Illustrating Income Mobility: Two New Measures*

John Creedy and Norman Gemmell

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Illustrating Income Mobility: Two New Measures[*]

John Creedy and Norman Gemmell[†]

Abstract

Jenkins and Lambert (1997) demonstrated that a number of measures of poverty could be combined and compared using the "Three Is of Poverty" (TIP) curve; the 'three Is' being the incidence, intensity and inequality of poverty. This paper takes the insights from the TIP curve and applies them to income growth based measures of mobility, proposing a "Three Is of Mobility", or TIM, curve. Similar analysis is then applied to re-ranking measures of mobility to yield a re-ranking ratio (RRR) curve. Illustrations are provided using income data from random samples of New Zealand income taxpayers over the period 1998 to 2010. It is argued that both curves represent simple graphical devices that nevertheless conveniently illustrate the "Three Is" properties of income mobility.

[†]Victoria Business School, Victoria University of Wellington, New Zealand.

1   Introduction

When comparing distributions of non-negative economic variables, such as annual income or consumption, the Lorenz curve is ubiquitous. With individual observations arranged in ascending order, this plots (within a box of unit height and base) the cumulative proportion of total income (the first-moment distribution function) against the corresponding cumulative proportion of individuals or units (the distribution function). A normalised area measure of the distance between the Lorenz curve and the line of equality gives rise to the equally famous Gini inequality measure.[1] Furthermore, the concept of 'Lorenz dominance' provides an immediate qualitative comparison between the inequality of two distributions, and this can be given a welfare interpretation when combined with the value judgement summarised by the 'principle of transfers'. The Lorenz curve thus provides a valuable diagrammatic summary, providing much more information than either the density function or the distribution function alone.[2]

Where concern is largely for those towards the lower end of the distribution – those below a poverty line – an alternative diagrammatic device involves, for incomes again arranged in ascending order, plotting the cumulative (absolute) poverty gap per person against the corresponding cumulative proportion of people. This gives rise to a TIP curve, named by Jenkins and Lambert (1997) for its ability to indicate the 'Three "I"s of Poverty', namely incidence, intensity and inequality. As with the Lorenz curve, dominance properties hold and the curve is a straight line (for those below the poverty line) only in situations where all the poor have equal incomes. In the case of Lorenz and TIP curves, comparisons involving intersecting curves lead to the need to impose more structure on evaluations, in the form of particular value judgements and quantitative inequality and poverty measures.

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[1]This measure can be related to an explicit social welfare function involving a 'reverse-rank' weighted average of incomes and an inequality measure based on an 'equally distributed equivalent income' measure.

[2]The so-called Pen Parade, following Pen (1971), is simply the distribution function rotated through 90 degrees, therefore showing income on the vertical axis and the cumulative proportion of people on the horizontal axis. It is used, along with the metaphorical parade of individuals aligned from poor to rich, mainly in popular presentations.

A similar challenge has arisen in the context of income mobility, where a number of different diagrams have been proposed to capture the key properties of mobility in an easily-perceived way. This is complicated by the variety of definitions and interpretation of different mobility concepts, such as those associated with individual income growth; positional change or re-ranking; impacts on the inequality of longer-term incomes; and 'income risk'; see Jäntti and Jenkins (2015).[3] As discussed in the next section, most illustrative devices for income mobility have focused on income growth measures. These include Trede (1998), Ravallion and Chen (2001), Bourguignon (2011), Van Kerm (2009) and Jenkins and Van Kerm (2006, 2011, 2016). Much of this recent analysis has focussed on the welfare dominance properties of alternative income mobility measures or illustrative devices. Surprisingly perhaps, none of those approaches explicitly examines or illustrates all three of the key 'positive' properties of incidence, intensity and inequality as captured by the TIP curve in the poverty context despite, as argued below, these properties being of similar interest for income mobility.

The present paper addresses those omissions by offering two new illustrative devices for income mobility. Firstly, a modification of income growth profiles is proposed to illustrate the 'Three "I"s of Mobility'. Like Bourguignon (2011) and Jenkins and Van Kerm (2016) this captures longitudinal dimensions. It plots the cumulative proportional income change *per capita* (not per head of the cumulated sub-group), with individuals ranked in ascending order of initial income, against the corresponding proportion of individuals. Since the diagram bears a close resemblance to the TIP curve it is described here as a 'Three "I"s of Mobility', or TIM curve. Secondly, a comparable device capable of illustrating the 'three "I"s' properties for a positional change measure of income mobility is developed. This considers the cumulative ratio of observed re-ranking to the maximum feasible re-ranking for each individual,

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[3]Jäntti and Jenkins (2015) suggest that the concept of income risk can be regarded as one aspect of longer term income inequlity, where changes in an income inequality measure over time have both permanent predictable and transitory unpredictable components. They label the latter as 'income risk'. In measuring aggregate income growth, Palmisando and Van de gaer (2016) combine individual income growth and initial rank position, to form a weighted average of individual growth with weights decreasing with initial rank.

ranked in ascending order of the initial income distribution. It is therefore labelled a 're-ranking ratio', or RRR, curve.

Existing illustrative devices for income mobility are discussed in Section 2. Subsequent sections propose and apply the two new illustrative devices for income mobility. Focusing first on the individual income growth class of mobility measures, allows the TIM curve concept to be introduced in Section 3, and illustrated using a longitudinal sample of individuals from New Zealand in Section 4. Section 5 introduces positional change mobility measures and the derivation of the RRR curve. This is also illustrated using New Zealand data in Section 6. Conclusions are in Section 7.

2 Illustrative Devices for Income Mobility

Despite not seeking explicitly to illustrate the 'three "I"s' properties of mobility – to be defined more fully below – a number of authors have sought to illustrate distributional dimensions of income mobility across a population or sample of individuals. This section reviews some of the more commonly used before considering the alternatives proposed here.

2.1 Quantile Profiles

Trede (1998), motivated by a desire to illustrate and summarise the information contained in a transition matrix, concentrated on the conditional distributions of income (relative to, say, the median) in one year, given incomes in an earlier year. He proposed the use of diagrams showing profiles of various quantiles of the conditional distributions, with relative income in the initial year on the horizontal axis. For incomes in t and $t-1$, the method involves non-parametric estimation of various quantiles of conditional distributions of x_t for given values of x_{t-1}. The quantile profiles are shown in a diagram with income in t on the vertical axis and x_{t-1} on the horizontal axis. Trede suggested translating incomes to relative values by dividing by the mean or median income in each period. A simplified example is shown if Figure 1, which illustrates just three quantiles.
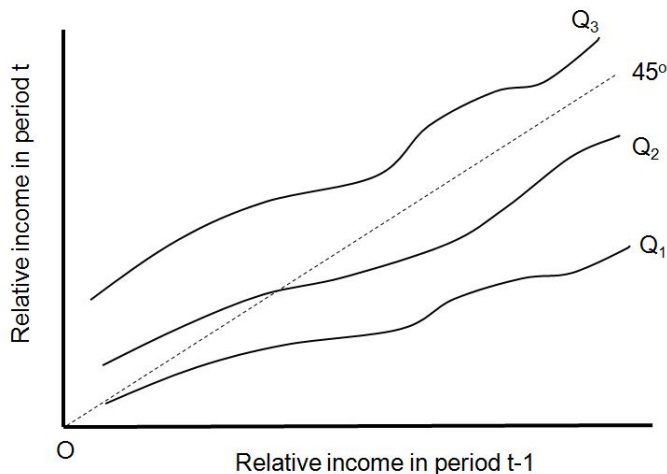
Figure 1: Quantile Regressions of Income in t on Income in $t-1$

Trede suggests the following interpretation of the quantile profiles, defining 'perfect mobility' as independence of income in $t-1$, whereby the quantile profiles become horizontal. The vertical distances between quantile profiles give an indication of the extent of income inequality in t.[4] The extreme of 'total [relative] immobility' produces quantile profiles that coincide: that is, all those with x_{t-1} have the same income in period t. If the (common) quantile profiles coincide with the 45-degree line in the diagram, there is no change in the (marginal) distribution when moving from $t-1$ to t. Thus, Trede (1998, p. 80) suggests that, 'both the distance from each other, and the slopes of the quantile lines provide information about income dynamics'.

However, the marginal distributions can remain unchanged even when the quantiles do not coincide and they are not 45-degree lines. For example, consider the simple mobility process in which there is regression towards the (geometric) mean and suppose that incomes, x, in two periods are jointly lognormally distributed, so that $y = \log x$ is jointly normally distributed.

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[4]However, the vertical distances refer to conditional distributions, not the marginal distribution in t.

Regression towards the (geometric) mean is described by:

$$y_t - \mu_t = \beta \left(y_{t-1} - \mu_{t-1} \right) + u_t \tag{1}$$

where u_t is a stochastic term with zero expected value and variance, σ_u^2. This case has received substantial attention in studies of income dynamics.[5] The coefficient, $\beta \leq 1$, indicates the degree to which those below the geometric mean experience, on average, a higher relative income increase than those above the geometric mean. Then:

$$\sigma_t^2 = \beta^2 \sigma_{t-1}^2 + \sigma_u^2 \tag{2}$$

There can therefore be no change in the inequality of the marginal distribution of y (so that $\sigma_t^2 = \sigma_{t-1}^2$) if:

$$\beta^2 = 1 - \frac{\sigma_u^2}{\sigma_{t-1}^2} \tag{3}$$

Hence, it is not necessary to have $\sigma_u^2 = 0$ and $\beta = 1$ for stability of relative inequality. Furthermore, in this case the conditional distributions of y_t are homoscedastic, so that the quantile profiles are all straight parallel lines. Letting ρ denote the correlation coefficient between log-income in the two periods, it is also known that:

$$\frac{\sigma_t}{\sigma_{t-1}} = \frac{\beta}{\rho} \tag{4}$$

Hence stability requires only that the correlation and regression coefficients are equal.

2.2 Growth Incidence Curves

An alternative approach, proposed by Ravallion and Chen (2003), uses cross-sectional data for two periods to produce a 'growth incidence curve' (GIC). This was extended by Bourguignon (2011) to recognise longitudinal aspects of individual income growth through what he refers to as 'non-anonymous

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[5]The process, with a number of extensions, is examined in detail in Creedy (1985).

growth incidence curves'. The GIC can easily display relative growth differences, by subtracting the overall income growth, plotting the growth rate from $t-1$ to t of each quantile or percentile of the distribution of initial income; that is, it is based only on the characteristics of the two relevant cross-sectional distributions (or longitudinal distributions in Bourguignon, 2011).

The Ravallion and Chen (2003) focus is specifically on 'pro-poor growth'; that is how far those initially below a poverty income threshold, $x(p)$, experience higher or lower income growth than those above the threshold.[6] Letting $H_t(x)$ be the distribution function of income at t, the pth percentile, $x_t(p)$, is given by:

$$x_t(p) = H_t^{-1}(p) \tag{5}$$

The growth rate, $\xi_t(p)$ of the pth percentile is:

$$\xi_t(p) = \frac{x_t(p)}{x_{t-1}(p)} - 1 \tag{6}$$

The GIC curve plots $\xi_t(p)$ against p. Since all percentiles are subject to some form of growth, the term 'incidence' is perhaps not the most appropriate: $\xi_t(p)$ rather shows the *extent* (or 'intensity') of growth of the pth percentile.

Ravallion and Chen (2003) show that $\xi_t(p)$ can be linked to the slopes of the two Lorenz curves. The Lorenz curve is obtained by plotting:

$$L(p) = \frac{1}{\bar{x}} \int_0^{H^{-1}(p)} u\,dH(u) \tag{7}$$

where $p = H(x)$ and $\bar{x}$ is arithmetic mean income, $\int_0^\infty x\,dH(x)$. The slope, $L'(p)$, is given by:

$$L'(p) = \frac{x(p)}{\bar{x}} \tag{8}$$

So that substituting for $x_t(p)$ and $x_{t-1}(p)$ in (6) gives:

$$\xi_t(p) = \frac{L_t'(p)}{L_{t-1}'(p)}(\gamma_t + 1) - 1 \tag{9}$$

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[6]A somewhat different approach to measuring the income growth of the poorest was suggested by Essama-Nssah and Lambert (2006). Instead of defining pro-poor growth in terms of the arithmetic mean growth rate of percentiles below a fixed poverty line, they used the concept of the elasticity of poverty with respect to a change in mean income.

where:
$$\gamma_t = \frac{\bar{x}_t}{\bar{x}_{t-1}} - 1 \tag{10}$$

is the growth rate of mean income. Hence if the Lorenz curve is unchanged, $\xi_t(p) = \gamma_t$ for all p: all percentiles grow at the same rate.

To concentrate on those in poverty, define x_P as the (constant) poverty line. Let $p_{x_P,t-1}$ denote the percentile corresponding to x_P for distribution $H_{t-1}(x)$: this is the headcount poverty measure at $t-1$. Then $p_{x_P,t-1} = \int_0^{x_p} dH_{t-1}(x) = H_{t-1}(x_P)$. Ravallion and Chen (2001) measure the pro-poor growth rate, PPG, as the mean growth rate 'for the poor':

$$PPG_t = \frac{1}{p_{x_P,t-1}} \int_0^{p_{x_P,t-1}} \xi_t(p)\, dp \tag{11}$$

Pro-poor growth, defined in this way, leads to a reduction in the Watts (1968) measure of poverty, W_t, defined in terms of a proportional poverty gap and given by:

$$W_t = \frac{1}{p_{x_P,t}} \int_0^{p_{x_P,t}} \log\left(\frac{x_P}{x_t(p)}\right) dp \tag{12}$$

Pro-poor growth therefore involves a change in the income distribution that is sufficient to lower the poverty measure. From (11), the PPG_t measure is directly related to the GIC curve: it is the area under the curve up to, $p_{x_P,t-1}$, the percentile associated with the poverty line.

However, PPG_t is the mean growth rate of percentiles below the fixed poverty line. It is not the growth rate of the mean income of those below x_P. Importantly, the wording can lead to misinterpretation, since it is also not the mean growth rate of those individuals who were below x_P in period $t-1$. Additionally, since the GIC is based purely on the two marginal distributions in t and $t-1$: the growth rate of the pth percentile, $\frac{x_t(p)}{x_{t-1}(p)} - 1$, does not refer to the growth rate between t and $t-1$ of the individual at the pth percentile in $t-1$. This latter dimension was addressed by Bourguignon (2011) and Jenkins and Van Kerm (2016), but without changing the nature of the GIC device.

2.3 Income Growth Profiles

Jenkins and Van Kerm (2016) define Income Growth Profiles, IGPs, which are similar to those developed by Van Kerm (2009) and Bourguignon (2011).[7] However much of their attention is addressed to assessing the welfare dominance properties of individual income growth based on an adaptation of the Atkinson and Bourguignon (1982) social welfare function where individual utilities are based on incomes in both the initial and final periods; see Jenkins and Van Kerm (2016, pp. 681-3). That is, their objective is to produce summary indices of income growth with consistent welfare foundations that are helpful for normative evaluations of alternative distributions of individual income growth.

Their objective is therefore rather different from the positive measurement or description of income mobility (growth) properties pursued in the present paper. Nevertheless, their profiles capture two properties that are similar to the TIM curves developed in Section 3. Firstly, the IGP involves plotting a measure of average income growth, $m(p)$, for the pth percentile, against p, where in their case $m(p)$ is a conditional expectation-based measure amenable to social welfare comparisons. The IGP bears a close resemblance to the Ravallion and Chen (2003) growth incidence curve, but is based on a longitudinal mobility concept, where the definition of $m(p)$ captures more than just initial incomes, $x(p)$. Secondly, Jenkins and Van Kerm (2016) propose a cumulative version of the IGP (a 'CIGP') in which a measure of average income growth for those with initial incomes below $x(p)$, $(\frac{1}{p}\int_0^p m(q)dq)$, is plotted against p.

Jenkins and Van Kerm (2016, p.685) suggest that the CIGP 'plots areas below the income growth profile – analogous to the way that a generalized Lorenz curve shows areas below a quantile function. The slope of the cumulative income growth profile may be positive or negative at different values of p'. As Section 3 shows, a closely related illustrative device to the CIGP (but without addressing social welfare properties) can readily be deployed to illustrate the 'three "I"s' distributional dimensions.

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[7]See also Grimm (2007).

3   The TIM Curve

This section begins in subsection 3.1 by summarising the key aspects of the TIP curve developed by Jenkins and Lambert (1997), It is then adapted in the income mobility context in subsection 3.2.

3.1   The TIP Curve

Jenkins and Lambert (1997) demonstrated that three important dimensions of poverty can be summarised by their TIP curve. These are: the *incidence* of poverty, as captured by the headcount poverty measure; the '*intensity*', as measured by the income gap, $x_p - x_i$, where x_p is the poverty line; and the '*inequality*' of poverty within the poor group, capturing how far the incomes of the poorest differ from those closer to the threshold, x_p.

Let x_i denote individual i's income, with $i = 1, ..., n$. Given x_p, the poverty gaps are defined by $g(x_i) = 0$ for $x_i > x_p$ and $g(x_i) = x_p - x_i$ for $x_i < x_p$. When incomes are ranked in ascending order, the TIP curve is obtained by plotting $\frac{1}{n} \sum_{i=1}^{k} g(x_i)$ against $\frac{k}{n}$, for $k = 1, ..., n$. That is, the total cumulative poverty gap per capita is plotted against the associated proportion of people.

A hypothetical example is shown in Figure 2. The slope at any point is equal to the average poverty gap, with a steeper slope indicating a larger poverty gap. Flattening of the curve therefore shows the extent to which the average poverty gap falls as income rises towards x_p. Thus, inequality among the poor is reflected in the curvature of the TIP curve. The curve becomes horizontal beyond H, since this fraction of the population is not in poverty, given a prior choice of p or $x(p)$. Poverty can be said to be unambiguously higher where a TIP curve lies wholly above and to the left of an alternative TIP curve.

3.2   Three "I"s of Mobility

To define the TIM curve, first define the logarithm of income, $y_i = \log x_i$, for individuals $i = 1, ..., n$. Hence $y_{i,t} - y_{i,t-1}$ is (approximately) person i's
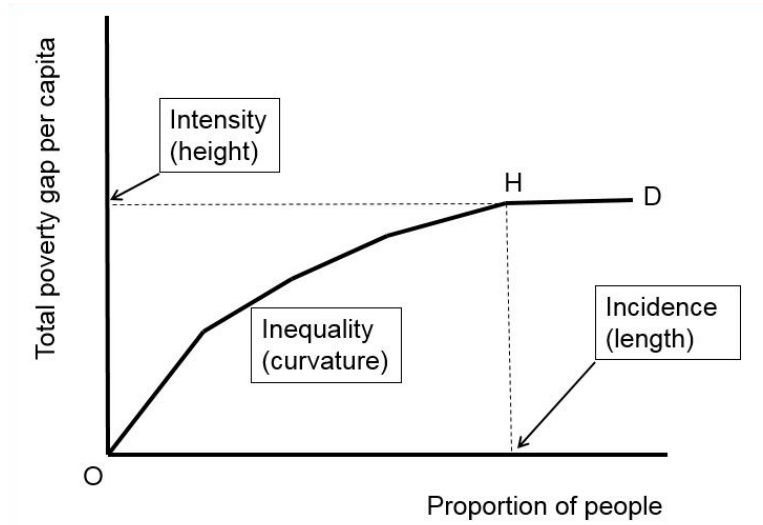
Figure 2: A TIP Curve

proportional change in income from period $t-1$ to t. With log incomes ranked in ascending order, plot $\frac{1}{n} \sum_{i=1}^{k} \left(y_{i,t} - y_{i,t-1} \right)$ against $\frac{k}{n}$, for $k = 1, ..., n$. Thus the TIM curve plots the cumulative proportional income change per capita against the corresponding proportion of individuals. One difference from the IGP curve, but shared with the TIP curve, is that the measure of mobility intensity on the vertical axis is obtained by dividing by n rather than k. This modification produces an alternative curve with valuable properties in term of its ability readily to illustrate important characteristics of mobility for specified population groups.

The 'Three Is' properties of poverty and the TIP curve have natural analogues in the context of income mobility. First, it is possible to consider the mobility of a particular group of low-income individuals. Thus consider those with incomes below $x(h)$, that is, for the proportion, h, of the population. In this framework h captures the *incidence* of the particular group of concern; just as the headcount poverty measures the incidence of poverty – the proportion below x_P. The TIM curve also reflects the *intensity* and *inequality* dimensions, analogous to the TIP curve, based on the 'mobility as income growth' concept.

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The TIM curve can be examined more formally as follows, ignoring i subscripts for convenience. Suppose incomes are described by a continuous distribution where $H(x_t)$ and $F(y_t)$ denote respectively the distribution functions of income and log-income at time t, with population size, n. For incomes ranked in ascending order, the TIM curve plots the cumulative proportional income changes, $y_t - y_{t-1}$, per capita, denoted $M_{h,t}$, against the corresponding proportion of people, h, where:

$$h = F(y_{h,t-1}) \tag{13}$$

Thus $y_{h,t-1} = F^{-1}(h)$ is the log-income corresponding to the hth percentile. Hence, the TIM curve plots $M_{h,t}$, given by:

$$M_{h,t} = \int_0^{y_{h,t-1}} (y_t - y_{t-1}) \, dF(y_{t-1}) \tag{14}$$

against h.[8]

Let μ_t denote the arithmetic mean of log-income (that is, the logarithm of the geometric mean, G_t, of income, x_t. Then equation (14) can be written as:

$$M_{h,t} = \int_0^{y_{h,t-1}} \left\{ (y_t - \mu_t) - (y_{t-1} - \mu_{t-1}) \right\} dF(y_{t-1}) + (\mu_t - \mu_{t-1}) F(y_{h,t-1}) \tag{15}$$

The term, $y_t - \mu_t$ is equal to $\log(x_t/G_t)$. Hence $(y_t - \mu_t) - (y_{t-1} - \mu_{t-1})$ is the proportional change in *relative* income. Thus, $M_{h,t}$ consists of the cumulative proportional change in income *relative* to the geometric mean, *plus* a component that depends only on the proportional change in geometric mean income.

Suppose the proportional change in the geometric mean, $\mu_t - \mu_{t-1}$, is equal to g. Furthermore, suppose the proportional change in relative income depends on income in $t - 1$, so that $(y_t - \mu_t) - (y_{t-1} - \mu_{t-1})$ can be written

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[8]For very large datasets it is convenient to plot values of the cumulative proportional change corresponding to percentiles, P_j, for $P_1 = 0.01$ and $P_j = P_{j-1} + 0.01$, for $j = 2, ..., 100$. Thus, obtain the cumulative sum $M_j = \frac{1}{n} \sum_{i=1}^{nP_j} (y_{i,t} - y_{i,t-1})$, where as above n is the number of individuals in the sample. Hence for $j = 2, ..., 100$: $M_j = M_{j-1}/n + \frac{1}{n} \sum_{i=nP_{j-1}+1}^{nP_j} (y_{i,t} - y_{i,t-1})$. The TIM curve is then plotted using just 100 values.

as the function, $g^*(y_{t-1})$. Then (15) can be expressed as:

$$M_{h,t} = \int_0^{y_{h,t-1}} g^*(y_{t-1}) \, dF(y_{t-1}) + gh \qquad (16)$$

If all individuals receive exactly the same relative income change, then relative positions are unchanged and $g^*(y_{t-1}) = 0$ for all y_{t-1}. Hence, $M_{h,t}$ plotted against h is simply a straight line through the origin with a slope of g. This means that the extent to which it is equalising or disequalising over any range of the income distribution can be seen immediately by the extent to which the TIM curve deviates from a straight line, which in turn depends on the properties of $g^*(y_{t-1})$. Appendix A considers a special case where $(y_t - \mu_t)$ is a linear function of $(y_{t-1} - \mu_{t-1})$, reflecting a systematic equalising tendency, and a random component: this is the regression to the (geometric) mean discussed briefly in subsection 2.1.

A hypothetical example of a TIM curve is shown in Figure 3. The particular curve illustrated, plotting $h = k/n$ on the horizontal axis, reflects a situation in which relatively lower-income individuals receive proportional income increases which are greater than that of average (geometric mean) income. Hence the TIM curve, OHG, lies wholly above the straight line OG.

If all incomes were to increase by the same proportion, the TIM curve would be the straight line OG. The height, G, indicates the average growth rate of the population as a whole, with the height, H, indicting the average growth rate for those below $x(h)$. Furthermore, just as with the TIP curve, inequality is reflected in the degree of curvature. For example, the curvature of the arc OH relative to the straight line OH indicates that lower income individuals have higher (more unequal) growth than those individuals to the left of, but closer to, h. If concern is for those below a poverty line, x_P, the corresponding percentile is $h_P = F(x_P)$, where, as defined above, $F(x)$ is the distribution function of x. The TIM curve gives an immediate indication of whether income changes have been pro-poor.

Suppose interest is focussed on those below the hth percentile, indicated in Figure 3. There is less 'inequality of mobility' - that is, less interpersonal dispersion of income changes - among the group below h, shown by the fact that the TIM curve from O to H is closer to a straight line than the complete
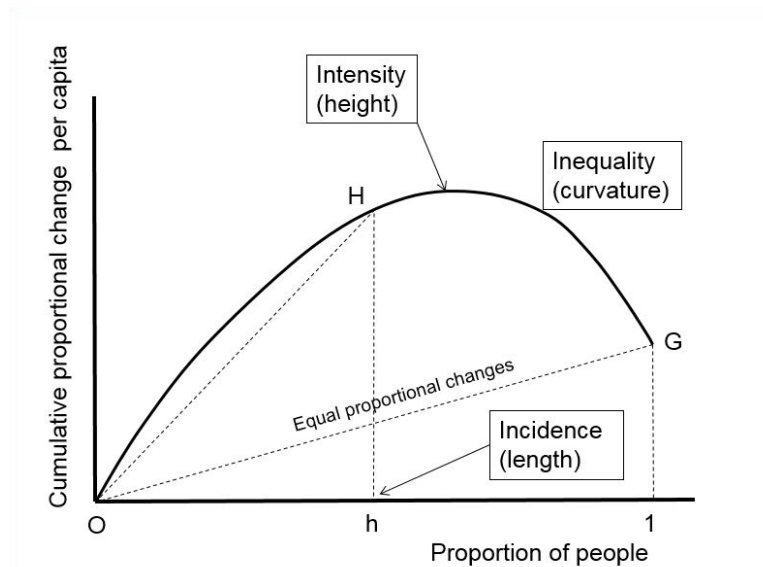
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Figure 3: A TIM Curve

curve OHG.[9] The TIM curve shows that the income growth of those below h is larger than that of the population as a whole. The average growth rate among the poor (the intensity of their growth) is given by the height, H.

Figure 4 illustrates a TIM curve reflecting a very different pattern of mobility. In this case the lower-income groups experience smaller proportional increases in income than those with higher incomes. If h_p is to the right of the intersection of the TIM curve with the line OG, average growth of those in poverty exceeds overall growth. Yet, in a manner analogous to the TIP curve, the TIM curve readily demonstrates that this reflects quite different experiences among the poor.

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[9]There is a potential ambiguity in the use of the term 'inequality' here since the TIP curve refers to a cross-sectional distribution whereas the TIM curve refers to income changes. To avoid confusion over nomenclature, when referring to the 'inequality dimension' of mobility (one of the three 'I's), the term 'interpersonal dispersion' of mobility will instead be used. The term 'inequality' is henceforth used only in reference to the inequality of incomes in a cross-sectional distribution, unless otherwise stated.
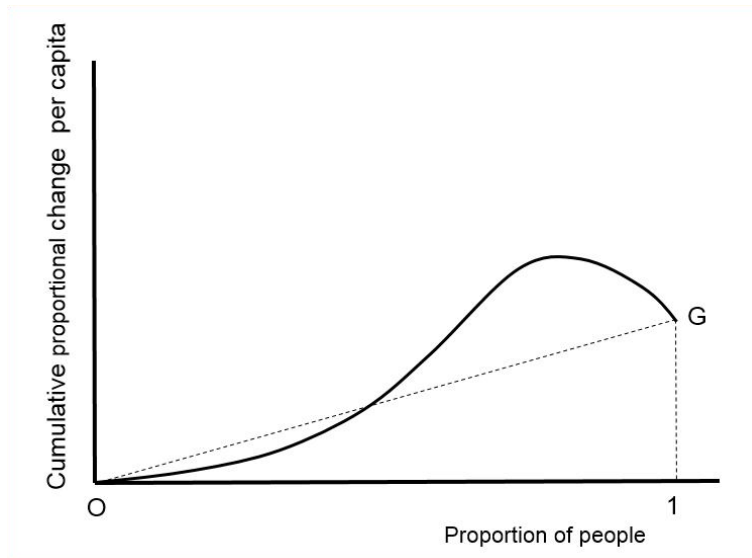
Figure 4: Example of Differential Mobility Pattern

4 New Zealand TIM Curve Examples

This section illustrates the TIM curves based on data for a 2% random sample of individual New Zealand Inland Revenue personal income taxpayers. Using data for 1998, 2002, 2006 and 2010, three separate panels were obtained for 1998-2002, 2002-06 and 2006-10, each (5-year) panel containing incomes for both years for the same taxpayers.

To avoid the exercise being contaminated by taxpayers with very low incomes (such as small part-time earnings of children, or small capital incomes of non-earners), individuals with annual incomes less than $1,000 were omitted from the sample. This yielded usable samples of 29,405, 31,355 and 32,970 individuals respectively for the three five-year panels. In each case individuals were ranked by their initial year incomes, with all of the diagrams below showing percentiles of the income distribution in the relevant initial year (1998, 2002, or 2006) on the horizontal axis.

Figure 5 shows three TIM curves corresponding to the three five-year periods (or four years of income growth). Growth rates shown on the vertical axis are measured over the entire period. The right-hand end of the TIM
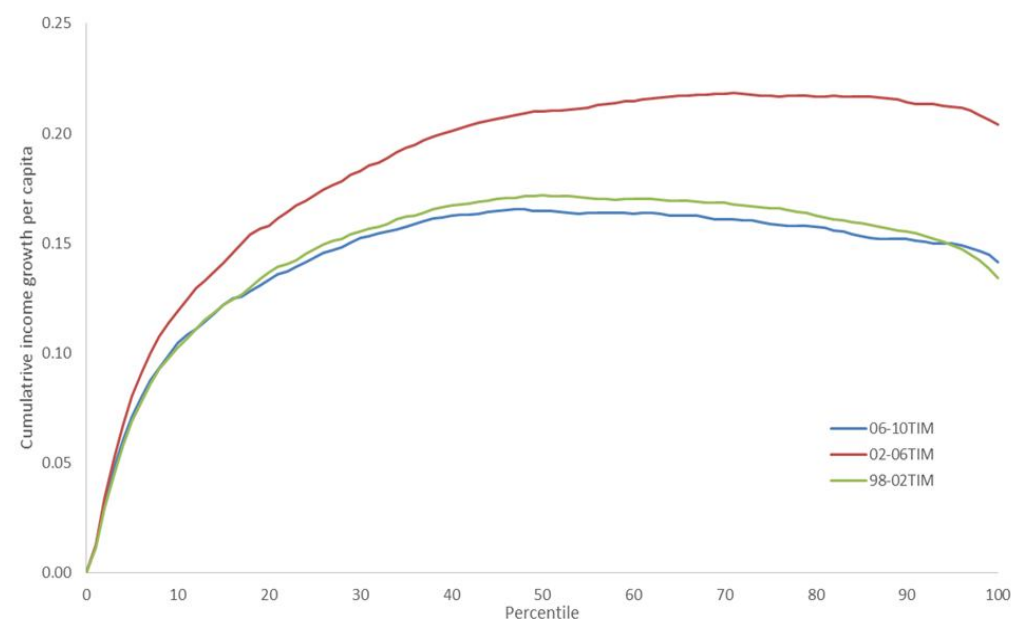
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Figure 5: Three TIM Curves for New Zealand

curve represents the average growth rate (over the five years) across all n individuals. While these growth rates were very similar, at around 15% over 1998 to 2002 and 2006 to 2010, it can be seen that growth was higher on average, around 20%, over the period 2002 to 2006.

All three curves tend to rise most steeply at the lowest income percentiles and flatten out at higher percentiles, suggesting greater equalising mobility especially among the lower percentiles. Also, if higher average income growth across the whole sample (as in the 2002 to 2006 case), is regarded as indicative of higher mobility on average, then mobility is clearly greater across the board in 2002 to 2006 compared to the other two periods. In this case it can be argued that the 2002 to 2006 TIM dominates the other two curves in the sense of indicating unambiguously greater mobility, though the detailed properties of the curves among approximately the lowest 5% are hard to identify on this scale.[10]

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[10]No welfare dominance can be inferred here from these TIM curve positions since this would require establishing a suitable social welfare evaluation framework of the sort developed by Jenkins and Van Kerm (2016) for their IGPs.
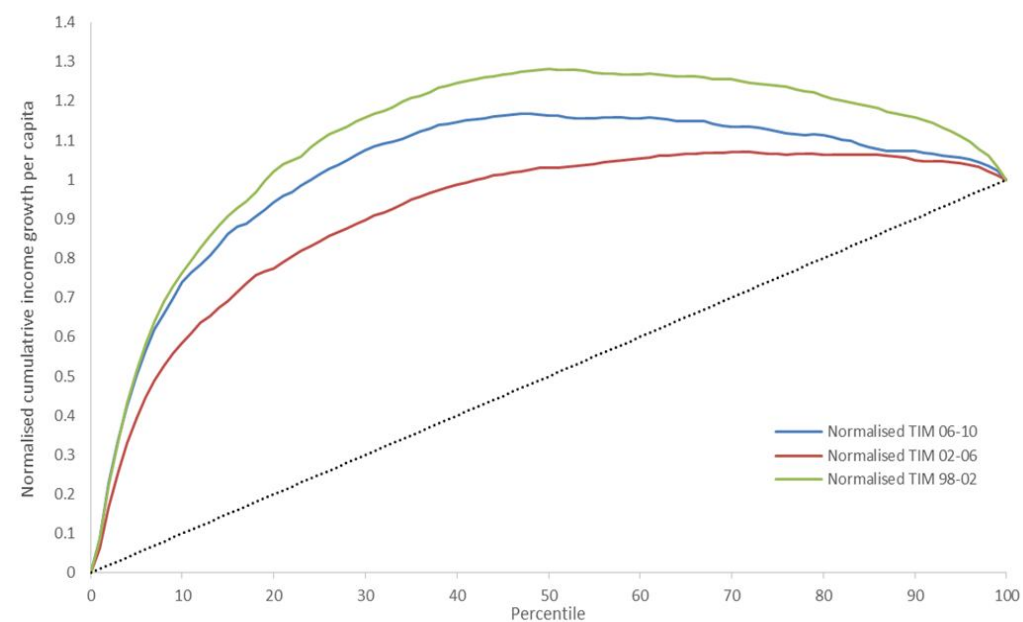
Figure 6: Normalised TIM Curves for New Zealand

If it is preferred to assess mobility from *relative* income growth rates, normalisation of the curves in Figure 5 is required. Figure 6 shows the equivalent 'normalised TIM' curves where each is normalised by the sample average growth rate in each panel, also allowing the curvature of each curve to be more readily compared. This reveals a quite different mobility pattern, with the 2002 to 2006 TIM revealing unambiguously lower mobility than the other two curves for all h. Indeed there would appear to be a clear ranking in the extent of mobility of 1998 to 2002 > 2006 to 2010 > 2002 to 2006. In addition, while similar patterns are evident in all three curves as h increases, clearly the 2002 to 2006 normalised TIM displays less curvature, implying less equalising mobility, at any selected h.[11]

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[11]If however, interest is in the interprersonal dispersion of mobility, *only* for those below h, then normalised TIM curves would be constructed based on average income growth of those below h. The relative curvature of *those* normalised TIM curves would then identify differences in interpersonal dispersion across the samples.

5 Positional Mobility

A widely used class of mobility measures is based on the idea of mobility as 'positional change', rather than relative income change. It is therefore useful to examine whether an equivalent to the TIM curve approach can be helpful in this context. This section considers re-ranking measures of mobility where changes in individuals' positions in the income distribution, rather then their income levels, are the focus of interest. Clearly individuals can move to higher or lower rank positions, and the explicit treatment of the direction of change is examined in detail below. In the following discussion, individuals are ranked in ascending order of incomes, x_i, so that ranks $i = 1, ..., n$ are for individuals from the lowest to the highest income. If the initial period is denoted 0, then define the initial ranks, $R_{i,0} = i$.

Defining a re-ranking mobility index requires, first, a choice regarding *whose* mobility is to be included. Second, it is necessary to decide whether negative re-ranking (dropping down the ranking) is treated symmetrically with positive re-ranking (upward movement within the ranking). Regarding the first choice consider, as in previous sections, the case where it is desired to measure the extent of mobility of a subset of individuals, $k \leq n$, with the lowest initial incomes.

On the treatment of positive and negative re-ranking, let $\Delta R_i = R_{i,1} - R_{i,0} = R_{i,1} - i$ denote the change in the rank order of the person who initially has rank, i. Three further options are possible, all related to how negative re-ranking is treated. Firstly, negative re-ranking could be treated symmetrically with positive re-ranking such that positional mobility is defined in *net* terms, that is, positive changes in rank net of any negative changes within group $i = 1, ..., k$.[12] This is referred to as '*net* re-ranking'. Secondly, negative movement in the ranking could be ignored, which simply involves setting $\Delta R_i = 0$ when $\Delta R_i < 0$. This is referred to as '*gross* re-ranking'. Thirdly, re-ranking may be measured in *absolute* terms in which all re-ranking is measured as a positive value. This is referred to as '*absolute* re-ranking'.

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[12]If individual changes in rank are simply aggregated to obtain an aggregate mobility index, then a change in rank of 50 places by one individuals is treated symetrically as 50 individuals each changing one ranking place.

The appropriate choice among these three measures depends on the question of interest. For example, if interest is focussed on those below the poverty line *as a group*, then it may be desired to balance any upward mobility by some of those in poverty with downward (negative) mobility of others in poverty, in order to gain an indication of the net impact on the group. This suggests a focus on net mobility in this case. Likewise, if movement *per se* is the mobility concept of interest, then a non-directional measure such as absolute re-ranking is more relevant. Gross re-ranking allows a focus on only those who are moving up.

The three re-ranking indices for an individual initially having rank order, i, (for $i = 1, \ldots, n$) can be defined as follows:

$$M_i^{net} = \Delta R_i \tag{17}$$

$$M_i^{gross} = \Delta R_i \big|_{\Delta R_i \geqslant 0} \tag{18}$$

$$M_i^{abs} = |\Delta R_i| \tag{19}$$

Aggregated across the k lowest income individuals in period 0, the corresponding aggregate re-ranking indices are then given by:

$$M_k^{net} = \sum_{i=1}^{k} M_i^{net} = \sum_{i=1}^{k} (R_{i,1} - R_{i,0}) \tag{20}$$

$$M_k^{gross} = \sum_{i=1}^{k} M_i^{gross} = \sum_{i=1}^{k} (R_{i,1} - R_{i,0}) \quad \text{for } \Delta R_i \geqslant 0 \tag{21}$$

and

$$M_k^{abs} = \sum_{i=1}^{k} M_i^{abs} = \sum_{i=1}^{k} |R_{i,1} - R_{i,0}| \tag{22}$$

This last absolute re-ranking case may be thought of as describing the extent of overall positional change within the relevant range of the income distribution. Over short periods of time this is often described as volatility, with the term 'income risk' applied to it. When measured over a longer time period it may be regarded as describing the flexibility of the income distribution, with less clear welfare associations.

To examine the 'three Is' properties similar to the TIM curve for the income growth case but based on the indices in (20), (21) and (22), one approach would be to plot the cumulative value of the relevant M_k index against the cumulative fraction of the population, $h = k/n$. However, a person's *opportunity* for re-ranking is partly determined by their initial position in the income ranking: someone among the lowest ranks has less opportunity to move down, other things equal, than someone higher up, and *vice versa*. It is therefore useful to consider the maximum re-ranking possible for each individual; actual re-ranking may then be compared with these maximum values for any given h.

To simplify the exposition, consider a population of $n = 100$ individuals, each with a different income level; hence each integer, $i = 1, ..., n$, represents a percentile of the distribution. They are ranked in period 0, $R_{i,0} = 1 \ldots 100$, representing the lowest to the highest incomes. Two polar cases are the maximum and minimum degrees of mobility possible. The former is defined here as a complete ranking reversal, $\Delta R_i(\text{max})$, such that in period 1, $R_{i,1}$ involves a lowest to highest ranking of $R_{i,1}(\text{max}) = n+1-R_{i,0} = 100, \ldots, 1$.[13]

Similarly, the minimum degree of re-ranking involves no change in the ranks such that $R_{i,1}(\text{min}) = R_{i,0}$ for all i, hence $\Delta R_i = 0$. It can be seen that maximum re-ranking implies:

$$M_i(\text{max}) = \Delta R_i(\text{max}) = R_{i,1}(\text{max}) - R_{i,0} = n + 1 - 2R_{i,0} \qquad (23)$$

which, for large n, can be approximated by $n - 2R_{i,0}$. Where it is desired to measure the extent of re-ranking of the subset of individuals, $k \leq n$, with the lowest incomes, the aggregate maximum re-ranking index for the net mobility

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[13]An alternative argument proposes that the relevant comparator should be defined as when the change in an individual's position in the ranking is purely random; see Jantti and Jenkins (2015; pp.8-9). That is, 'maximum' mobility involves independence from initial positions, rather than complete reversals. In that case, given $R_{i,0}$, maximum mobility requires an actual ordering in period 0 to be compared with a random ordering in period 1. Jantti and Jenkins reject the use of 'maximum' when mobility is based on origin independence because, they suggest, "it is difficult to argue that origin independence represents 'maximum' mobility in the literal sense".

case , $M_k^{net}(\max)$, is given by:

$$M_k^{net}(\max) = \sum_{i=1}^{k} M_i^{net}(\max) = \sum_{i=1}^{k}(n + 1 - 2R_{i,0}) \qquad (24)$$

Using the sum of an arithmetic progression:

$$\sum_{i=1}^{k} R_{i,0} = 1 + 2 + 3 + \ ... \ + k = \frac{k(k+1)}{2} \qquad (25)$$

equation (24) becomes:

$$\begin{aligned} M_k^{net}(\max) &= \sum_{i=1}^{k}(n+1-2R_{i,0}) = k(n+1) - k(k+1) \\ &= k(n-k) \qquad (26) \end{aligned}$$

Hence, in the $n = 100$ example above, if interest focuses only on the poorest individual ($k = 1$), maximum re-ranking is given by $M_k^{net}(\max) = (100 - 1) = 99$; when $k = 2$, $M_k^{net}(\max) = 2(100 - 2) = 196$; and so on. More generally, since maximum re-ranking (complete ranking reversal) involves all those below the median individual changing positions with those above the median, it follows from (26) that the maximum value of $M_k^{net}(\max)$ as k increases is obtained for $k = n/2$, yielding $M_k^{net}(\max) = n^2/4$.[14]

This measure is therefore not 'scale independent': larger populations imply larger $M_k^{net}(\max)$ and M_k^{net}. It could be 'normalised' to create a form of per capita index by dividing by n^2 such that the index becomes: $m_k^{net}(\max) = h(1 - h)$. The maximum value is reached at $h = 0.5$, where $m_k^{net}(\max) = 0.25$.

For the gross re-ranking case, $M_k^{gross}(\max)$, the value of $M_k^{gross}(\max)$ also reaches a maximum as k increases of $M_k^{gross}(\max) = n^2/4$ for $k = n/2$, since all individuals below $n/2$ experience positive re-ranking in this (maximum) case. However, above $k = n/2$, as more above-median individuals are included within k, their re-rankings are now given by $\Delta R_i = 0$, such that the cumulative value of $M_k^{gross}(\max)$ remains unchanged as $k \longrightarrow n$.

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[14]Strictly, for small n, the median individual is $k = (n+1)/2$, and $M_k^{net}(\max)$ is given by $(n+1)(n-1)/4$.

Finally, for the absolute re-ranking case in (22), $M_k^{abs}(\max)$, it can be shown that, as with the other cases, this increases as k increases from $k = 1$ to $k = n/2$ to reach $M_k^{abs}(\max) = n^2/4$. However, this represents a point of inflection rather than a maximum, since inclusion of the absolute value of above-median individuals' re-ranking in $M_k^{abs}(\max)$, ensures that $M_k^{abs}(\max)$ continues to increase for $k > n/2$, reaching $M_k^{abs}(\max) = n^2/2$ at $k = n$.

5.1 Maximum Re-Ranking profiles

Cumulative profiles for the three maximum re-ranking cases discussed above, where $M_k^{net}(\max)$, $M_k^{gross}(\max)$, and $M_k^{abs}(\max)$ are plotted against $h = k/n$, are illustrated in Figure 7. This uses the sample of $n = 32,970$ New Zealand taxpayers between 2006 and 2010 described in section 4. However, maximum re-ranking profiles, identical in shape to those in Figure 7, could of be obtained for any values of n. The New Zealand values here are helpful when comparing with actual M_k values in the next sub-section.

Figure 7 depicts the cumulative percentile of the population on the horizontal axis and the cumulative value of the re-ranking index on the vertical axis. Hence the maximum value of $M_k^{net}(\max)$ at the 50th percentile is $n^2/4 = 271,155,225$. As noted above these profiles could be 'normalised' by dividing all values by n^2, or by $n^2/4$ to give an index that lies between 0 and 1.

The Figure shows the distinct non-linear shape of the maximum profiles, whichever definition of positional mobility is adopted (net, gross or absolute re-ranking). As expected, the net re-ranking profile displays a parabolic shape which, from differentiation of (26), has a slope of $(n - 2k)$. The equivalent gross re-ranking profile reaches a maximum, as expected, at the 50th percentile and remains constant thereafter, while the absolute re-ranking profile displays a sigmoid shape, reaching a local point of inflection at the 50th percentile but then rising at an increasing rate till the cumulative value of $M_k^{abs}(\max)$ has doubled at $k = n$.
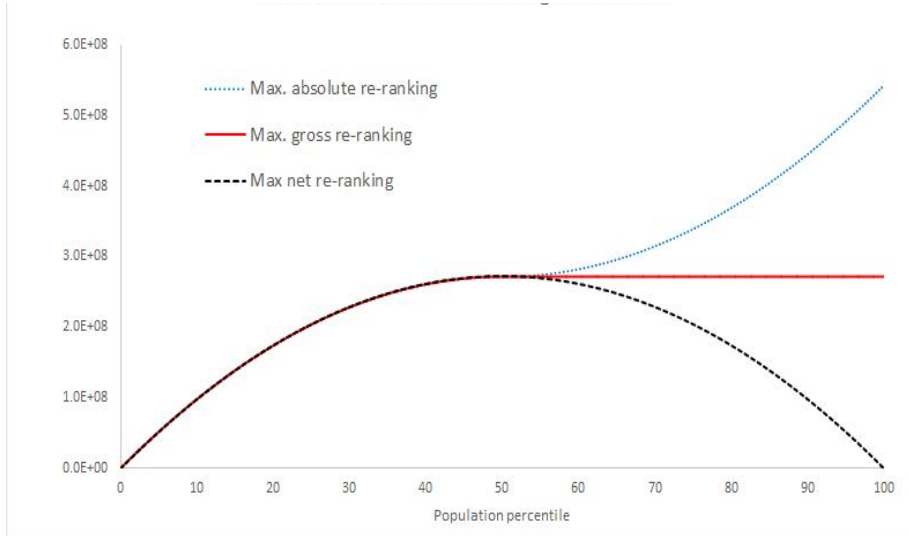
Figure 7: Maximum Re-ranking: Three Cases

6 New Zealand Re-ranking Profiles

An assessment of how much actual positional mobility occurs, and the roles of incidence, intensity and interpersonal dimensions, is facilitated by plotting cumulative M_k profiles equivalent to the $M_k(\text{max})$ profiles in Figure 7. These are shown in Figure 8, which again illustrates the nonlinear nature of the profiles. In each case, these profiles could contain concave, linear or convex segments, reflecting the degree of re-ranking being experienced as k is increased to include higher income individuals. A greater amount of re-ranking tends to generate profiles that are more concave. That is, unlike the poverty TIP curve, but like the TIM curve, greater (concave) curvature implies more-equalising positional mobility. Convexity implies disequalising re-ranking, with neutrality captured by linear segments.

To assess the incidence, intensity and interpersonal aspects of these re-ranking measures, Figure 8 should be interpreted as follows. For a given definition of positional mobility (net, gross or absolute re-ranking), select a value of $h = k/n$ representing the sub-set of low income individuals of interest (the incidence dimension). The point on the profile on the vertical axis at this value of h represents the intensity of re-ranking for this group; namely
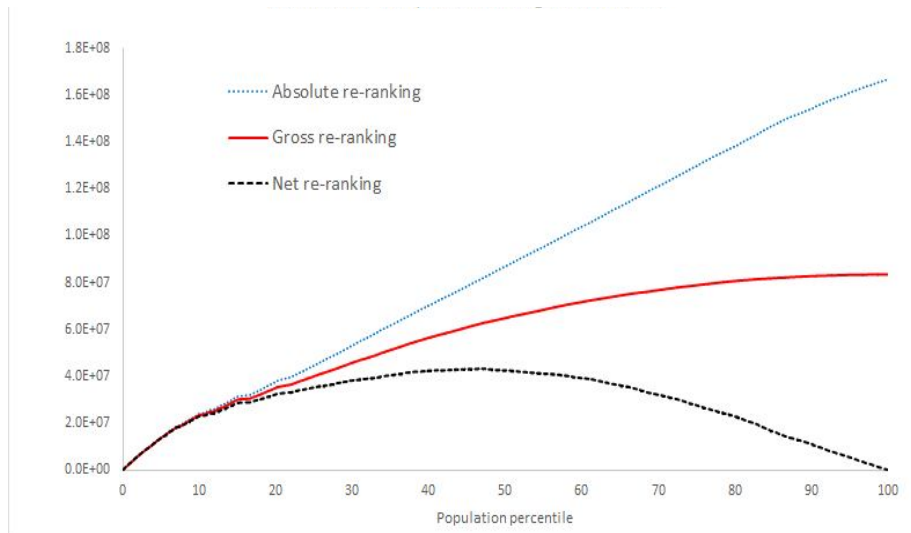
22

Figure 8: Actual Re-ranking: Three Cases

how much re-ranking they have experienced on average (or cumulatively). The section of the profile to the right of h becomes irrelevant, equivalent to the flat section of the TIP curve, to the right of the percentile corresponding to the poverty line in the Jenkins-Lambert (1997) analysis.

The deviation from linearity of the cumulative M_k profile, from the origin to its value at the selected h, provides a measure of the degree of equalising (concave) or disequalising (convex) re-ranking within h. That is, the actual profile may be compared to a straight line from the origin to the value of M_h – the value of M_k at h. In Figure 8 for example, the profile for absolute re-ranking appears to be remarkably linear, at least above the 10th percentile. This suggests that, at least for this sample and measure, the extent of re-ranking is relatively constant across the income distribution.

Of course, as noted above, while some groups may experience higher re-ranking in Figure 8, their movements may be more or less constrained by the maximum re-ranking possible. An alternative means of determining whether some individuals or groups experience more or less mobility than others is, therefore, to compare their actual re-ranking to the maximum re-ranking achievable. The differences between the actual cumulative

M_k, and the relevant $M_k(\text{max})$, can be identified by considering the ratio $M_h/M_h(\text{max})$ as $h \longrightarrow k$. This is referred to below as the 're-ranking ratio', $RRR_h = M_h/M_h(\text{max})$, for which relevant profiles are shown for the three re-ranking measures in Figure 9.
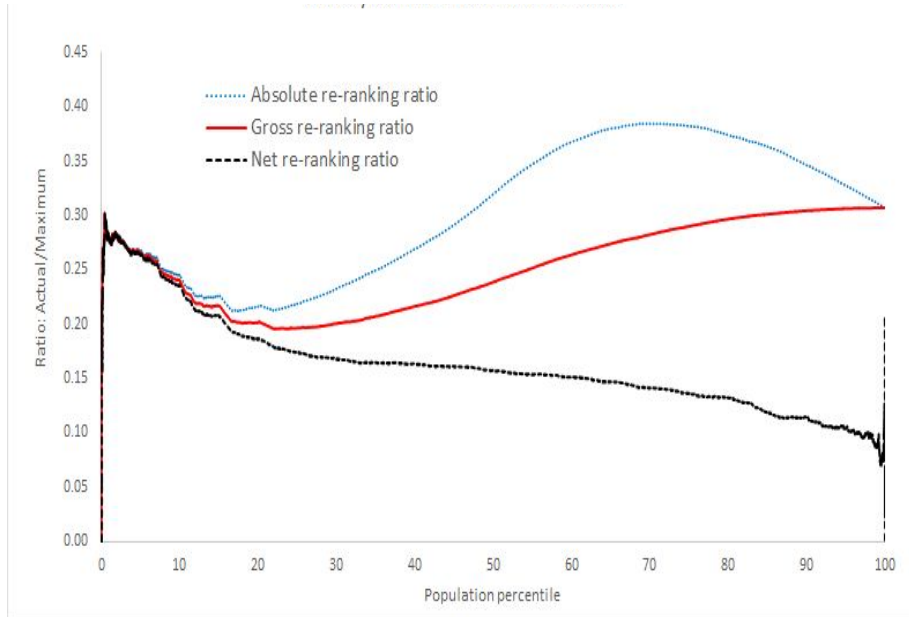

Figure 9: Ratios of Actual to Maximum Re-ranking: Three Cases

This indicates that, for all three re-ranking measures in the New Zealand case, the extent of mobility relative to the maximum achievable, is relatively high for the lowest income individuals (low h), at around $0.25 - 0.3$. This steadily declines, as h is increased, to a minimum of approximately 0.2 at around the 20th to 25th percentile, except in the case of the M_h^{net} profile which continues to decline for $h > 0.2$, though at a somewhat slower rate than for $h < 0.2$.[15] Thereafter, the M_h^{abs} RRR rises to around the 70th percentile (and to the 100th percentile in the case of the M_h^{gross} profile). From this it may be inferred that the group experiencing absolute re-ranking that is

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[15]The strong fluctiations in the M_H^{net} curve as h approches 1, relfect the fact that the value of both the actual and maximum net re-ranking measures equal zero at $h = 1$. Hence the ratio can be quite unstable in the vacinity of $h = 1$. (and is, of course, undefined at $h = 1$).

closest to the maximum achievable are the 'middle income' group between approximately the 50th and 70th percentiles. For gross re-ranking, actual and maximum re-ranking are generally closest for the lowest and highest population percentiles, reaching around $RRR_h^{gross} = 0.3$.

It can also be seen that the RRR_h^{gross} and the RRR_h^{abs} profiles reach the same value for $h = k/n = 1$. This is not a coincidence. It has already been shown that $M_h^{abs}(\max) = n^2/2$, while $M_h^{gross}(\max) = n^2/4$, at $k = n$; that is, $M_{h=1}^{abs}(\max) = 2M_{h=1}^{gross}(\max)$. This same relationship holds for the actual measures: $M_{h=1}^{abs} = 2M_{h=1}^{gross}$. This can be seen by noting that:

$$M_{h=1}^{abs} = \sum_1^n |R_{i,1} - R_{i,0}|$$

However, at $k = n$ the sum of positive ranking movements must equal the sum of negative ranking movements, so that:

$$M_{h=1}^{abs} = 2\sum_1^n (R_{i,1} - R_{i,0})\Big|_{\Delta R_i \geqslant 0} \tag{27}$$

The term after the summation in (27) is simply the gross re-ranking measure, $M_{h=1}^{gross}$. Hence the RRR_h for both the gross and absolute re-ranking measures are equal at $h = 1$.

Considering the three profiles in Figure 9 it is clear that the measure of net movement, RRR_h^{net}, indicates a persistent downward trend as h increases towards 1. This would seem to suggest that the lowest income individuals generally experienced more movement in their incomes (relative to the maximum achievable) over this period than those on higher incomes. It is presumably a re-ranking analogue of the 'regression to the mean' in income levels observed above.

7   Conclusions

Almost two decades ago, Jenkins and Lambert (1997) introduced new insights into the poverty measurement literature by demonstrating that various extant measures of the incidence, intensity and inequality of poverty could be integrated and illustrated by their 'Three "I"s of Poverty' (TIP) curve.

This paper has suggested that, despite a wide range of income mobility concepts and measures available in the mobility literature, these three important dimensions of mobility – incidence, intensity and inequality – are also not readily or simultaneously identifiable from current measures or illustrative devices.

However, based on an analogue of the TIP curve, this paper has proposed that a 'Three "I"s of Mobility', or TIM, curve can provide a useful means of combining and illustrating these three concepts within a single diagram. For income mobility measured as relative income growth, this plots the cumulative proportion of the population (from lowest to highest incomes) against the cumulative change in log-incomes per capita over a given period.

For mobility measures based on positional changes, or the extent of re-ranking of individuals over a given period, it was shown that an equivalent re-ranking mobility curve can illustrate the incidence, intensity and inequality of re-ranking. This plots the cumulative degree of re-ranking against the cumulative proportion of the population (from lowest to highest incomes). Additionally, since for any given fraction of the population there is a maximum possible extent of re-ranking, it is useful to consider the cumulative re-ranking ratio of actual-to-maximum re-ranking against the cumulative proportion of the population.

Illustrations for both of these mobility concepts – relative income growth and re-ranking – were provided based on three panels of New Zealand incomes from 1998 to 2010. These showed that income growth rates within the lower part of the income distribution were quite substantially higher than those observed higher up the income distribution, reflecting in part a relatively high degree of regression towards the mean. Van Kerm (2009) and Jenkins and Van Kerm (2011, 2016) report similar regression to the mean patterns in their income growth profiles for the UK and a selection of other European countries.

Evidence on the extent of re-ranking of individual incomes across a five year period also suggested a relatively high degree of positional mobility, compared to the maximum possible, among the lowest income individuals and also among those around the 50th to 70th percentiles. The evidence also

suggested that some conclusions regarding the extent of re-ranking depends crucially on the re-ranking measure adopted – gross, net or absolute.

With numerous mobility concepts, measures and associated illustrative devices already available in the literature, new diagrammatic methods should be proposed with some caution and demonstrable value added. The suggestion in this paper is that while existing devices for both relative income growth and positional change measures of mobility have various merits, they do not readily capture each of the incidence, intensity and 'inequality' (or interpersonal dispersion) dimensions of mobility in ways that can be easily presented in a single diagram. Yet, as with poverty mesurement, these three dimensions of mobility are frequently of interest when seeking to compare *who* is affected and by *how much*. For this, the TIM and re-ranking ratio curves provide, for income growth and positional change mobility measures respectively, readily constructed and interpreted devices illustrating these 'three Is' properties.

Appendix A: The TIM Curve and Regression to the Mean

It was shown in section 4 that the TIM curve can be written as:

$$M_{h,t} = \int_0^{y_{h,t-1}} g^* (y_{t-1}) \, dF (y_{t-1}) + gh \tag{28}$$

where $g^* (y_{t-1}) = (y_t - \mu_t) - (y_{t-1} - \mu_{t-1})$ represents the proportional change in relative income. A simple special case is to suppose that:

$$g^* (y_{t-1}) = -\gamma (y_{t-1} - \mu_{t-1}) + u_t \tag{29}$$

where u_t is a stochastic term with expected value of zero. For $\gamma > 0$, those with $y_{t-1} > \mu_{t-1}$ experience a systematic relative reduction in income plus a random proportional change. Conversely, those below the geometric mean experience systematic relative income increases. Hence, letting $1 - \gamma = \beta$, it can be seen that:

$$y_t - \mu_t = \beta (y_{t-1} - \mu_{t-1}) + u_t \tag{30}$$

which is the same as (1) above. The extent to which β is less than 1 indicates the degree to which those below the geometric mean experience, on average, a higher relative income increase than those above the geometric mean. This process therefore represents Galtonian regression towards the (geometric) mean. If, instead, $\gamma < 0$, clearly $\beta > 1$ and there is regression away from the geometric mean.

Substituting for $y_t - \mu_t$ from (30) into (15) gives:

$$
\begin{aligned}
M_{h,t} &= \left[(\beta - 1) \int_0^{y_{h,t-1}} (y_{t-1} - \mu_{t-1}) \, dF (y_{t-1}) \right] \\
&+ \left[(\mu_t - \mu_{t-1}) F (y_{h,t-1}) \right] \\
&+ \left[\int_0^{y_{h,t-1}} u_t dF (y_{t-1}) \right]
\end{aligned} \tag{31}
$$

The height of the TIM curve, at any value of h, is thus made up of three components, each contained within square brackets. The first term is $(\beta - 1)$ multiplied by the sum, up to $y_{h,t-1} = F^{-1} (h)$, of the differences between log-income and mean log-income in period $t - 1$ (or the sum of the logarithms

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of relative income, x_t/G_t). The second term is h multiplied by the overall growth rate of (geometric mean) income: this term has a linear profile. The third term is the sum of the stochastic terms. For values of $y_{t-1} < \mu_{t-1}$ (incomes below the geometric mean), the slope of $M_{h,t}$ is positive. A turning point occurs for $y_{h,t-1} = \mu_{t-1}$, after which the slope is negative, since $y_{t-1} > \mu_{t-1}$ and $\beta < 1$.

Consider the component of the TIM curve, $M_{h,t}^R$, say, that reflects only the systematic component of relative income changes, the regression towards the mean. Then:

$$M_{h,t}^R = (\beta - 1) \int_0^{y_{h,t-1}} \left(y_{t-1} - \mu_{t-1}\right) dF\left(y_{t-1}\right) \tag{32}$$

Furthermore, let $F_1(y)$ denote the first moment distribution function of log-income, the proportion of total log-income obtained by those with log-income below y. Hence a graph of $F_1(y)$ plotted against $F(y)$ gives the Lorenz curve of log-income, with $F_1(y) \le F(y)$. Then:

$$M_{h,t}^R = (1 - \beta)\mu_{t-1}\left\{F\left(y_{h,t-1}\right) - F_1\left(y_{h,t-1}\right)\right\} \tag{33}$$

Given that $F(0) = F_1(0)$ and $F(\infty) = F_1(\infty)$, this component of the TIM curve starts and ends at zero. Differentiating:

$$\frac{dM_{h,t}^R}{dF\left(y_{h,t-1}\right)} = (1 - \beta)\mu_{t-1}\left(1 - \frac{dF_1\left(y_{h,t-1}\right)}{dF\left(y_{h,t-1}\right)}\right) \tag{34}$$

The slope of $M_{h,t}^R$ therefore depends on the degree of regression, $1 - \beta$, and the slope of the Lorenz curve of income in $t - 1$ at the corresponding value of $h = F\left(y_{h,t}\right)$. Up to the arithmetic mean of log-income, the slope of the Lorenz curve, dF_1/dF, is less than 1, and above the mean the slope is greater than 1. The curvature is given by:

$$\frac{d^2 M_{h,t}^R}{dF\left(y_{h,t-1}\right)^2} = -(1 - \beta)\mu_{t-1}\frac{d^2 F_1\left(y_{h,t-1}\right)}{dF\left(y_{h,t-1}\right)^2} \tag{35}$$

More regression, resulting from a lower value of β, means that the profile is concave and deviates further from a straight line, and lies everywhere above the profile obtained from a higher β. The maximum height of this component

of the TIM curve is obtained by setting (34) equal to zero, and recognising the well-known property of a Lorenz curve that its slope, $\frac{dF_1\left(y_{h,t-1}\right)}{dF\left(y_{h,t-1}\right)}$, equals 1 at the point on the curve corresponding to the mean, μ_{t-1}.[16] This height is thus equal to:

$$(1 - \beta)\,\mu_{t-1}\left\{F\left(\mu_{t-1}\right) - F_1\left(\mu_{t-1}\right)\right\} \tag{36}$$

The term in curly bracket is clearly positive, given that the Lorenz curve lies below the diagonal of equality, and hence low β is associated with a higher maximum height of the TIM curve. The term in curly brackets is the maximum vertical distance between the Lorenz curve of log-income and the diagonal of equality.

The slope of a ray from the origin to a point on the M_h^R component of the TIM curve is:

$$(1 - \beta)\,\mu_{t-1}\left(1 - \frac{F_1\left(y_{h,t-1}\right)}{F\left(y_{h,t-1}\right)}\right) \tag{37}$$

and this of course is always positive. This slope depends on the extent of regression towards the mean, and on the slope of a ray from the origin to the corresponding point on the Lorenz curve of log-income in $t-1$.

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[16]The tangent to the Lorenz curve corresponding to μ_{t-1} is parallel to the 45 degree line of equality.

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About the Authors

John Creedy is Professor of Public Economics and Taxation at Victoria
Business School, Victoria University of Wellington, New Zealand.
Email: john.creedy@vuw.ac.nz

Norman Gemmell is Professor of Public Finance at Victoria Business School,
Victoria University of Wellington, New Zealand.
Email: norman.gemmell@vuw.ac.nz

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