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# Income Dynamics, Pro-Poor Mobility and Poverty Persistence Curves\*

John Creedy and Norman Gemmell†

## Abstract

This paper explores poverty income dynamics in the form of income mobility by the poor and poverty persistence, making use of simple diagrams. It seeks to illustrate (a) the extent to which income mobility is pro-poor; and (b) when mobility is associated with persistence below, or movement across, a poverty line over a specified time period. While statistical measures can be used to examine detailed characteristics of income dynamics, two simple diagrams are shown to capture the extent of pro-poor mobility and poverty persistence respectively in ways that allow convenient comparisons. These are referred to as a ‘three I’s of mobility’ (or TIM) curve, and a ‘poverty persistence curve’. The curves are illustrated using anonymised Inland Revenue longitudinal individual income data for New Zealand over 2006-10.

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# 1 Introduction

This paper explores the relationship between income dynamics, pro-poor income mobility, and poverty persistence, introducing two new, simple diagrams. Measures of so-called ‘pro-poor growth’ and the income mobility of the initially poor has been a focus of interest of recent literature. Similarly, poverty persistence is an important mobility characteristic, reflecting the extent to which people remain below a designated poverty line over a specified time period.

The existing literature has, of course, produced a range of statistical measures of income mobility and poverty dynamics. There is also value in having simple diagrams, such as Growth Incidence Curves (Ravallion and Chen, 2003) and Growth Income Profiles (Jenkins and Van Kerm, 2016), as well as illustrative devices such as transition matrices. However, similarly simple diagrams illustrating poverty persistence seem to be lacking. This paper offers two new diagrams to illustrate pro-poor income mobility and poverty persistence. The first adapts the more general ‘Three Is of Mobility’ (TIM) curve, proposed by Creedy and Gemmell (2017) specifically to illustrate pro-poor mobility, while the second is a ‘poverty persistence curve’. Each aims to capture their respective aspects of poverty dynamics in ways that allow convenient comparisons.

In the context of cross-sectional poverty comparisons, a well-known diagrammatic device is the ‘Three Is of Poverty’ (TIP) curve proposed by Jenkins and Lambert (1997). For those below a poverty line and for incomes ranked in ascending order, this plots the cumulative poverty (income) gap per person against the corresponding cumulative proportion of people. As with the famous Lorenz curve, which plots the cumulative proportion of total income against the corresponding cumulative proportion of individuals, it provides valuable information that is not apparent from either the density function or the distribution function alone.

One approach to mobility measurement, proposed by Ravallion and Chen (2003), uses cross-sectional data for two periods, rather than longitudinal information about individuals, to produce a ‘growth incidence curve’ (GIC). This plots the growth rate, between two periods, of each quantile or percentile of the distribution in the initial period. It can easily display relative growth differences, by subtracting the overall income growth, and can be used to examine whether or not income growth is said to be ‘pro-poor’. A distinction needs to be drawn between such a curve, which is based on the growth of quantiles rather than of individual incomes, and one which uses longitudinal data. Bourguignon (2011) developed a longitudinal version of the GIC, labelled somewhat obscurely as a ‘non-anonymous growth incidence curve’.

A related approach, proposed by Jenkins and Van Kerm (2016), defines Income Growth Profiles (IGPs), which are similar to those developed by Bourguignon (2011) and aim to facilitate normative social welfare judgements.<sup>1</sup> The IGP involves plotting a measure of average income growth,  $m(p)$ , for the  $p$ th percentile, against  $p$ , where in their case  $m(p)$  is a conditional expectation-based measure amenable to social welfare comparisons.<sup>2</sup> Jenkins and Van Kerm (2016) also propose a cumulative version of the IGP, called the CIGP, in which a measure of average income growth, for those with initial incomes below  $x(p)$ , is plotted against  $p$ .

The present paper first uses a modification of the CIGP to indicate aspects of pro-poor income mobility, reflected in differential income growth of those below, and above, a poverty threshold. This is the ‘Three Is of Mobility’ (TIM) curve proposed by Creedy and Gemmell (2017), which may be used to compare different poverty sub-groups to identify their relative income mobility. Secondly, the paper focuses on income changes which generate movements out of, and into, poverty (that is, movements across a designated poverty line which is assumed to remain constant in real terms). Consideration of such changes then gives rise to a ‘poverty persistence curve’ to illustrate poverty dynamics, and from which the extent of poverty persistence can readily be measured.

Before formally introducing these pro-poor TIM and poverty persistence curve, Section 2 briefly discuss the GIC and an associated elasticity measure of pro-poor growth. Section 3 introduces the TIM curve and its application to pro-poor growth comparisons, then considers the modification leading to the poverty persistence curve, relating to movements across a poverty line. Section 4 uses longitudinal income data for New Zealand, from administrative taxpayer records, to construct examples of pro-poor income mobility and poverty persistence curves. Brief conclusions are in Section 5.

## 2 Alternative Measures of Pro-Poor Growth

### 2.1 Growth Incidence Curves

The Growth Incidence Curve, introduced by Ravallion and Chen (2003), refers to the change in percentiles of the income (or other welfare metric) distribution from  $t - 1$  to  $t$ ; that is, it is based only on the characteristics of the two relevant cross-sectional distributions. Where

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<sup>1</sup>The IGP bears a close resemblance to the Ravallion and Chen (2003) growth incidence curve, but is based on a longitudinal mobility concept.

<sup>2</sup>See also Grimm (2007) and Van Kerm (2009). Jenkins and Van Kerm (2016) examine the welfare-dominance properties of individual income growth based on a social welfare function for which individual utilities are based on incomes in both the initial and final periods. That is, their objective is to produce summary indices of income growth with consistent welfare foundations that are helpful for normative evaluations of alternative distributions of individual income growth.

$H_t(x)$  is the distribution function of income at  $t$ , the  $p$ th percentile,  $x_t(p)$ , is given by:

$$x_t(p) = H_t^{-1}(p) \quad (1)$$

The growth rate,  $\xi_t(p)$  of the  $p$ th percentile is:

$$\xi_t(p) = \frac{x_t(p)}{x_{t-1}(p)} - 1 \quad (2)$$

The GIC curve plots  $\xi_t(p)$  against  $p$ . Since all percentiles are subject to some form of growth, the term ‘incidence’ is perhaps not the most appropriate: rather,  $\xi_t(p)$  shows the *extent* of growth of the  $p$ th percentile.

Ravallion and Chen (2003) show that  $\xi_t(p)$  can be linked to the slopes of the two Lorenz curves in  $t - 1$  and  $t$ . The Lorenz curve is obtained by plotting:

$$L(p) = \frac{1}{\bar{x}} \int_0^{H^{-1}(p)} u dH(u) \quad (3)$$

where  $p = H(x)$  and  $\bar{x}$  is arithmetic mean income,  $\int_0^\infty x dH(x)$ . The slope,  $L'(p)$ , is given by:

$$L'(p) = \frac{x(p)}{\bar{x}} \quad (4)$$

So that substituting for  $x_t(p)$  and  $x_{t-1}(p)$  in (2) gives:

$$\xi_t(p) = \frac{L'_t(p)}{L'_{t-1}(p)} (\gamma_t + 1) - 1 \quad (5)$$

where:

$$\gamma_t = \frac{\bar{x}_t}{\bar{x}_{t-1}} - 1 \quad (6)$$

is the growth rate of mean income. Hence if the Lorenz curve is unchanged,  $\xi_t(p) = \gamma_t$  for all  $p$  and all percentiles grow at the same rate.

Let  $p_{x_p, t-1} = \int_0^{x_p} dH_{t-1}(x) = H_{t-1}(x_p)$  denote the headcount poverty measure at  $t - 1$ , where  $x_p$  is the constant poverty line. Hence,  $p_{x_p, t-1}$  is the percentile corresponding to  $x_p$  for distribution  $H_{t-1}(x)$ . Ravallion and Chen (2003) go on to measure the pro-poor growth rate, PPG, by the mean growth rate:

$$PPG_t = \frac{1}{p_{x_p, t-1}} \int_0^{p_{x_p, t-1}} \xi_t(p) dp \quad (7)$$

Pro-poor growth, defined in this way, leads to a reduction in the Watts (1968) measure of poverty,  $W_t$ , defined in terms of a proportional poverty gap and given by:

$$W_t = \frac{1}{p_{x_p, t}} \int_0^{p_{x_p, t}} \log \left( \frac{x_p}{x_t(p)} \right) dp \quad (8)$$

Pro-poor growth therefore involves a change in the income distribution that is sufficient to lower the poverty measure. From (7) it is clear that the  $PPG_t$  measure is directly related to the GIC curve: it is the area under the curve up to,  $p_{x_p, t-1}$ , the percentile associated with the poverty line.

However, the mean growth rate of percentiles below the fixed poverty line,  $x_p$ , is not the growth rate of the mean income of those below  $x_p$ . It is also not the mean growth rate of those individuals who were below  $x_p$  in period  $t - 1$ . The GIC is based purely on the two marginal distributions in  $t$  and  $t - 1$ : the growth rate of the  $p$ th percentile,  $\frac{x_t(p)}{x_{t-1}(p)} - 1$ , does not refer to the growth rate between  $t$  and  $t - 1$  of the individual at the  $p$ th percentile in  $t - 1$ .

## 2.2 An Elasticity Measure of Pro-Poor Growth

Instead of defining pro-poor growth in terms of the arithmetic mean growth rate of percentiles below a fixed poverty line, Essama-Nssah and Lambert (2006) use the concept of the elasticity of poverty with respect to a change in mean income. Letting  $D(x)$  denote a ‘deprivation’ measure, with  $D(x) = 0$  for  $x \geq x_p$  and  $D(x)$  is a decreasing convex function of  $x$  for  $x < x_p$ , they consider the set of ‘average deprivation’ poverty measures defined by:

$$P = \int_0^{x_p} D(x) dH(x) \quad (9)$$

Define  $\eta_{P, \bar{x}}$  as the elasticity of  $P$  with respect to changes in arithmetic mean income,  $\bar{x}$ . Then:

$$\eta_{P, \bar{x}} = \frac{\bar{x}}{P} \frac{dP}{d\bar{x}} = \frac{\bar{x}}{P} \int_0^{x_p} \frac{dD(x)}{dx} \frac{dx}{d\bar{x}} dH(x) \quad (10)$$

Define  $\eta_{x, \bar{x}}$  as the elasticity of  $x$  with respect to changes in mean income. The  $\eta_{x, \bar{x}}$ s describe the individual income dynamics over the period. Then (10) can be written, using  $D'(x) = \frac{dD(x)}{dx}$ , as:<sup>3</sup>

$$\eta_{P, \bar{x}} = \frac{1}{P} \int_0^{x_p} x D'(x) \eta_{x, \bar{x}} dH(x) \quad (11)$$

It is important to recognise that Essama-Nssah and Lambert (2006) implicitly assume that the individual income changes are such that there are no movements across the poverty line,  $x_p$ . For the headcount poverty measure, which requires  $D(x) = 1$  for  $x < x_p$ , equation (11) gives  $\eta_{P, \bar{x}} = 0$ . However, consider the simple case where  $D(x) = 1 - x/x_p$ , for which:

$$P = H(x_p) \left( 1 - \frac{\bar{x}_p}{x_p} \right) \quad (12)$$

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<sup>3</sup>Is is possible to introduce another elasticity  $\eta_{D, x}$ , so that  $\eta_{P, \bar{x}} = \int_0^{x_p} D'(x) \eta_{D, x} \eta_{x, \bar{x}} dH(x)$ .

As above,  $H(x_p)$  is the headcount poverty measure and  $\bar{x}_p$  is the arithmetic mean income of those below the poverty line. Using  $D'(x) = -\frac{1}{x_p}$ , the elasticity becomes:

$$\eta_{P,\bar{x}} = -\frac{1}{H(x_p)(x_p - \bar{x}_p)} \int_0^{x_p} x\eta_{x,\bar{x}} dH(x) \quad (13)$$

Define  $\hat{x}_p$  as the weighted average income of those in poverty, with weights  $\eta_{x,\bar{x}}$ . Hence  $\hat{x}_p = \frac{1}{H(x_p)} \int_0^{x_p} x\eta_{x,\bar{x}} dH(x)$  and:

$$\eta_{P,\bar{x}} = -\frac{\hat{x}_p}{x_p - \bar{x}_p} \quad (14)$$

The elasticity,  $\eta_{P,\bar{x}}$ , can be rewritten as:

$$\eta_{P,\bar{x}} = \frac{1}{P} \int_0^{x_p} xD'(x) dH(x) + \frac{1}{P} \int_0^{x_p} xD'(x) (\eta_{x,\bar{x}} - 1) dH(x) \quad (15)$$

The first term in (15) is the elasticity that would result from uniform income growth of  $\eta_{x,\bar{x}} = 1$  for all  $x$ . The second term reflects the contribution of the deviation from uniform growth. Essama-Nssah and Lambert (2006) define the extent of pro-poor growth,  $\pi$ , as:

$$\pi = P \left( \eta_{P,\bar{x}} \Big|_{\eta_{x,\bar{x}}=1} - \eta_{P,\bar{x}} \right) \quad (16)$$

where  $\eta_{P,\bar{x}} \Big|_{\eta_{x,\bar{x}}=1}$  is the elasticity resulting from uniform income growth, that is, the first term in (15). Hence:

$$\pi = - \int_0^{x_p} xD'(x) (\eta_{x,\bar{x}} - 1) dH(x) \quad (17)$$

For example, in the above case where  $D(x) = 1 - x/x_p$ , it can be seen that:

$$\pi = -H(x_p) \left( \frac{\hat{x}_p - \bar{x}_p}{x_p} \right) \quad (18)$$

In the case where the poorest of the poor experience relatively larger (and positive) income changes compared with those closer to the poverty line,  $\hat{x}_p < \bar{x}_p$  and  $\pi$  is positive.

### 3 Income Dynamics

This section first examines the income changes of the poor versus the non-poor, illustrating those via a TIM curve in sub-section 3.1. It then examines the income changes needed for individuals to escape and avoid entering poverty between two periods, depending on whether they are below or above a poverty line in an initial period. Consideration of such income dynamics leads to the concept of a poverty persistence curve in sub-section 3.2.



### 3.1 The TIM Curve

The ‘Three "I"s of Mobility’, or TIM, curve, was adapted to a mobility context from Jenkins and Lambert’s (1997) TIP curve, by Creedy and Gemmell (2017). This aims to illustrate the ‘three Is’ characteristics (incidence, intensity and inequality) of income mobility across a given population analogous to the equivalent poverty characteristics stressed by Jenkins and Lambert.

The TIM curve is defined as follows. Let  $x_i$  denote individual  $i$ ’s income, with  $i = 1, \dots, n$ . Define  $y_i = \log x_i$  as the logarithm of income of person  $i$ . Hence  $y_{i,t} - y_{i,t-1}$  is (approximately) person  $i$ ’s proportional change in income from period  $t - 1$  to  $t$ . With incomes ranked in ascending order, plot the cumulative mobility index,  $M_k = \frac{1}{n} \sum_{i=1}^k (y_{i,t} - y_{i,t-1})$  against  $\frac{k}{n}$ , for  $k = 1, \dots, n$ . Thus the TIM curve plots the cumulative proportional income change per capita,  $M_k$ , against the corresponding proportion of individuals. This differs from the CIGP, which plots the average income change for the sub-group,  $k$ , rather than  $n$ , on the vertical axis; that is, the CIGP plots  $\frac{1}{k} \sum_{i=1}^k (y_{i,t} - y_{i,t-1})$  against  $\frac{k}{n}$ .

An example is shown in Figure 1. This curve reflects a situation in which relatively lower-income individuals receive proportional income increases which are greater than that of average (geometric mean) income. If all incomes were to increase by the same proportion, the TIM curve would be the straight line OG. The height, G, indicates the average growth rate of the population as a whole,  $M_n$ . Furthermore, inequality in growth rates (rather than static income inequality) is reflected in the degree of curvature. Further details are given in the Appendix.

In the case shown in Figure 1, the TIM curve is everywhere above OG, reflecting the case where those initially below mean income experience faster income growth than those above; that is, they display regression towards the mean. However, as Creedy and Gemmell (2017) show, TIM curves could lie below OG, where income growth rates are ‘pro-rich’, or the TIM curve can cross the line OG (from above or below) depending on the pattern of individual income growth rates across the initial income distribution.

To examine pro-poor mobility, suppose interest is focussed on a proportion of the population, for example those below the  $h$ th percentile, as indicated in Figure 1. There is less inequality of income changes among the group below  $h$ , shown by the fact that the TIM curve from O to H is closer to a straight line than the complete curve OHG. The TIM curve also shows that the income growth of those below  $h$  is larger than that of the population as a whole, captured by the slopes of the lines OH and OG. The average growth rate among the poor (the intensity of their growth) is given by the height, H ( $= OM_h$ ). If concern is largely for those below a poverty line,  $x_p$ , the corresponding percentile is  $h_p = H(x_p)$ , where  $H(x)$  is the distribution function of  $x$ . It is clear that, for alternative values of  $h$ , the TIM

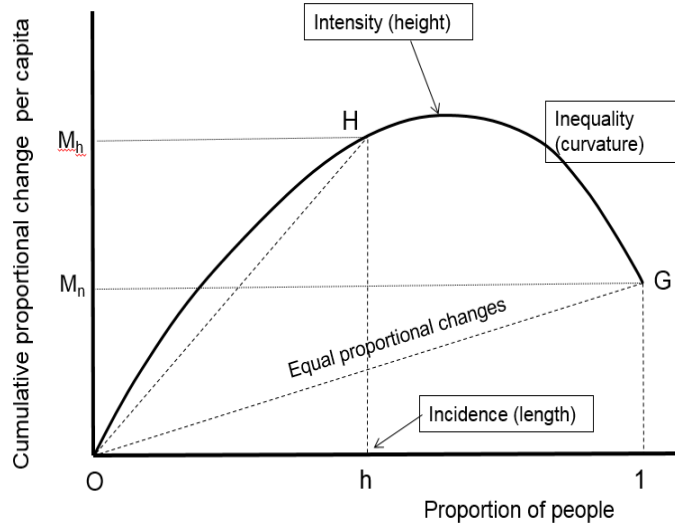


Figure 1: A TIM Curve

curve gives an immediate indication of whether income changes have been ‘pro-poor’ (that is, whether the slope of OH exceed that of OG).

From the definition of  $M_k$  above and Figure 1, it is clear that the slope of OH is given by  $M_h/h$ , while the slope of OG is simply given by  $M_n$ . Hence a measure of the extent of pro-poor mobility can be obtained as:

$$M_{ProP} = \frac{M_h}{hM_n} \quad (19)$$

which measures the extent to which the cumulative proportional income growth per capita for those below  $h$  exceeds that of the population,  $n$ , as a whole. Having defined a fraction of the population below the poverty line,  $h_p$ , if the average income growth of the poor is the same as average income growth across the population (that is, income mobility is neither pro-poor nor pro-rich) then  $M_{h_p}/h_p = M_n$  and  $M_{ProP} = 1$ .<sup>4</sup> In principle the maximum extent of pro-poor mobility by this measure is infinite; for example,  $M_{ProP} \rightarrow \infty$  as  $M_n \rightarrow 0$  (zero income growth across the population on average) or  $M_{h_p}/h_p \rightarrow \infty$  (infinitely high income growth for the poor, such as when their initial incomes are zero). On the other hand, maximum pro-rich income mobility implies  $M_{ProP} \rightarrow -\infty$ .<sup>5</sup> Using

<sup>4</sup>There may nevertheless be differing degrees on ‘inequality of mobility’ (as represented by the curvature of the TIM curves up to  $h$ ) within the poor group compared to that within the population as a whole.

<sup>5</sup>An alternative measure of the extent of pro-poor income mobility could be based on the *difference* in slopes of OH and OG in Figure 1. That is:  $M_h/h - M_n$ . In this case mobility-neutral growth (neither pro-poor nor pro-rich) implies  $M_{ProP} = 0$ .

equation (19) and New Zealand income growth data, section 4 illustrates values of  $M_{ProP}$  for different values of  $h_p$ .

### 3.2 Poverty Persistence

A natural question to ask is whether the TIM curve can illustrate poverty persistence, reflecting the extent to which upward income mobility shifts individuals from below, to above, a given income poverty threshold,  $x_p$ ? Since the TIM curve illustrates the *cumulative* extent of mobility for those below  $h_p$ , it can address whether the incomes of an initially poor group *on average* grew sufficiently to escape from poverty, but it cannot directly illustrate the extent to which *individuals* below  $h_p$  escape from poverty. Suppose first that poverty is measured in relative terms. If the TIM curve up to  $h_p$  were to lie below the straight line OG of Figure 1, then it is clear that income growth for those below  $h_p$  is insufficient to lift this *group*, in poverty at  $t - 1$ , above the poverty line at  $t$ . That is, had the income growth experienced in aggregate by those below  $h_p$  been redistributed among those individuals to maximise the numbers above  $x_p$  at  $t$ , there is no reallocation that could have lifted all of them out of poverty.

Nevertheless, some individuals within this group at  $t - 1$  may experience sufficient income growth between  $t - 1$  and  $t$  to raise their income levels above  $x_p$ . Assume further that the poverty income threshold is constant in both years. The condition required for those individuals for whom  $x_{i,t-1} < x_p$  to move out of poverty is given by:

$$g_i > \frac{x_p}{x_{i,t-1}} - 1 \quad (20)$$

where  $g_i = \frac{dx_i}{x_{i,t-1}}$  is individual  $i$ 's proportional income growth between  $t - 1$  and  $t$ . More generally, all individuals can be allocated to one of four groups based on their values of  $g_i$  and  $x_{i,t-1}$ , as shown in Table 1.

Table 1: Poverty Persistence

	In poverty	Out of poverty
Move	$g_i < \frac{x_p}{x_{i,t-1}} - 1;$ $x_{i,t-1} > x_p$	$g_i > \frac{x_p}{x_{i,t-1}} - 1;$ $x_{i,t-1} < x_p$
Persist	$g_i < \frac{x_p}{x_{i,t-1}} - 1;$ $x_{i,t-1} < x_p$	$g_i > \frac{x_p}{x_{i,t-1}} - 1;$ $x_{i,t-1} > x_p$

The two conditions separating the four groups are  $g^* = \frac{x_p}{x_{i,t-1}} - 1$  and  $x_{i,t-1}^* = x_p$ . These can be illustrated by a variant of the TIM curve. Whereas the TIM curve plots, for incomes in ascending order, cumulative proportional income changes per capita against the

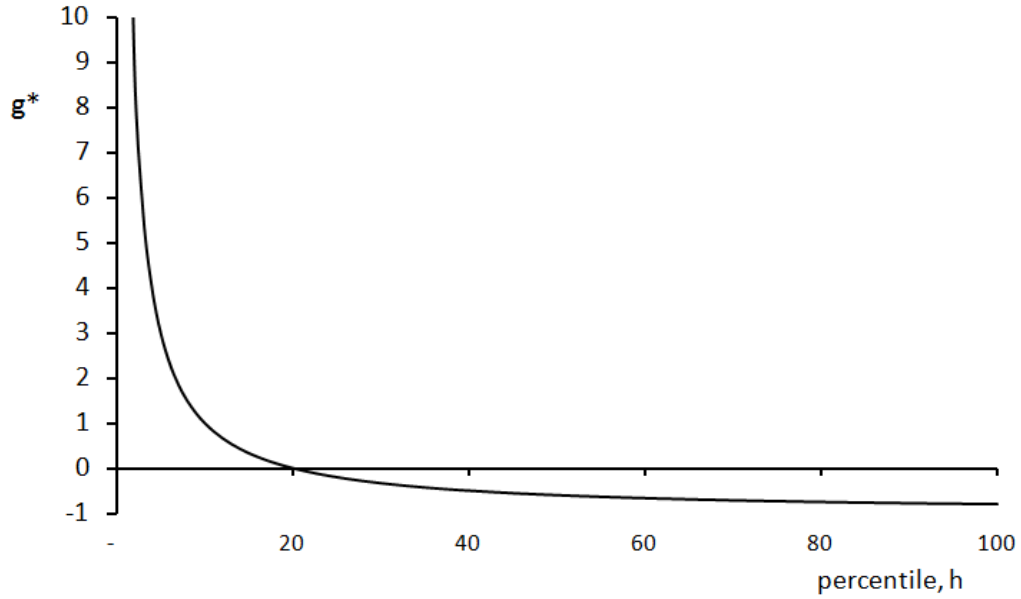


Figure 2: Income Growth Rates Required to Escape and Avoid Entering Poverty

corresponding proportion of people,  $h$ , in this case individual income growth rates,  $g_i$  and  $g^*$ , may be plotted against  $h$ , for any given income poverty threshold,  $x_p$ , and associated  $h_p$ .

Figure 2 plots the values of  $g^*$  against  $h$ , for a poverty income threshold,  $x_p$ , such that  $h_p = 0.2$  (20 per cent), and the  $g^*$  profile crosses the  $x$ -axis at  $h = 0.2$ . This is a poverty persistence curve,  $g^*(h_p)$ , which is defined for a given value of  $h_p$ .

To the left of  $h_p = 0.2$  actual growth rates greater than  $g^*$  are sufficient to move the individual out of poverty; that is, above  $h_p$ . Conversely, to the right of  $h_p = 0.2$  actual growth rates less than  $g^*$  are sufficiently negative to move the individual into poverty, below  $h_p$ . The profile of critical values,  $g^*$ , asymptotes at  $-1.0$  ( $-100$  per cent); that is, as incomes become very large relative to  $x_p$ , the required (negative) growth rate to move such individuals into poverty approaches  $-100$  per cent.

### 3.3 Movements Across the Poverty Line

For an individual with initial income of  $x$ , experiencing a proportional change of  $dx/x$ , poverty is avoided if:

$$x \left( 1 + \frac{dx}{x} \right) > x_p \quad (21)$$

Hence the income change must be such that:

$$\frac{dx}{x} > \frac{x_p}{x} - 1 \quad (22)$$

Suppose the dynamics can be described by the following function:

$$\frac{dx}{x} = g + f(x) + u \quad (23)$$

where  $g$  denotes the general growth in incomes,  $u$  is a stochastic term distributed as  $N(0, \sigma^2)$ , and the function  $f(x)$  describes the relative income movements. For example, for the basic regression towards the mean model,  $f(x) = -(1 - \beta) \log(x/G)$ , where  $\beta$  is the regression coefficient and  $G$  is the geometric mean income in the initial period. Hence to avoid poverty, it is required that:

$$g + f(x) + u > \frac{x_p}{x} - 1 \quad (24)$$

or:

$$u > \frac{x_p}{x} - (1 + g) - f(x) = s(x, x_p, g) \quad (25)$$

The probability of avoiding poverty in the second period,  $P(x_t > x_p)$  is thus:

$$P(x_t > x_p) = \int_{s(x, x_p, g)}^{\infty} dN(u|0, \sigma^2) = 1 - \int_{-\infty}^{s(x, x_p, g)} dN(u|0, \sigma^2) \quad (26)$$

The probability of being in poverty in the second period,  $P(x_t < x_p)$ , is:

$$P(x_t < x_p) = N\left(\frac{s(x, x_p, g)}{\sigma} \middle| 0, 1\right) \quad (27)$$

As above,  $H(x_p)$  is the initial headcount poverty measure, the headcount poverty measure in the second period,  $H_t(x_p)$ , is a weighted average of individual probabilities, given by:

$$H_t(x_p) = \int_0^{\infty} N\left(\frac{s(x, x_p, g)}{\sigma} \middle| 0, 1\right) dH(x) \quad (28)$$

## 4 New Zealand TIM and Poverty Persistence Curves

The data used in this section to illustrate pro-poor TIMs and poverty persistence curves are based on a 2 per cent random sample of individual New Zealand personal income taxpayers using administrative data from the Inland Revenue Department. The panel dataset contains incomes for both 2006 and 2010 for the same taxpayers. To avoid the exercise being contaminated by taxpayers with very low incomes (such as small part-time earnings of children, or small capital incomes of non-earners), individuals with 2006 or 2010 incomes less than \$1,000 were omitted from the sample. This yielded a usable sample of 32,970 individuals.

Before applying the earlier analysis, it should be acknowledged that since these data are based on incomes of *individuals*, as opposed to *households*, the notion of a poverty line is less meaningful. Clearly many individuals could, and in the dataset do, experience substantial year-to-year changes in income without this necessarily implying that the households of which they are members move into, or out of, poverty. Nevertheless the dataset has the particular advantage of providing more reliable estimates of individuals' incomes from matched tax records, and has much wider coverage than more limited longitudinal household data based on survey methods available for New Zealand.<sup>6</sup> For present purposes it serves to illustrate the conceptual aspects of interest.

#### 4.1 Pro-Poor TIM curves

To construct a TIM curve for income growth over 2006-10, individuals were first ranked by their incomes in 2006. Cumulative income growth rates for the five year period were then obtained as described in section 3 and plotted against the cumulative population shares. The curve is therefore constructed using the full sample of 32,970 individuals, rather than based on averages within percentiles.

Figure 3 shows the resulting TIM curve. The vertical height of the point G in the Figure indicates the average growth rate across the full sample of around 0.14 (that is, 14 per cent income growth over the period 2006 to 2010). It can be seen that the TIM curve is everywhere above OG indicating higher growth rates at lower income levels such that as initial incomes rise (moving from left to right in the Figure), the cumulative growth of the lower income sub-group remains above the overall average at G.

This generally also holds at each threshold adopted for  $h_p$ ; for example, the slope of line OH, for  $h_p = 0.2$  is steeper than the line OK for  $h_p = 0.4$ , which in turn is steeper than the line OG. Hence the degree of pro-poor mobility in these data would seem to suggest greater pro-poor growth the lower the threshold adopted to define the (initially) poor.

Figure 4 shows the values of  $M_{ProP} = M_{h_p}/h_p M_n$ , using equation (19), for different percentile values of  $h_p$  from 0.05 to 0.5, in steps of 0.05. At  $h_p = 0.05$  income mobility of the poorest 5 per cent is approximately ten times that of the full sample and could therefore be considered to be highly pro-poor. As  $h_p$  rises towards 0.5, the extent of this pro-poor mobility falls, but remains above one, reaching  $M_{ProP} = 2.3$  at  $h_p = 0.5$ .<sup>7</sup>

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<sup>6</sup>Statistics New Zealand collected longitudinal household income data 2001-2010 in the SoFIE dataset, in 8 waves. While SoFIE initially involved around 11,000 households, this declined progressively across waves with an attrition rate reaching 54 per cent by wave 7; see Statistics New Zealand (2011).

<sup>7</sup>By definition,  $M_{ProP} = 1.0$  at  $h_p = 1.0$ .

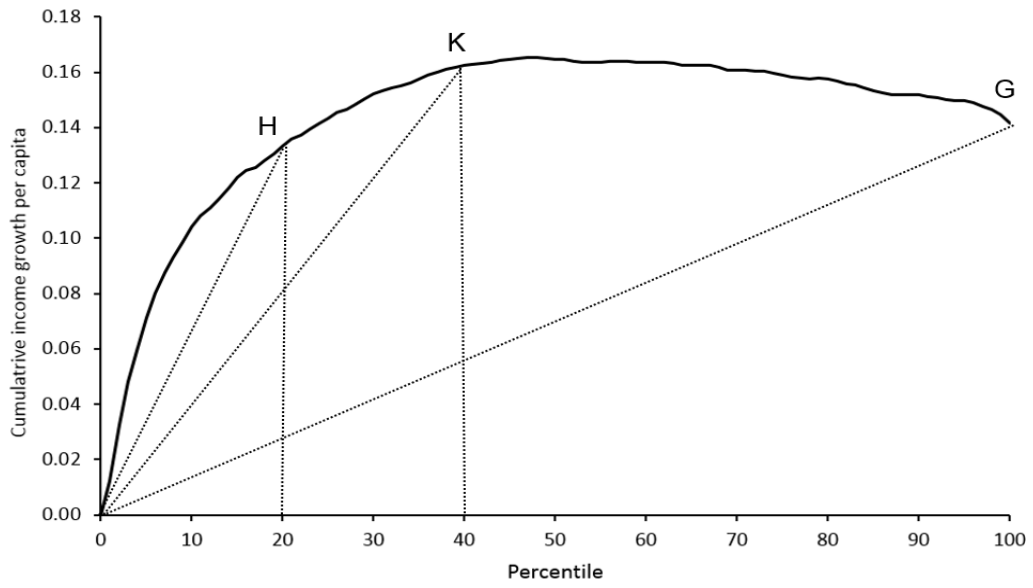


Figure 3: The New Zealand TIM Curve and Poverty Income Dynamics: 2006 to 2010

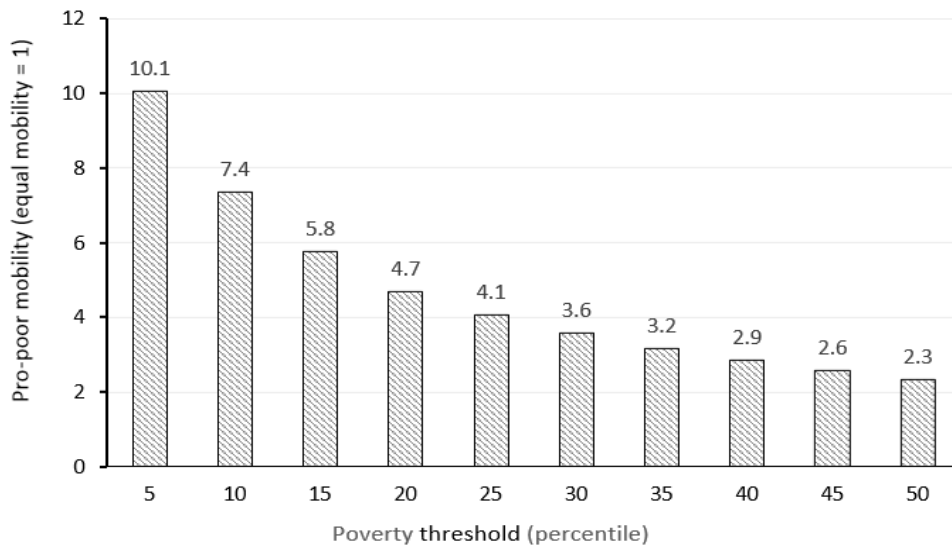


Figure 4: Pro-Poor Mobility: 2006 to 2010

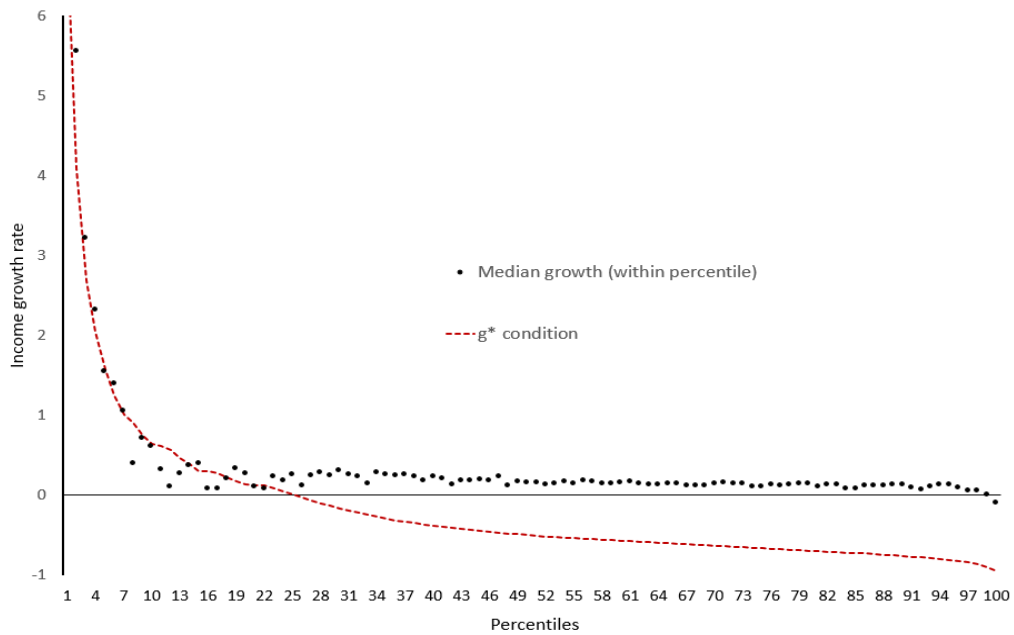


Figure 5: Poverty Persistence in New Zealand: 2006 to 2010

## 4.2 Poverty Persistence Curves

As mentioned above, identifying poverty persistence requires a choice of poverty income threshold,  $x_p$ . For illustrative purposes this is set here at 50 per cent of median income in 2006: thus, where  $x_{median} = \$34,087$ , the poverty line is  $x_p = \$17,044$ . In this case it turns out that  $h_p = 0.25$  (or 25 per cent).<sup>8</sup> Figure 5 shows both the critical growth rate,  $g^*$  (the dashed curve) and median actual growth rates within each percentile.<sup>9</sup>

As required, the critical growth rate,  $g^*$ , crosses the  $x$ -axis at  $h_p = 0.25$ . To the left of that point, any median growth rate for a given percentile which is above the  $g^*$  curve implies that ‘on average’ (that is, for at least half of the individuals in the percentile) actual income growth was sufficiently large that their income in 2010 exceeded  $x_p$ .

It can be seen from Figure 5 that this condition is satisfied for about 10 of the 25 percentiles below  $h_p$ . By contrast, above  $h_p$  there are no percentiles for which median growth is sufficiently negative (that is, lying below the  $g^*$  curve) to push median individuals below  $x_p$  in 2010. This suggests a high degree of poverty avoidance over the 2006 to 2010 period for those not initially in poverty, and somewhat less persistence in poverty for those initially below the poverty line.

<sup>8</sup>This unusual property results from the fact that the distribution function over the relevant range is approximately a straight line through the origin.

<sup>9</sup>Given the sample size, there are 329 or 330 individuals in each percentile.



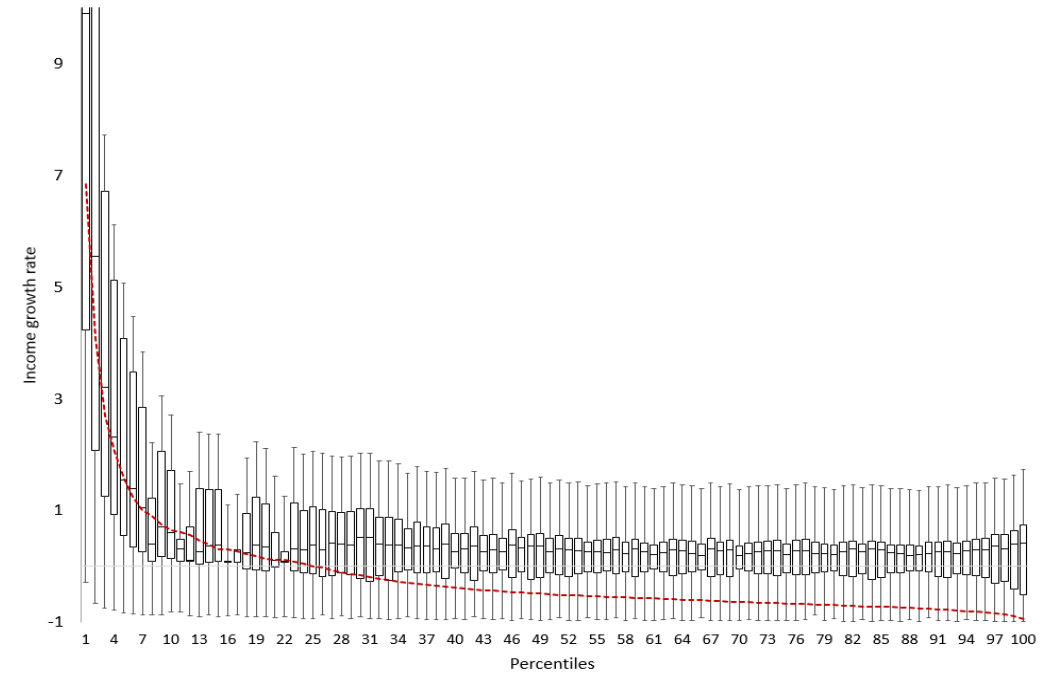


Figure 6: Details of Poverty Persistence in New Zealand: 2006 to 2010

However, the median percentile growth rates cannot capture the diversity of experience within each percentile. Figure 6 replaces the percentile medians with box plots for actual percentile growth rates where each ‘box’ shows the median growth rate and inter-quartile range. The ‘whiskers’ record the maximum and minimum income growth rate within each percentile.

Figure 6 reveals a richer pattern of movement into and out of poverty. In particular, the whiskers indicate a wide range of growth rates within each percentile of the initial income distribution, such that, for example, every percentile above  $h_p$  includes at least one person who moved into poverty. Similarly, for those percentiles initially below  $h_p = 0.25$ , there is evidence of many individuals in almost all the lower percentiles moving out of poverty. There are numerous cases of individuals lying between the median and upper quartile (of the percentile distribution) who are observed to move out of poverty. This is consistent with a known property of New Zealand personal income taxpayer data, namely the high degree of volatility in individual taxpayer incomes from year to year.

This volatility can also be observed when numbers for New Zealand are inserted into the four way classification in Table 1. These are shown in Table 2 for three alternative definitions of the poverty line,  $x_p$ : at 0.33, 0.50, and 0.67 of median income. Values in Table 2 for  $x_p/x_{median} = 0.50$  are for the case shown in Figure 6.

Table 2: Poverty Persistence: 2006 to 2010

	2006 headcount (%)	
	In poverty	Not in poverty
$x_p/x_{median} = 0.33$		
Move	9	5
Persist	3	82
$x_p/x_{median} = 0.50$		
Move	11	8
Persist	14	67
$x_p/x_{median} = 0.67$		
Move	12	9
Persist	21	58

The table shows that, for the lowest poverty line (0.33), 14 per cent of individuals move: 9 per cent move out of poverty by 2010 while 5 per cent move into poverty from above the 2006 poverty line. The extent of poverty persistence is sensitive to the poverty line, rising from only 3 per cent or one-quarter of those in poverty, 3/12, (at  $x_p/x_{median} = 0.33$ ) to 14 per cent (at 0.50) and 21 per cent or around two-thirds of those in poverty (at 0.67).

Hence in this New Zealand case, those initially more deeply in poverty – those below one-third of 2006 median income – appear to experience the least persistence of the three poverty group definitions. A similar picture emerges for those initially above the poverty line, but moving below it: the percentage moving into poverty is also sensitive to poverty line definitions, ranging from 82 per cent (at 0.33) to 58 per cent (at 0.67). These values for ‘movers’ suggest that, at least among individual New Zealand income taxpayers, there is substantial movement into and out of the lowest income levels. Since these lowest income levels are typically a few thousand dollars of annual income, this may capture, *inter alia*, the effects of moving into, or out of, the labour market and/or part-time earning.<sup>10</sup>

## 5 Conclusions

This paper has suggested two new illustrative devices for poverty income dynamics. To examine pro-poor mobility in the form of relative income growth, it has proposed using TIM curves applied to individuals in poverty. In addition to highlighting the ‘three I’s’ properties (incidence, intensity and inequality) of mobility for alternative poverty definitions, this allows the relative mobility of each poverty group to be compared with mobility by the

<sup>10</sup>Excuding taxpayers with less than \$1000 of taxable income does not preclude zero labour market earnings since capital income (such as property income, interest, and dividends) are included within the taxable income definition.

population as a whole.

To examine poverty persistence, the paper suggested that a poverty persistence curve can readily identify both the extent of movement across a poverty threshold and the particular poor and non-poor incomes for which persistence or movement is prevalent.

Applying these concepts to New Zealand income data for individual income taxpayers showed that income dynamics were especially pro-poor during 2006-10, with much faster income growth for those on the lowest incomes than those higher up the income distribution. For example, a mobility index based on cumulative income growth rates for those with the lowest five per cent of 2006 incomes, on average was around ten times higher than the equivalent index for all taxpayers combined. For the lowest twenty-five per cent the equivalent index was around four times higher than mobility across all taxpayers.

On poverty persistence, average growth rates within each percentile of the distribution suggested relatively little movement into poverty but somewhat more movement out of poverty. However considering all individuals within each percentile (around 330 individuals per percentile in this case) revealed a relatively mobile population overall, with some individuals observed within all percentiles above the 2006 poverty threshold moving into poverty over the five year period examined.

These New Zealand figures are of course purely illustrative, depending here on the particular poverty threshold chosen and relating to individual, rather than household, incomes. But, suitably applied to household- or family-level income data, the devices proposed here would seem to offer convenient illustrations of the nature and extent of poverty income dynamics that are readily constructed and interpreted.

## Appendix A: Further Properties of the TIM Curve

The appendix provides a more formal treatment of the TIM curve defined in section 3. As above,  $y = \log(x)$  denotes the logarithm of income. Suppose  $H(x_t)$  and  $F(y_t)$  denote respectively the distribution functions of income and log-income at time  $t$ , with population size,  $n$ . For incomes ranked in ascending order, the TIM curve plots the cumulative proportional income changes,  $y_t - y_{t-1}$ , *per capita*, denoted  $M_{h,t}$ , against the corresponding proportion of people,  $h$ , where:

$$h = F(y_{h,t-1}) \quad (\text{A.1})$$

Thus  $y_{h,t-1} = F^{-1}(h)$  is the log-income corresponding to the  $h^{\text{th}}$  percentile. Hence, the TIM curve plots  $M_{h,t}$ , given by:

$$M_{h,t} = \int_0^{y_{h,t-1}} (y_t - y_{t-1}) dF(y_{t-1}) \quad (\text{A.2})$$

against  $h$ .<sup>11</sup> Let  $\mu_t$  denote the arithmetic mean of log-income, so that if  $G_t$  is the geometric mean,  $\mu_t = \log G_t$ . Then (A.2) can be written:

$$M_{h,t} = \int_0^{y_{h,t-1}} \{(y_t - \mu_t) - (y_{t-1} - \mu_{t-1})\} dF(y_{t-1}) + (\mu_t - \mu_{t-1}) F(y_{h,t-1}) \quad (\text{A.3})$$

The term,  $y_t - \mu_t$  is equal to  $\log(x_t/G_t)$ . Hence  $(y_t - \mu_t) - (y_{t-1} - \mu_{t-1})$  is the proportional change in *relative* income. Thus,  $M_{h,t}$  consists of the cumulative proportional change in income *relative* to the geometric mean, *plus* a component that depends only on the proportional change in geometric mean income. If all individuals receive exactly the same relative income change,  $g$ , say,  $M_{h,t}$  plotted against  $h$  is a straight line with a slope of  $g$ . This means that the nature of mobility – the extent to which it is equalising or disequalising over any range of the income distribution – can be seen immediately by the extent to which the TIM curve deviates from a straight line.

Suppose the proportional change in the geometric mean,  $\mu_t - \mu_{t-1}$ , is equal to  $g$ . Furthermore, suppose the proportional change in relative income depends on income in  $t-1$ , so that  $(y_t - \mu_t) - (y_{t-1} - \mu_{t-1})$  can be written as the function,  $\hat{g}(y_{t-1})$ . Then (A.3) can be expressed as:

$$M_{h,t} = \int_0^{y_{h,t-1}} \hat{g}(y_{t-1}) dF(y_{t-1}) + gh \quad (\text{A.4})$$

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<sup>11</sup>For very large datasets it is convenient to plot values of the cumulative proportional change corresponding to percentiles,  $P_j$ , for  $P_1 = 0.01$  and  $P_j = P_{j-1} + 0.01$ , for  $j = 2, \dots, 100$ . Thus, obtain the cumulative sum  $M_j = \frac{1}{n} \sum_{i=1}^{nP_j} (y_{i,t} - y_{i,t-1})$ , where as above  $n$  is the number of individuals in the sample. Hence for  $j = 2, \dots, 100$ :  $M_j = M_{j-1}/n + \frac{1}{n} \sum_{i=nP_{j-1}+1}^{nP_j} (y_{i,t} - y_{i,t-1})$ . The TIM curve is then plotted using just 100 values.

If all individuals receive exactly the same relative income change, then relative positions are unchanged and  $\hat{g}(y_{t-1}) = 0$  for all  $y_{t-1}$ . Hence,  $M_{h,t}$  plotted against  $h$  is a straight line through the origin with a slope of  $g$ . This means that the extent to which it is equalising or disequalising over any range of the income distribution can be seen immediately by the extent to which the TIM curve deviates from a straight line, which in turn depends on the properties of  $\hat{g}(y_{t-1})$ .

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