A z_{t-1} + B z_{t-2} + C z_{t-3} + \varepsilon_t \tag{A.4}$$

the aim is again to recover values of the basic parameters. Since, now $A = \beta + \rho$, $B = \gamma - \rho\beta$ and $C = -\rho\gamma$, the only term that differs from the previous case is C. Using $\beta = A - \rho$ and $\gamma = -C/\rho$, substitution gives $-C/\rho - \rho\,(A - \rho) = B$ which can be rearranged to get the cubic in ρ, $C + B\rho + A\rho^2 - \rho^3 = 0$. Again, empirical estimates result in only a single real root of this cubic, which is positive and slightly below 1. This approach thus produces a value for β which is negative. Another route to solving the equations is to get the cubic in β, given by $(AB + C) + (A^2 - B)\,\beta - 2A\beta^2 + \beta^3 = 0$, but it produces exactly the same value of β as obtained by the previous route. The specification in the present section is thus observationally equivalent to that in the previous section, but produces individual parameters for which it is hard to give a meaningful interpretation. Therefore, the *a priori* assumption is made that the data generation process is described by the model of Section 3.

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