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Plotting Labour Force Status Shares: Interdependence and Ternary Plots

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Abstract

Data on the proportions in each of the labour force status categories sum to one and form a composition which can be displayed with a ternary diagram. However the points lie in a small region and need to be scaled or transformed if ternary diagrams are to be a useful tool. This paper uses both scaling and transformation to study labour force status. The simple graphic illustration of patterns of movement over time emphasizes the multivariate character of the data and the changing interaction of employment, unemployment and non market work. It shows the importance of heterogeneity in the population and raises many issues about the relative magnitude of different sources of variation.

JEL: E24, J64

Keywords: ternary plots, labour force status, composition models, employment, labour force plots, unemployment.

1 Introduction

Ternary graphs are a plot on an isosceles triangle showing the proportions of three parts which sum to one. They are widely used in the physical sciences and in some social science areas. By emphasizing the multivariate character of the data, and the fact it lies in a simplex, the interrelationship of the parts is clearly portrayed. The labour force status data lies in a small region near the boundary of the plot, so it is necessary to develop larger scale views of a segment of the graph. This is not commonly emphasized with ternary graphs, and an important feature of the paper is description and use of three alternative ways of presenting detailed graphs of a part of the sample space.

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Graphical presentation of the CPS data on the status variables in this way is rich in insights.

Three categories are used internationally for describing the personal relationship to the labour market. Those with a job (E), those searching for a job, referred to as the unemployed (U) and all others who we will describe as doing non-market work (N). We use non-market work as a descriptor, because the great majority of those in that group are making essential contributions to the activity in society and enjoyment of living. Compositions are a set of numbers giving the proportions of a set of parts in a total. In this paper the numbers in all three parts are converted to proportions $E + U + N = 1$. This conversion is referred to as closure of the parts, and in general is replacement of the parts by the values divided by their sum. The compositions literature focuses on providing an appropriate analysis of data subject to this constraint. In this paper, U is *always* a proportion of the population, and we do not use the conventional ratio $U/(E + U)$. This means U is treated the same as the other parts, and unemployment rates are lower than the common convention, but they have a direct and immediate interpretation as a share of the population with that status.

The labour market involves the total population, the number of jobs, and changes in both. Some problems require analysis of the totals, but for others a focus on the shares within the population give a great deal of insight to social issues and change. Elsby and others(2015) have argued that changes in participation in the labour market are important. The graphs in this paper provide evidence of the ways in which changing participation has been central to behaviour in the United States over the last forty years. Many other recent papers have explored how heterogeneity has contributed to the trends. Heterogeneity raises many problems, some of which are illustrated here using gender differences. While graphical tools cannot provide detailed quantitative analysis, good graphics can provide an overview of major features and properties of the data.

The constraint $E + U + N = 1$ makes it meaningless to talk about change in one part without describing how the change is related to the others. If employment changes there must be a change in unemployment or non market work. The bias which occurs with common bivariate plots of the parts of a constant sum has been known for over a century, but is still nearly always ignored. It was not until the work of Aitchison(1986) that the recent development of the analysis of compositions began. He stressed the way in which a composition such as the proportions in three status categories should be treated as a multivariate descriptor. The shares lie in a two dimensional space and a ternary diagram provides a means of portraying them simply. For convenience compositions are usually described using proportions, but percentage shares are also used, and treated as synonyms. Aitchison's work led to development of orthogonal coordinate systems by Egozcue and others, (EPMB)(2003) which avoid the biases when using original proportions.

Their work provides new ways of detailed analysis of the data with many categories and is used for analysis of labour force status in New Zealand in Jackson(2015) and for the USA in Jackson and Khaled(2017). Coordinates using the the log ratios of the parts and of search if you do not have a job give a two dimensional representation of the data conveying all the variance. The graphical tools in this paper assist in developing the intuition important with that framework for interpreting the data.

2 Ternary Graphs and Segment Plots

A three dimensional diagram can be used to portray the sequence of numbers (E,U,N) in a population over time. In that diagram points on a straight line through the origin are spoken of as a composition, since the ratios between the three components are constant. The ray through (E,U,N) can be associated with the closure formed by (E/P,U/P,N/P) with $P = E + U + N$. Each ray from the origin passes through the triangle which intersects each axis in R_3 at the unit point. A ternary graph is that triangle in the 3D diagram. Points must be interior to the triangle for it to be a composition of three parts with all components positive. The vertices of the triangle are limits as the magnitude of the other components tend to zero, but are not themselves a composition. The sum of the distances from a point to each of the sides is one and is the same for all points. The distances to each side are the proportions in each of the three categories.

Figure 1 shows the main features with vertices on the axes of the 3D diagram. A grid can be put on the triangle simply by ruling equally spaced lines parallel to each side of the triangle. Lines parallel to NU have equal percentages of the part E. These percentages are displayed along the side EU. Lines parallel to EN have equal percentages of part U, shown along side UN. Lines parallel to EU have equal percentages of N, shown along NE. A trace of points in the triangle displays changes in the pattern of proportions for the parts over time.

Any interior point has three coordinates. For example (60,20,20) is the highest interior point on the grid of the diagram. One of the two adjacent interior grid points is (40,20,40) which has the same share for U, but a different ratio for E and N. There are just two degrees of freedom, so any point is completely defined by two coordinates, and the third is defined by the requirement that they sum to 100.

With the grid it is easy to establish the shares for any point. The tools for analysis of compositions require that the components are strictly non-negative, but the linear transforms used with the graphics also handle values on the sides of the triangle.

Figure 1 illustrates the features we have described, and adds data points for monthly observations of the subpopulations of males and females in the

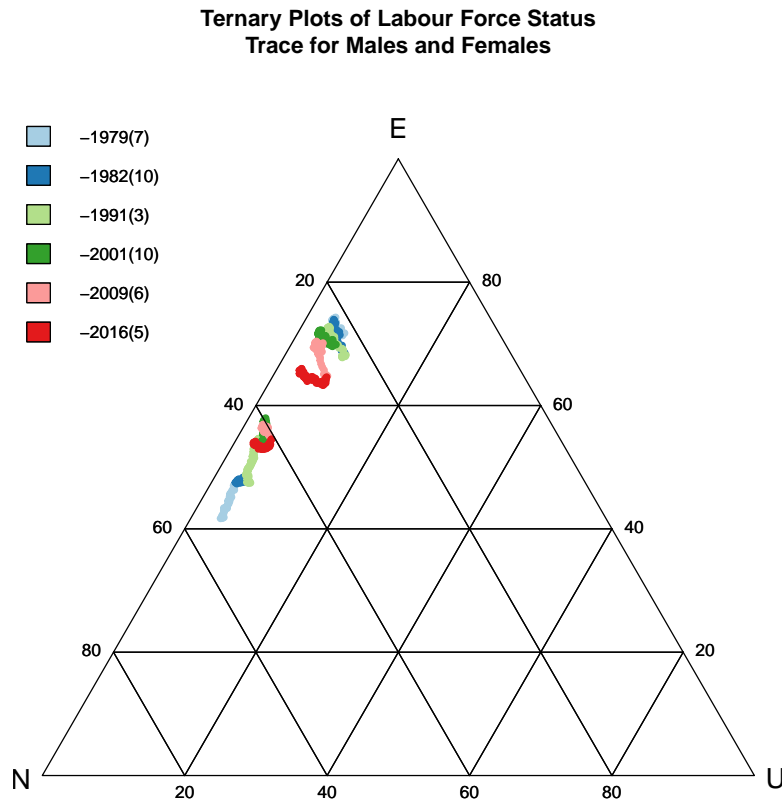


Figure 1: Ternary Diagram of Percentages in Labour Force Status Categories

US Current Population Survey (CPS) for the period from 1975(1) to 2016(5). It is clear that the changes over forty years have been within a small part of the space and there was no overlap of the upper triangle male and lower triangle female observations. The diagram is informative but its scale is too small for analysis of the patterns in the data.

Two broad approaches for dealing with the scale of the graph are discussed by von Eynatten, Pawlowsky-Glahn and Egozcue (vEPE)(2002). The first is to use a triangular segment of the plot as in the work of DeCelles and others(1998). The second which vEPE develop is to use the ratio structure of the data as emphasized by Aitchison, and the perturb operation to translate the data to the center of the triangle. If the second method is used, a third alternative using powering has been used by Boogaart and Tolasano-Delgado(2013) to vary the scale. We find that all three are useful, and combining them adds further tools. In this section we discuss and compare the data on the two gender groups separately and then in their

aggregate using segments of the diagram. In the next section we explore the ratio structure using transforms, the fourth section explores a further transform called powering and a final section reviews the tools and their application.

2.1 Scaling using a segment of the plot

Grid lines assist interpretation of the diagrams. In Figure 1 the points for each group lie in a segment of the diagram with sides indicating a 20 percent change. We can select any isosceles triangular segment to portray a subset of points. A common problem is to find the minimal triangle which contains all the points. That triangle preserves the property that the sum of all three parts is constant but restricts the observed range to an equal length interval for each part.

To find the minimal triangular segment let

$$z = (E^*, U^*, N^*) \tag{1}$$

with

$$(E^* < E_{min}, U^* < U_{min}, N^* < N_{min})$$

where a min subscript denotes the minimum of the values. We refer to z as the base of the segment. This defines a lower left vertex

$$(E^*, U^*, 1 - (E^* + U^*)) \tag{2}$$

of a region containing all the points. From the definitions let N_{max} be a bound on the maximum attainable value of N irrespective of whether it occurs. The simplex property implies $N_{max} < 1 - (E_{min} + U_{min})$, so the range k in a segment of the ternary diagram with

$$k = N_{max} - N^* = 1 - (E^* + U^* + N^*)$$

contains all the data. For notational convenience it is useful to convert the minima to integer percentages. To retain integer tic marks for a grid, k should be chosen as a multiple of one plus the number of grid lines desired. For the data points x use $x^* = \frac{x-z}{k}$ as coordinates for a ternary graph. A grid with lines corresponding to the original coordinates provides scales for the region of the space containing all the data.

Note that in the new diagram, the sum of the grid coordinates is 100, and k gives the percentage share divided between the parts within the plot. The plot retains all of the additive properties of the original complete ternary diagram.

A real advantage of this chart is that every time there are changes all of the features of the shares of the change are represented in the new point. Further it gives an immediate picture of the proportions of the population

affected by the change. Many macro economic models are based on sums, and cash flows, so a concern with the absolute changes is appropriate. However micro models have a focus on the ratios of components and their prices. This diagram shows changes in absolute terms, but as directions to the vertices of the original triangle are lost it does not portray the changes in the ratio structure of the parts studied in Section 3.

2.2 Employment and Job Search by Males

Following the procedure outlined we get Figure 2 for males. For the male data a diagram with $k = 15$ percent, and a lower bound for (E, U, N) of $(63, 2, 20)$ percent gives a new diagram.

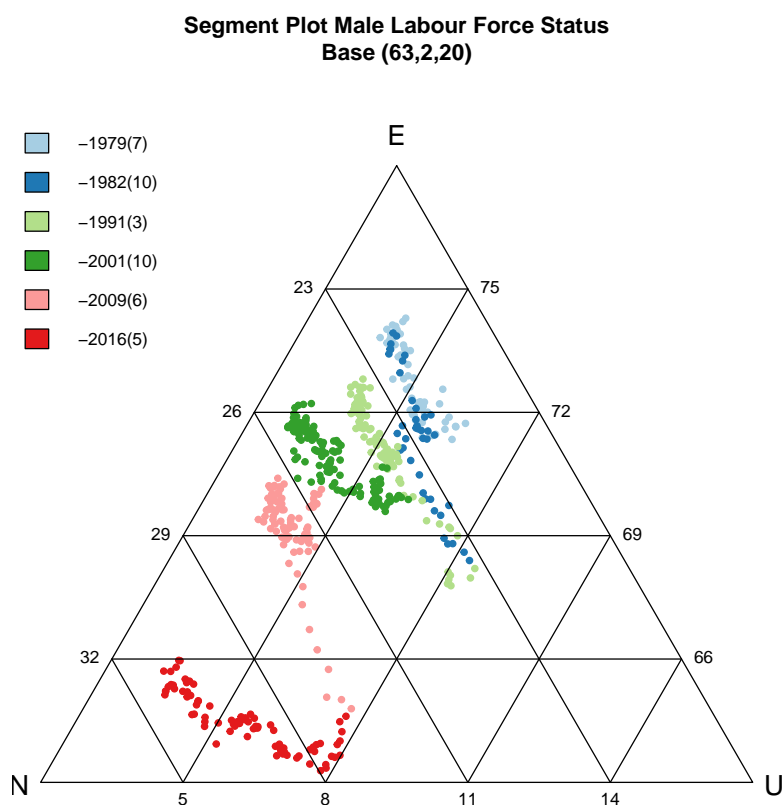


Figure 2: Segment of Ternary Diagram of Male Labour Force Status

Early analysis of this data led to using business cycle trough dates to define periods of change, and analysis using the tools in Jackson and Khaled(2017) suggested a change in behaviour prior to the trough in July 1980. The set of periods used for that paper is also used here. Colour is

used to identify points within each cycle.

For Males:

1. Treating the points as true multivariate items identifies each cycle as being in a different region of the space with little overlap. Each cycle has its own character and pattern. This is not easily observed with separate time series plots.
2. A major feature is the trend of decreasing participation in employment without a trend in searching for work over time. There has been a nearly continuous increase in the share in N as employment participation of males has fallen.
3. Employment participation varies markedly within the cycles, with a range of about six percent of the male population in the 80's and with the global financial crisis (GFC) and a smaller range in the others. In each case this displays as changes in employment and an immediate direct effect on unemployment. These changes appear to have been largely orthogonal to the trend described in the previous point.
4. The grid lines for N are lines of constant labour force participation. Movements parallel to the grid are changes in the balance between employment and unemployment, without any change in the proportion of the population involved, though they may refer to quite different individuals or subgroups at different times. In cycles 1 and 3 there was a nearly constant conventional labour force participation. Each of the other cycles showed a changing pattern of participation within the cycle. The long period of growth in cycle 4 had a decline in participation early, followed by further decline in period 5 and a very sharp decline after the GFC. In the final period after continued decline there appears to be a reversion to constant labour force participation behaviour in the most recent data, but at the lowest historical level. The concern of Elsbey et al(2015) about movement involving N appears as a shift between cycles for the first four periods, but shows much larger effects within the two periods in this century.
5. These patterns show that the main change usually occurred close to the business cycle trough. Even post GFC the main participation effects appear to have occurred over a short period though they have continued further into the recovery. The reasons for the trends have been the subject of much recent debate, with some focus on supply. These are such large changes, that demand side changes in traditionally male jobs must also be considered as a major covariate.
6. The pattern of job search relative to the level of employment has had some similarity over each of the cycles. It has been maintained across

major changes in total participation and a long term decline. The very rapid response to change within the cycles suggests we need to rethink many aspects of the analysis and interpretation of this data.

This pattern is consistent with a model of job search being related to the number who have not attained their employment objectives, and the available jobs rationed by employer hiring choices. The need to replace income lost from the reduced number of jobs is seen in higher levels of job search as the extent of rationing increases.

In cycle 5 the very sharp decline in employment was associated with a decline in labour force participation. Job search is costly in time, cash and psychological terms, and if the probability of success longer term is low it makes sense to reduce the level of search. This decline continued with the low levels of employment in the most recent cycle. The economy moved into a region of lower levels of male employment than previously experienced in our data period. It would be surprising if there was not some change in participation with a change of this magnitude. Even if you want work, if there are too few jobs search is not worth while. Models in which labour force participation is related to employment levels are commonly referred to as ‘discouraged worker effects’, but may have long term effects as households must adjust to a new employment demand situation.

2.3 Employment and Job Search by Females

Figure 1 showed that the pattern for females is in a different part of the diagram from that of males. Figure 3 does not overlap Figure 2. The diagram for females is very different from the pattern for males. Understanding the reasons behind this vast difference in behaviour is obviously important.

1. Each of the cycles was over a distinct part of the space, but with a very strong trend. Within the 21st century, experience has trended back to lower employment over previous values, and the GFC led to the levels of unemployment of females only previously observed in the 80’s. Cycle 5 was the first with substantial overlap, and the period since the GFC has generated different behaviour.
2. There was a very strong trend movement over the first four cycles, with large increases in female employment. These were in the opposite direction to the trend for males. What was driving increasing participation by females, and decreasing participation by males? Female participation in E peaked about 2000 and total participation at about the same time. Over the first four cycles, N showed nearly continuous decrease, broadly matching the increase in employment. The cycles were closer to constant unemployment, than constant labour

force with the preponderant movement in participation, not changes in the levels of search.

3. Job search appears less volatile within the cycle than for males. That is partly because the level of participation of females is lower. It barely got outside the band from two to five percent. The nearly continuous increase in participation over the first four cycles was much larger. The pattern in this data was for limited change in unemployment level and the main change a movement from non-market work to employment but the post GFC changes have led to more variation in unemployment.

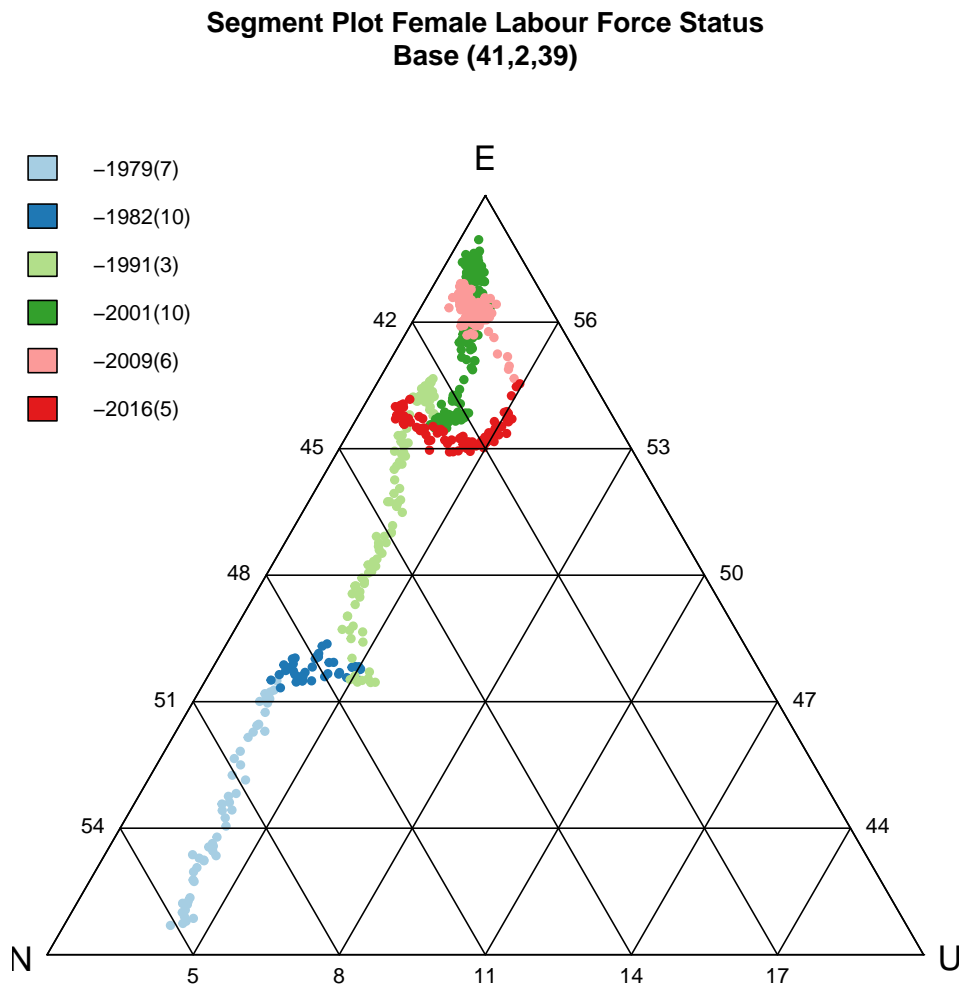


Figure 3: Ternary Diagram of Female Labour Force Status

- The last two cycles have shown similar patterns to males but with much smaller absolute changes. This is quite different from the first four cycles.

These diagrams make clear that changes in the female participation in employment have played a major part in the history portrayed by this data. It obviously involves change on both sides of the market, but it is difficult to see why it would happen unless there were advantages for employers in modifying the gender balance of their staff perhaps because of the difference in wage rates, changes in technology, attitudes towards work, changes in industry demand, or other changes in structure of demand. There has been a considerable change in some income levels over the period of the data, and that may have had effects on the participation rates in market activity.

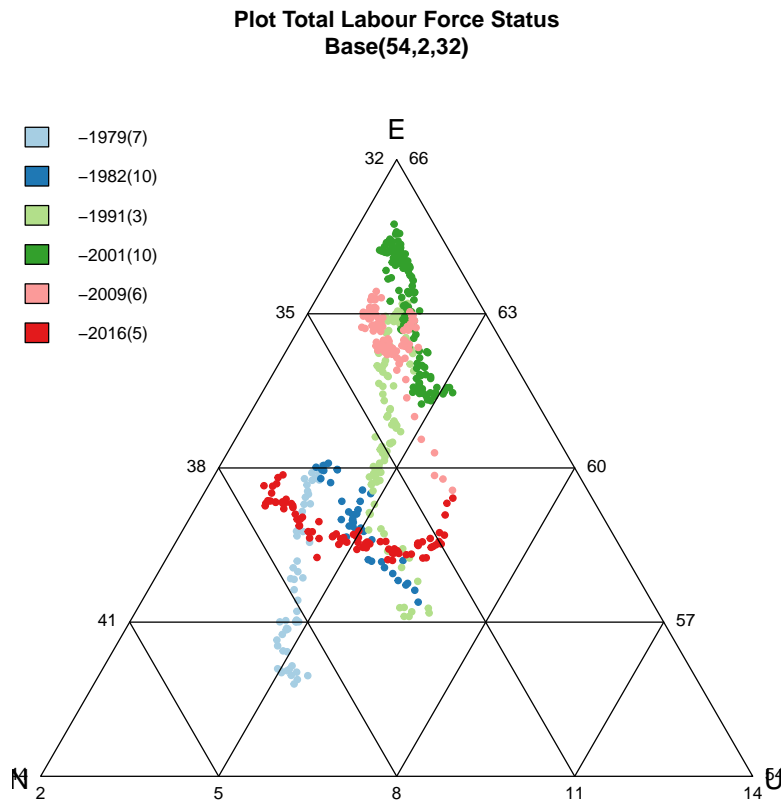


Figure 4: Ternary Diagram of Total Labour Force Status

2.4 A Total Population View

Aggregating to a total population view, gives a picture of the interaction of households with the employment market in Figure 4.

1. The different cycles overlap but show different behaviour, with a movement in the GFC back to aggregate participation and employment levels of the 80's, but with a different gender balance. Whereas there were continuing trends in the opposite direction with both males and females, Figure 4 shows that the combined effect was an increasing total participation in employment until the turn of the century and then a downward movement affecting the total, amplified by the GFC. A 4 percent move in participation is large.
2. Within the cycles the pattern is a combination of the employment pattern for males, and the participation changes for females.
3. It also shows that the first and final cycles in the data are in the same part of the ternary space, but with quite different behaviour, and at levels of employment much lower than near the turn of the century.
4. The gender patterns in the new century have been much more similar than during the first four cycles. In those cycles the male pattern showed more cyclical movement, and the females much more participation movement. The last two cycles have had experience of both males and females moving in similar directions.
5. It is essential to understand the gender detail to construct an adequate foundation for analysis of the aggregate picture.

The method of increasing the scale over a segment retains the simplex property of constant sum of the data. That is important in any analysis of totals and income and other effects dependent on them. However the diagrams show the importance of studying this market using a multivariate descriptor, at least incorporating the six categories generated by gender differences.

3 Centering Data

The closure converting the data to proportions of the whole, puts the focus on the ratios of the parts. To focus on the ratios of the parts as in many economic allocation models, we need additional descriptors. For ratios of the parts the mean is the geometric mean and in this section references to the mean always refer to the geometric mean. Aitchison showed that compositions form a linear vector space, with two operations. In the usual real space, the operations are addition and multiplication. In the simplex

space of a composition, one is called perturbation, and the other powering. In this section we explore perturbation, and in the next section powering.

To convert data to deviations from its mean we need to convert the data to ratios to the mean. If we multiply the coordinates (x_1, x_2, x_3) by scale factors (y_1, y_2, y_3) with $y_i > 0$ and form the closure $C(x_1y_1, x_2y_2, x_3y_3)$ any point in the triangle is translated to a new position. This is called perturbation of x by y . The center of the triangle is at

$$y = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

which is called the neutral point. Perturbing all points by the neutral point leaves them all unchanged.

To place the geometric mean of the columns of the data (g_1, g_2, g_3) at the center of the triangle perturb the data by the vector

$$y = \left(\frac{1}{g_1}, \frac{1}{g_2}, \frac{1}{g_3}\right)$$

This expresses every point as ratios to the mean value for each part. By perturbing the data in this way, the points are displayed as deviations from their geometric mean.

In the segment plots Figures 2, 3 and 4 a small difference can hide a very large change in the ratios of the parts. In the graph perturbed to center on the neutral point, equal distances correspond to equal absolute changes in the ratios of the parts. Just as with logarithms, small changes near one are approximated by nearly linear behaviour. This has a striking effect, and in the graphs in this section it is clear that there is a nearly linear relationship between the ratio changes of all three of the parts. Aitchison constructed a measure of distance for ratio data in a simplex, and the perturbation operation translates the data in the space, but does not change the Aitchison distance between the points. vEPE showed that perturbation is a linear operation, moving any line in the triangle to a new line. The grid of the perturbed data is a transform of the grid of the original data. vEPE used the grid to aid interpretation of the data. To show its effect we illustrate Figure 5 which provides a transformed grid for the grid lines in Figure 1 with the mean across all the data at the neutral point. Among the nine triangles to the left of the figure, the points for males lie in the third from the top, and females in the fifth.

In plots in this section, three transformed grid lines through the mean are shown on the diagram. In Figure 6 the red line shows points with the employment at the mean. At its right end is the employment participation at the mean, 69.9 percent. The blue line shows points with the mean unemployment, with the mean of 4.7 on the unemployment scale. The green line is points with mean participation in non-market work, 25.3 shown at its left margin on the non market work scale. The fact that the red and green

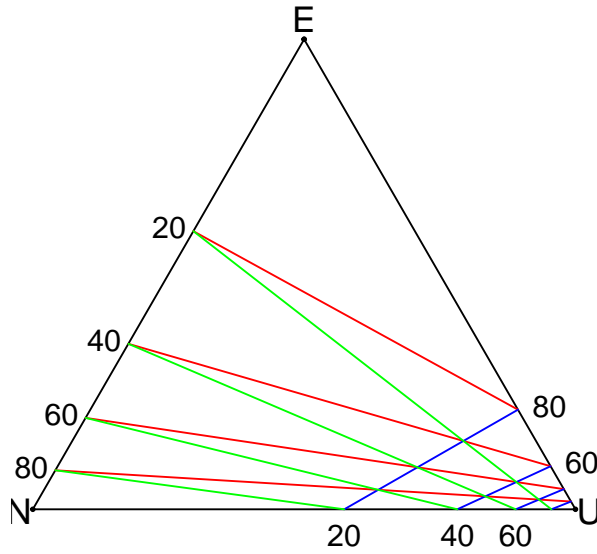


Figure 5: Scaled Grid from Figure 1

lines are close to each other, results from the property that unemployment is generally a small part relative to the other two parts. It indicates that when employment changes, there are related changes in the ratio of employment with unemployment, and the ratio of non-market work with unemployment.

3.1 Centered Graphs for Males

The new values are illustrated in Figure 6 and for this data are all close to the centre of the triangle. Again we have a problem with the scale of the diagram. There are two ways of scaling the data about the mean. One is to use a centralised segment of the transformed ternary plot. This preserves all the information near the centre of the data. A segment about the mean is shown on Figure 6 and used to display the data in Figure 7.

Figure 2 and Figure 7 are generated from the same data and can be translated from one to the other and back, so they contain the same information but with a different emphasis. Note how the line for constant unemployment is almost orthogonal to the lines of constant shares in employment and non market work.

1. The cycles are still clearly distinguishable from each other. They appear as shifts in the mean from the overall mean at the centre of the triangle.
2. Within each cycle there is a strong nearly linear relationship between the points. The large changes in U are associated with changes in both the other variables.

3. The changes within each cycle which are perhaps orthogonal to between cycle trends, but generate a large within cycle pattern. The ordering of decreasing mean employment is preserved.
4. The significance of changes in unemployment is highlighted, with much larger relative changes than in the other variables. That is exactly what you would expect but the graph emphasizes it.
5. These diagrams are drawn separately for each cycle in the appendix. They show that cycles one through four broadly follow the green line of constant labour force participation, but after the GFC, N rose and in the last cycle the points have displayed both changes in employment and in the labour force.

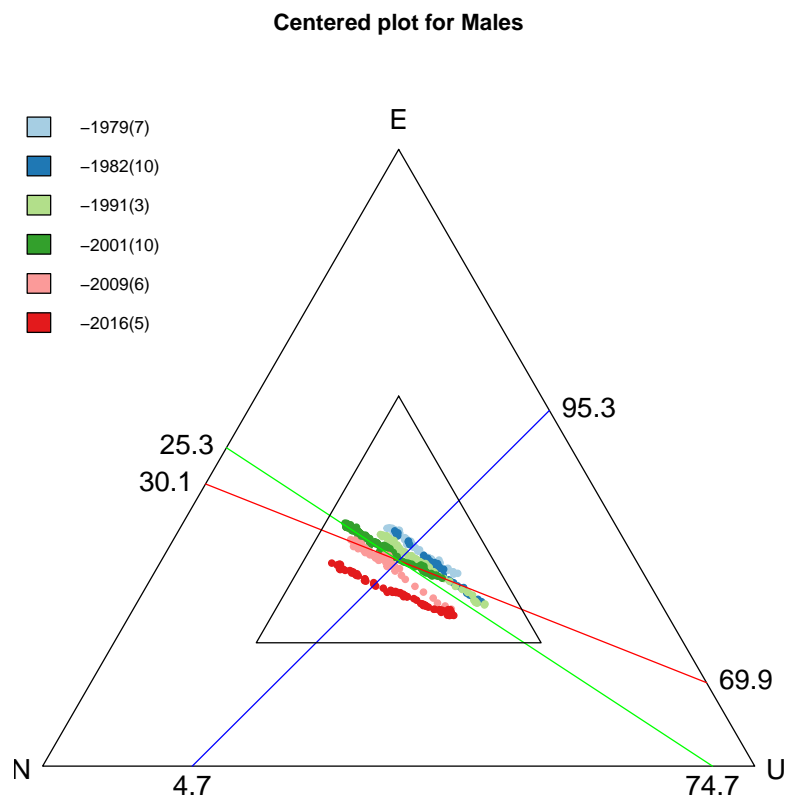


Figure 6: Centered Ternary Diagram of Male Labour Force Status Vectors

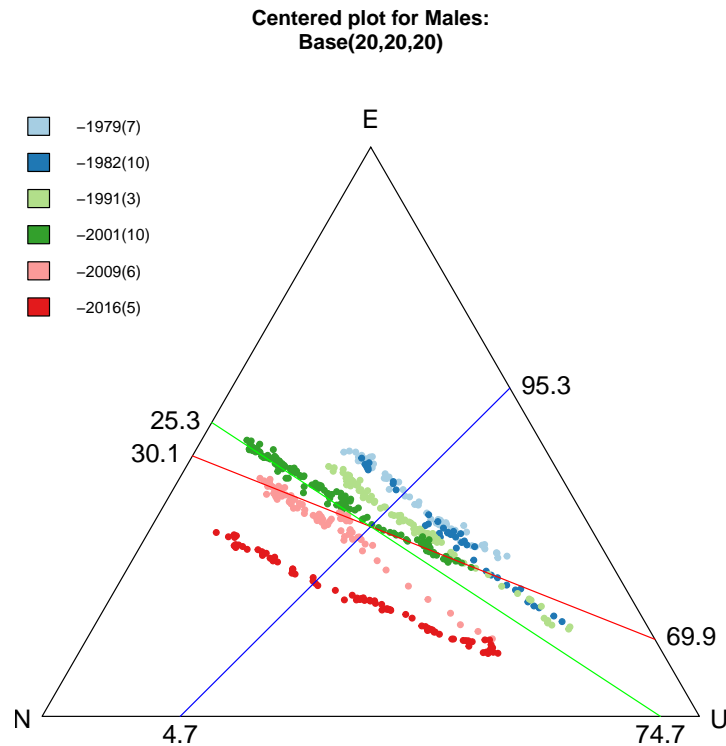


Figure 7: Centered Ternary Diagram of Male Labour Force Status Vectors

3.2 Centered Graphs for Females

Many of the features outlined for males carry over to Figure 8, but the differences are even more significant. Figure 8 displays the large difference between gender patterns discussed in Section 2, but from a different perspective.

1. The cycles are clearly distinguished, and the differences in mean stand out in the same way, but the changes in participation which lead to those differences are shown especially in periods 1 and 3.
2. Variation within the other cycles again appears orthogonal to the shifting employment trend.
3. The large relative variation in unemployment is still a feature.
4. The pattern of a nearly linear relationship between U and the other variables is maintained, even in the cycles 1 and 3 when the gradient is different.

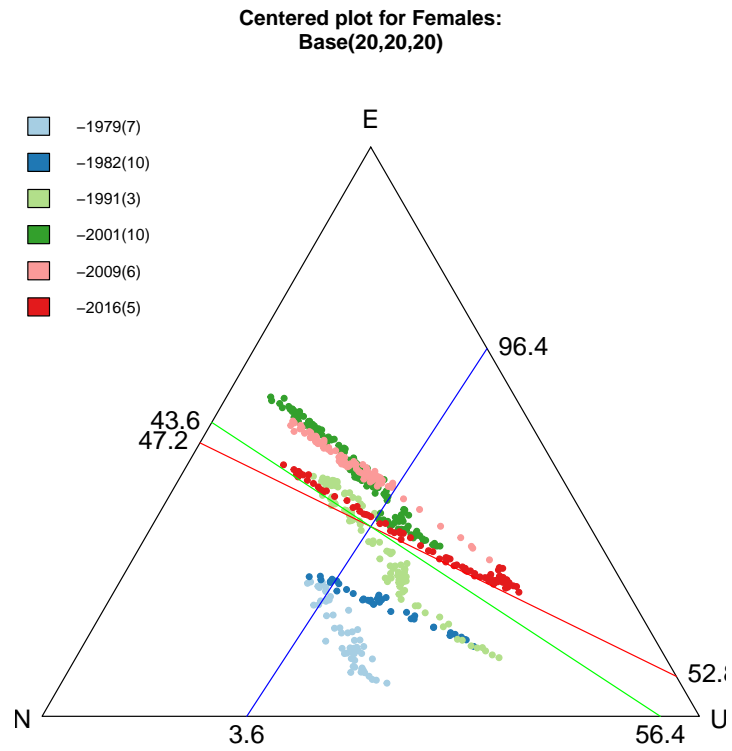


Figure 8: Centered Ternary Diagram of Female Labour Force Status Vectors

5. Appendix diagrams give a clearer indication of changes in the relative balance between unemployment and non market work in each period.

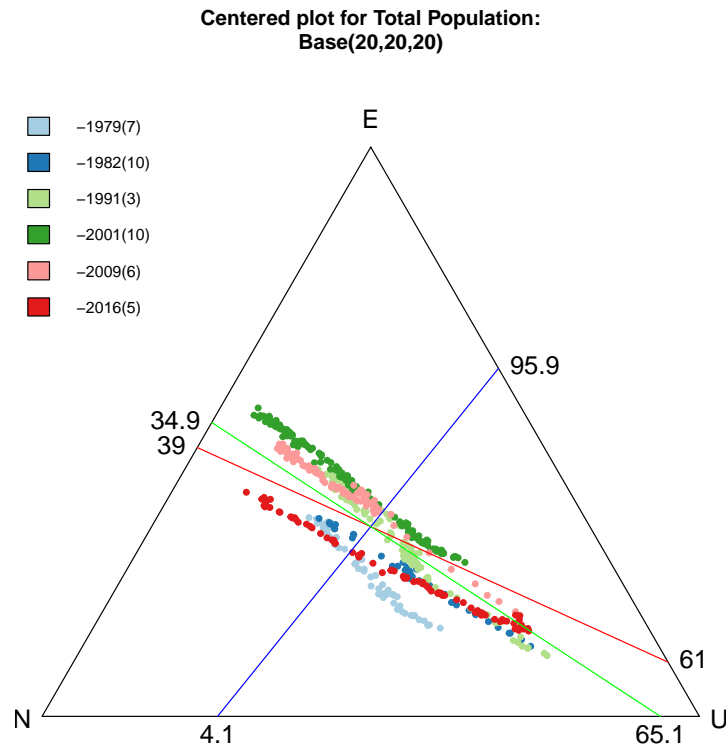


Figure 9: Centered Ternary Diagram of Total Labour Force Status Vectors

3.3 Centered Graph for The Total Population

It is common practise to look at the total data. This diagram shows how the aggregate has performed. There are smaller differences between the cycles. The first and third cycles show the effects of the rapid change in gender participation, and the final cycle the post GFC decline in participation.

When the parts are aggregated to ratios for the total, it is clear that the large changes in unemployment are taking place in an environment where there are limited relative changes in employment and non market work. Changes in unemployment are taking place within the same period as the changes in employment and non market work because of the simplex constraint.

There have been smaller relative changes in aggregated data than in either of the male or female parts. It is very clear that within each cycle there are nearly linear changes in the ratios of all the parts. The patterns would be consistent with an exogenously changing employment level impacting on households with a relatively stable intermediate term desired participation in both employment and non market work.

4 Powering to spread the data

In Section 3 we used perturbing the data about the neutral point to change the location and segment plots to generate spread. Within the linear vector space of points x in the simplex, the set of points $C(x_1^a, x_2^a, x_3^a)$ for $a \in \mathbb{R}$ generates a line through the neutral point. The vector x is called the leading direction. For any point y , a line through y is associated with a vector x specifying a leading direction by forming $C(y_1x_1^a, y_2x_2^a, y_3x_3^a)$. It is referred to as a line since it is generated by repeated application of a current direction to change from the current position. If those points are perturbed back to center on the mean of the data, it generates a line in the original units representing the behaviour within the cycle.

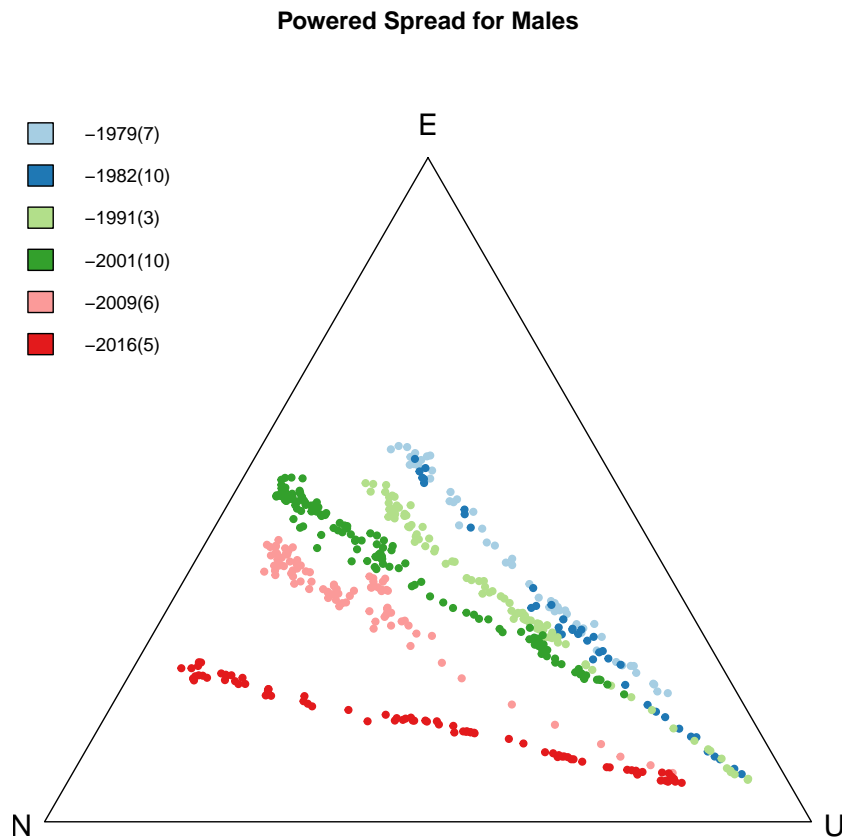


Figure 10: Centered Powered Diagram of Male LFS

Each point is associated with a unique direction from the mean, and raising it to a power of a increases the distance moved from the mean. The larger the absolute value of the power the further the points will be from

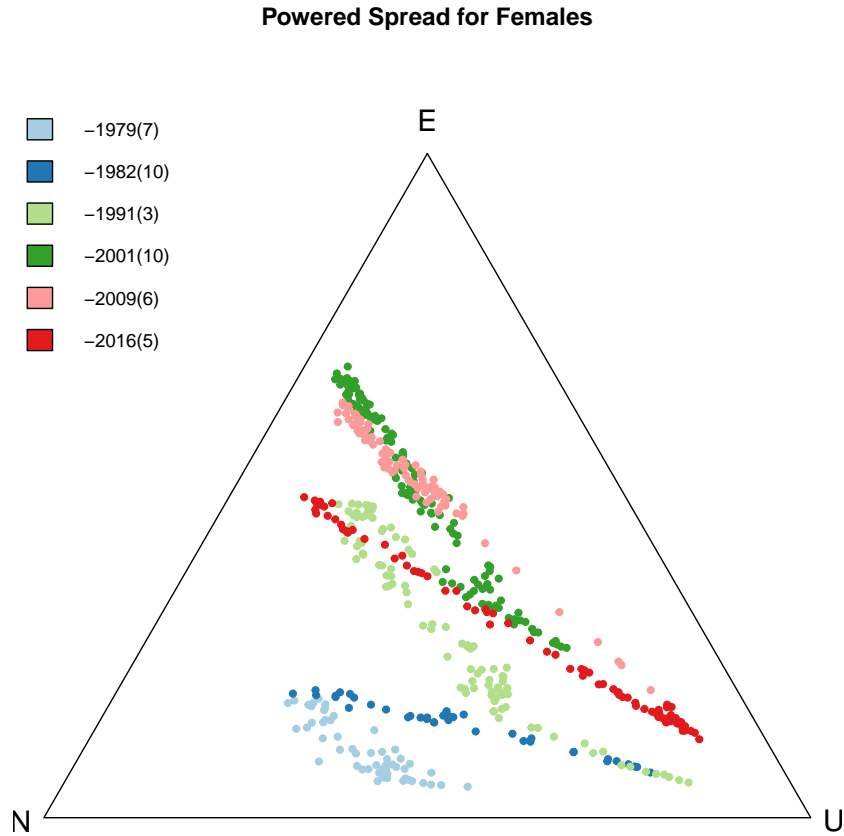


Figure 11: Centered Powered Diagram of Female LFS

the mean. For a set of points powering changes the variance. The powering operation includes use of closure after powering, so at large powers every point moves towards the vertices associated with the largest and smallest shares in the leading direction. Powering is not a linear operation and creates curved lines in the ternary diagram of the original proportions but for points close to the neutral point it remains nearly linear. However it does give a better picture of longer term effects of continuation of a current pattern of change.

In Section 3 the points were still concentrated near the center of the triangle. It is common to standardise a variable by moving to the mean and dividing by the standard deviation. To standardise this data we use the square root of the Aitchison variance. The Aitchison variance is itself an important characteristic of a group of points, but having a method of comparing behaviour independently of the scale of that variance is also useful. To construct a diagram at unit variance raise all components to the

power of the inverse of the square root of the Aitchison variance. This increases the ratios associated with points further away relatively more and may introduce curvature.

The figures in the previous section illustrate that the ratio changes in unemployment are larger than for the other parts, and that as the level of any one of the parts changes there are consistent patterns of change in ratios relative to the other parts. It is striking to observe the way in which within each business cycle there has been a nearly linear pattern in the ratios between unemployment, employment and non market work.

Using a small segment of the plot as in Section 2 gives a useful increase in scale but does not alter the relative position of the points. The log ratios of all points are changed by a constant. Powering has the effect of increasing the distance between points that are further away from the mean. Already large effects are amplified and it spreads the periphery but the closure operation compresses parts of the space further from the mean. The space of a central segment is mapped on the whole triangle.

Figure 10 gives a plot of the data for males. Unlike Figure 7 which is a segment of the ternary diagram, Figure 10 has expanded the data in a manner which translates all points of the original diagram to a point within the triangle, and has spread the points towards the margins. The main features of the diagram are similar to those of Figure 7. For each cycle, the mean of the points in each cycle is distinct from the overall mean, separating the patterns, and there is a nearly linear relationship between the changes in the ratios of the parts.

Powering the data generates curvature in the representation of a grid on the original diagram, and compresses the regions furthest from the neutral point $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. The curvature is more evident in Figure 11 for females. The large change in employment participation gives these curves a quite different aspect to Figure 8. The last two cycles provide another picture of the change in behaviour of this period with a changed balance between job search and non market work.

5 Discussion and Conclusions

The compositions literature has emphasized the need to consider the data as essentially multivariate. Two approaches to the analysis have been developed, the first exploring patterns using the simplex and its geometry, and the second converting the data to orthogonal coordinate systems of one dimension lower than the coordinate vector for the simplex. This paper has focused on staying within the simplex and using it to emphasize the multivariate character of the data. The second approach provides a wide range of additional tools which can assist exploration of the relationships within the data and is used in Jackson and Khaled(2017) . Links between the two

approaches and research suggestions are outlined in this section.

In Section 2 we used segment plots to show many of the main patterns of change. The trend in participation in employment and the rapid shorter term fluctuations in unemployment in a direction different from the trend show the links between the parts. There can only be two independent coordinates, and the plots suggest ways we may wish to develop concepts to describe them. The concept of a 'labour force' gives a useful summary for males with movement along constant N within several of the cycles but in others and between cycles the whole relationship appears to move. For females, until the turn of the century the 'labour force' showed constant change. In spite of large changes in participation in the labour force, the range of values of unemployment appears to be quite constrained. The segment plots show different patterns of change in subgroups within the population, but aggregating the results to the population requires weighted sums and suggests aggregate employment may affect each of the parts.

The centered plots in Section 3 show a striking linearity in the patterns within each cycle. Focusing on the ratios of the parts shows unemployment varying rapidly relative to the other parts. The concept of perturbation provides a way of describing the relative change between two points. As in Section 2 there are important differences between the cycles. The Appendix diagrams centered about the cycle mean values show that in each cycle variation is about that mean, but there are differences in the leading direction of change. The location of points in the narrow region between the lines for constant employment participation and constant non market participation demonstrate that factors driving change impact primarily on the relative share of the unemployed.

There has been a large literature about the 'ins and outs' of unemployment. It is meaningless to talk about changes in unemployment, without talking about the associated changes in employment and non market work. Causality can be in any direction. Shimer(2012) states that 'Ninety percent in the fluctuations in unemployment since 1987 were a consequence of movements in the job finding probability' and he contrasts this with the finding of Blanchard and Diamond(1990) and Davis and Haltiwanger(1990,1992) that recessions are 'characterised by a high exit rate from employment'. Both can be occurring simultaneously. The diagrams of Section 3 of this paper show that within each cycle over the last forty years there has been a nearly linear relationship in the ratios between employment, unemployment and non-market work. It applies over both the growth and decline of the proportion employed. The 'ins' and 'outs' both respond to changes in the level of employment. The diagrams have shown that this can occur in the presence of other factors shifting the overall participation in search for market work and under some circumstances those changes can dominate the outcome.

The very short period between the peak and the trough for most cycles,

generates the ‘high exit rate from employment’, making it a period when employment is contracting rapidly and unemployment takes up a major part of the change, while some move to non-market work. However there is also change with some in non-market work finding their best option is to start searching for work, and perhaps find a job. Within the cycle the time of minimal employment appears to be the time when change in aggregate behavioural patterns occurs.

The general pattern in this data is that search only increases as the level of employment falls, and decreases as the level of employment rises. Search represents the failure to achieve personal goals and its level reflects the rationing informally executed by employer decisions. The large swings during each business cycle represent a response to the changing level of activity of employers, in a situation where socially accepted wage levels for a range of tasks are broadly used to guide personal decisions about search, and the overall balance of the desire for market work and balance with non-market work changes more slowly. These diagrams illustrate the need for a wider consideration of adjustment mechanisms in the labour market, as emphasized by Solow(1990).

The changes in choice of non-market work would be expected to be influenced by the likelihood of success of search, as well as many other factors. Figures 7 and 8 show a remarkable similarity in the pattern of change for over 40 years of business cycle experience. The Appendix diagrams illustrate that for males, the pattern about a local mean is remarkably similar in each cycle. We can look at these points from the perspective of each of the parts. The pattern shows the main relative variance is in U and it moves in a consistent way relative to the variation of E and N . The transforms have shifted the grid on the triangle so that movement in the diagram is associated with much less change in the ratios between E and N when U changes. When employment changes the points have tended to follow a stable labour force pattern, but that is largely because of the declining male share in employment. One impact of loss of work is that households may seek alternative ways of recovering their market income and additional persons currently without a job may also begin searching for work. The current options for the household change as one of those searching finds a job. As the proportion with jobs increases, some come from those searching for work, and there is less need for those doing non market work to seek work. Changes in these ratios is consistent with the work of Elsbj, Hobijn and Sabin(2015) and contrasts with the concept of a relatively fixed labour force which plays a part in much of the conventional wisdom.

As the share of the population with a job increases, those in N , still doing non market work must have a higher relative advantage contributing to lower levels of search. In these diagrams the number searching for work maxes out at about eight percent for males, and about five for females. Beyond those levels we do not know the extent to which individuals may be

prepared to enter the labour force.

As unemployment changes, the change is split in at most a slowly varying way between E and N . This shows immediately as a sharp change in U/N and U/E . While unemployment shows the greatest variance of the three components, unemployment and its associated search behaviour are not driving the system. As employment rises the proportion unemployed falls and the proportion seeking work also falls. That is what we would expect if employment is determined outside this market and what is being observed is a response to the continuing process of change, with personal decisions aimed at maintaining the social position of the household and its members given the rationed number of jobs.

Using these diagrams is just one approach to applying composition analysis tools to the labour market status data. The second is to develop orthogonal coordinate systems and explore their structure and response to changing exogeneous variables. Jackson and Khaled(2017) consider the data using those tools and the broader picture arising from the ‘gross flow’ data which gives more detail about the changes in peoples decisions. That analysis reinforces the view that a model in which employment is determined outside this market and that the observed status data is a response. The patterns observed raise many issues for interpretation of micro data and the difficulty of measuring features of labour supply.

Many aspects of these diagrams are like the observed behaviour in the old longshoreing industry but writ at a macro level. The number of jobs depended on when the ships came in, what exactly was on board to unload and what was to be loaded. The range of skills required varied. At the daily muster there were a range of rules and expectations about who got the jobs. Numbers went home on most days as there were not enough jobs. If there was not much work around, many of those available just didn’t turn up.

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Appendix

5.1 Scaled Plots for Each Cycle

To increase clarity, plots for each of the six cycle periods are provided. These diagrams provide detail on males and females for each individual cycle period. By graphing the data about the mean for that cycle we observe the similarities between cycles, and obtain a clearer view of the deviations from a labour force or participation margin.

The mean of the data for each cycle can be read from the scales for each of the vertices of the triangles. E the value on the E scale closest to vertex U, U the value on the U scale closest to vertex N, and N the value on the N scale closest to vertex E.

Males 1975(1)–1979(7)

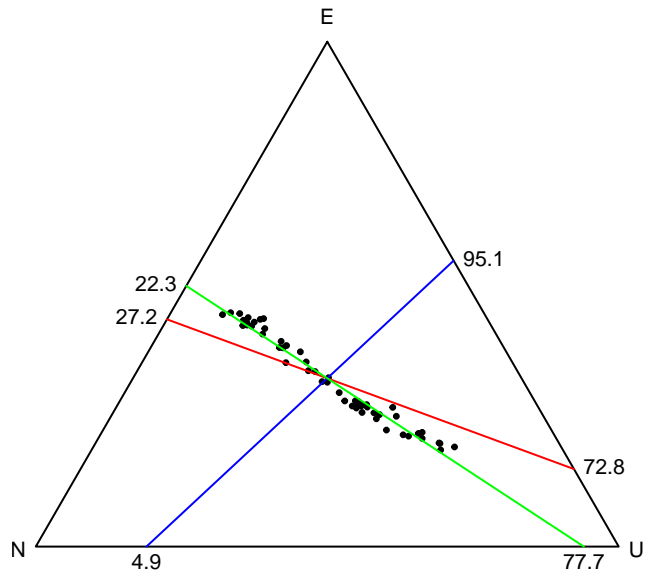


Figure 12: Scaled Plot Males Cycle 1

Males 1978(8)–1982(10)

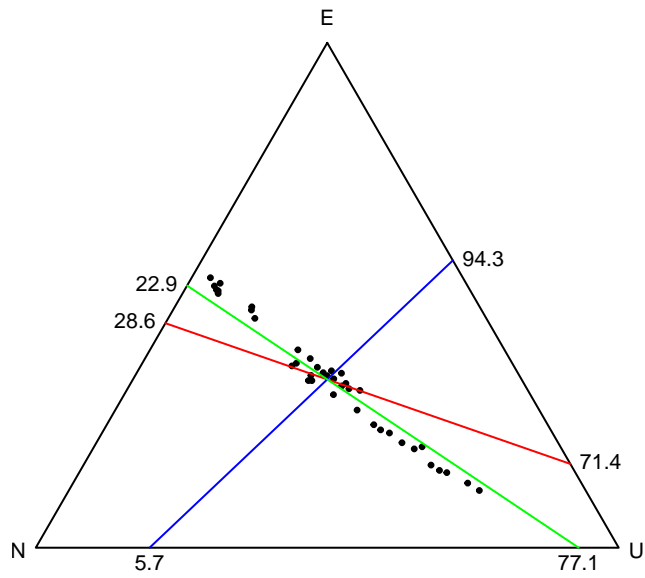


Figure 13: Scaled Plot Males Cycle 2

Males 1982(11)–1991(3)

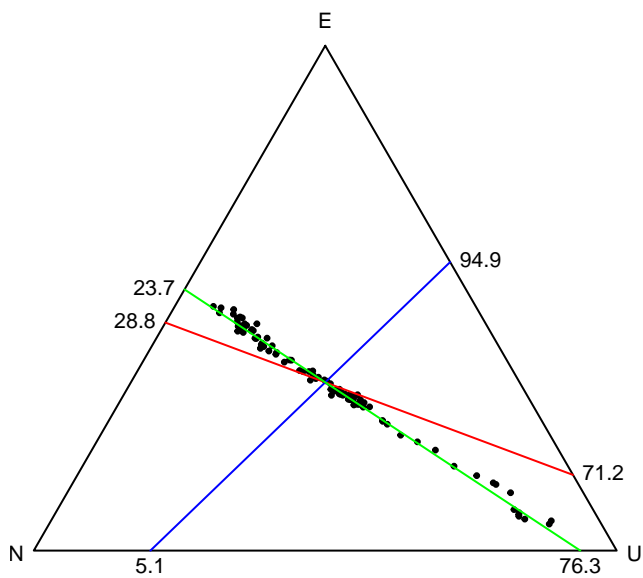


Figure 14: Scaled Plot Males Cycle 3

Males 1991(4)–2001(10)

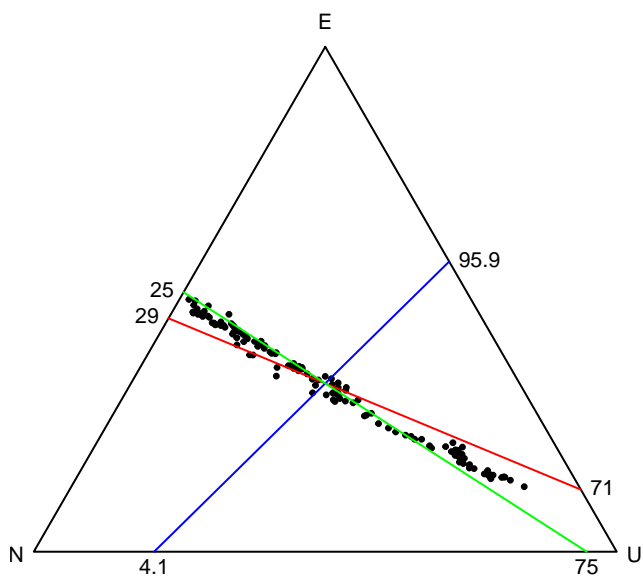


Figure 15: Scaled Plot Males Cycle 4

Males 2001(11)–2009(6)
Min 23 percent

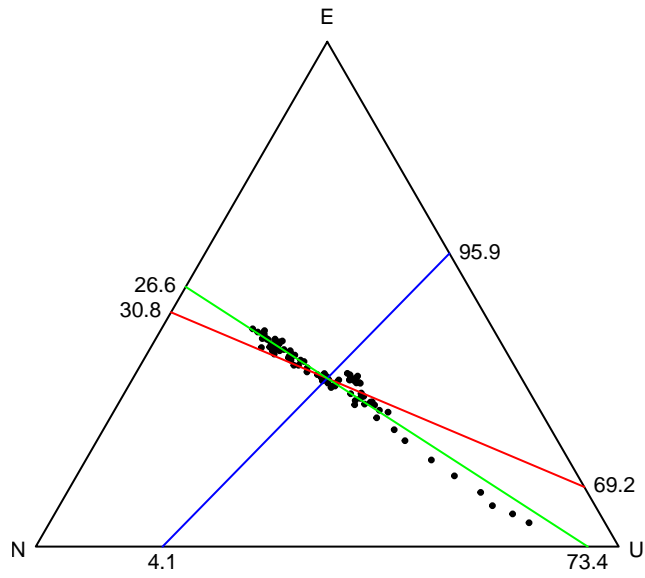


Figure 16: Scaled Plot Males Cycle 5

Males 2009(6)–2016(5)
Min 22 percent

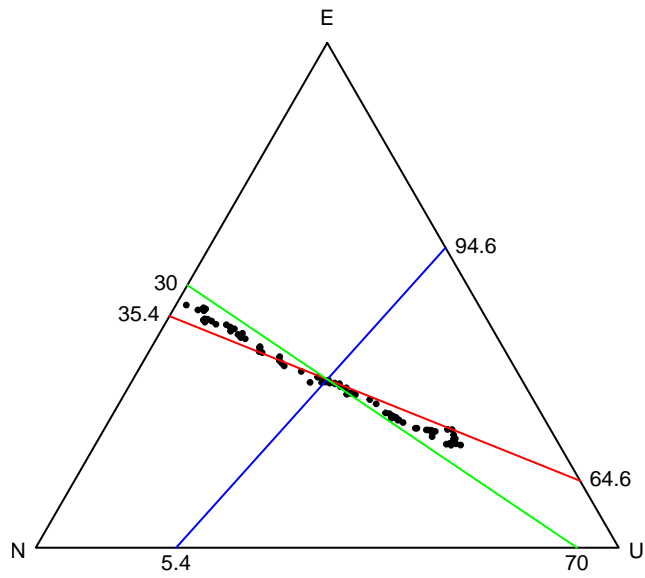


Figure 17: Scaled Plot Males Cycle 6

Females 1975(1)–1979(7)

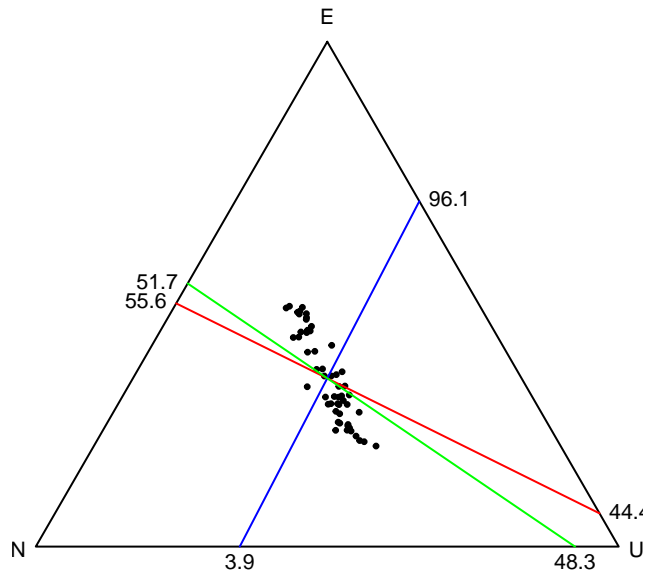


Figure 18: Scaled Plot Females Cycle 1

Females 1978(8)–1982(10)

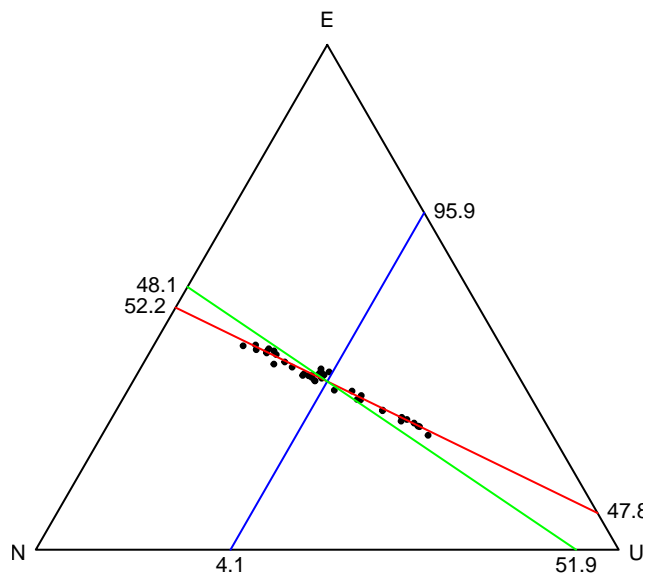


Figure 19: Scaled Plot Females Cycle 2

Females 1982(11)–1991(3)

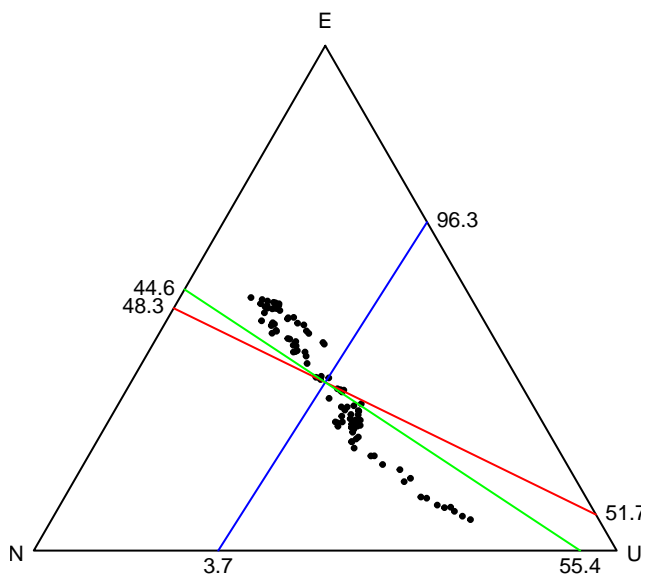


Figure 20: Scaled Plot Females Cycle 3

Females 1991(4)–2001(10)

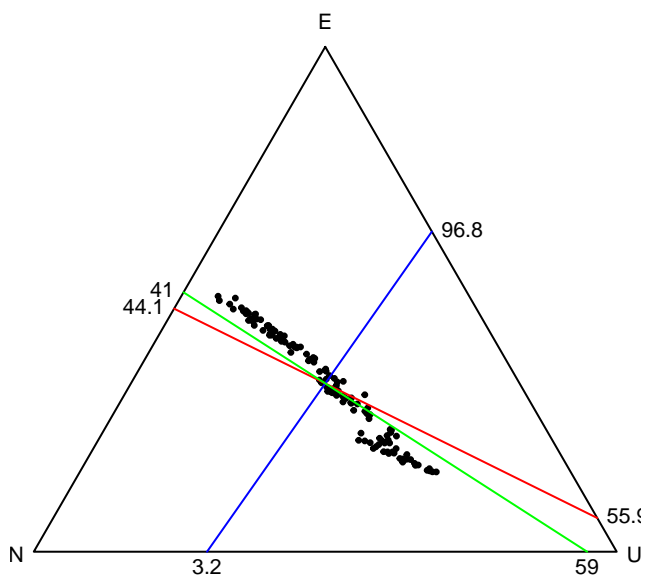


Figure 21: Scaled Plot Females Cycle 4

Females 2001(11)–2009(6)

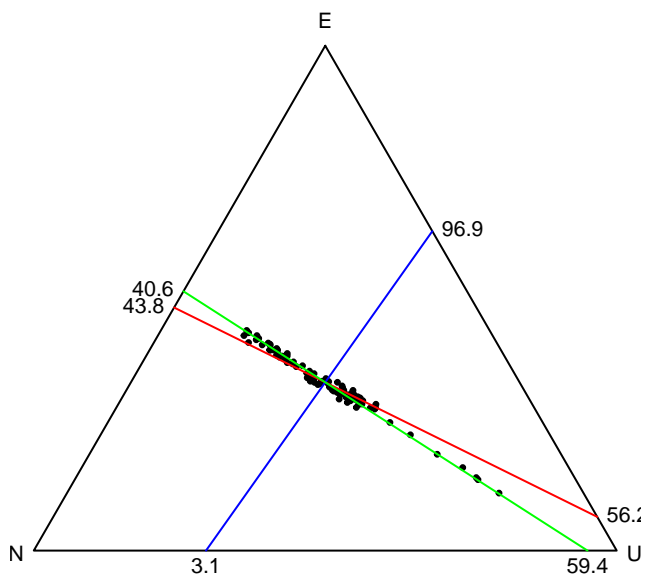


Figure 22: Scaled Plot Females Cycle 5

Females 2009(6)–2016(5)

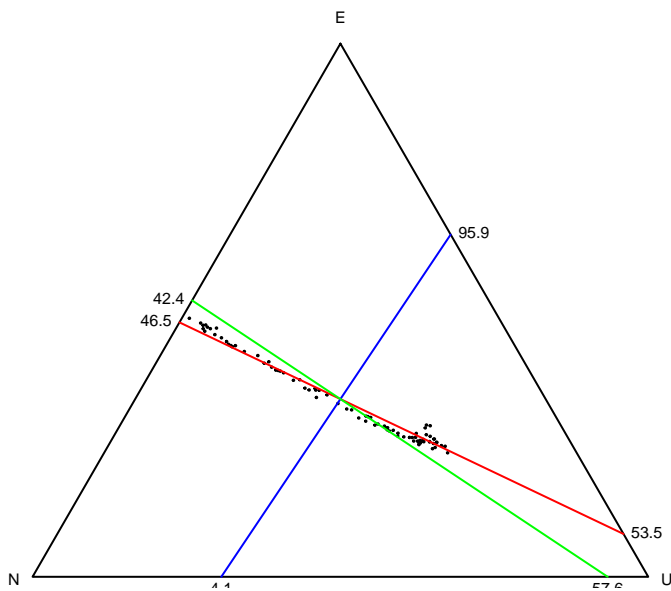


Figure 23: Scaled Plot Females Cycle 6



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