# Labour Supply Elasticities in New Zealand

John Creedy and Penny Mok

**Working Papers in Public Finance**

VICTORIA
UNIVERSITY OF WELLINGTON
TE WHARE WĀNANGA
O TE ŪPOKO O TE IKA A MĀUI

Chair in Public Finance
Victoria Business School

# Labour Supply Elasticities in New Zealand*

John Creedy and Penny Mok
Victoria University of Wellington and MBIE, Wellington

**Abstract**

The aim of this paper is to explore alternative labour supply elasticity concepts in cross-sectional contexts and to present estimates for New Zealand. Emphasis is placed on the elasticity of hours worked with respect to a change in the gross wage rate, though it is shown that the gross wage elasticity is usually sufficient when considering labour supply responses to effective marginal tax rate changes. The elasticities presented here, for both intensive and extensive margins and for a range of demographic groups, are based on simulated labour supply responses to a proportional change in gross wage rates using the New Zealand Treasury's behavioural microsimulation model, Taxwell-B. This uses a discrete-hours random-utility specification of preferences. Comparisons are made with the only previous estimates for NZ. As for other countries, elasticities at the extensive margin are found to be larger than at the intensive margin.

**JEL Codes**: J2; H3; D2
**Keywords**: Labour supply; gross wage elasticity; behavioural microsimulation

# 1 Introduction

The aim of this paper is to explore alternative labour supply elasticity concepts in cross-sectional contexts and to present estimates for New Zealand.[1] Emphasis is placed on the elasticity of hours worked with respect to a change in the gross wage rate. However, in practical policy debates interest is often concerned with the likely responses of labour supply and hence taxable income to changes in the direct tax structure. Responses to changes in the marginal tax rate, the net wage rate and the gross wage rate are necessarily closely related, and it is shown here that the gross wage elasticity is usually sufficient when considering labour supply responses to effective marginal tax rate changes.

Elasticities have long been ubiquitous in economic analysis following their formal exploration by Alfred Marshall.[2] This is despite the famous dismissive comment by Samuelson (1947, p. 125) that, 'economists have developed a fondness of certain dimensionless expressions called elasticity coefficients. On the whole, it appears that the importance is not very great except possibly as mental exercise for beginning students'.[3] Elasticities are certainly open to abuse. Marshall himself warned against the assumption that elasticities are constant, and the old distinction between 'point' and 'arc' elasticities is, it seems, often neglected.[4]

The context of labour supply presents its own special set of problems, since a wage rate increase involves an increase in both the price of leisure

---

[1]The paper does not consider elasticities with respect to non-wage components of income, though a similar approach could be taken. Furthermore, in view of the single-period perspective, the concept of the Frisch elasticity, which relates to multi-period contexts and the intertemporal elasticity (trading consumption and work over time), is not examined here. For a review of estimates and the use of the Frisch elasticity, see Whalen and Reichling (2017).

[2]On the development of the concept from Cournot and Whewell to Marshall, see Creedy (1992, pp. 24-37).

[3]Samuelson (1947) made the points that elasticities, though without dimensions, are not invariant with respect to origin and scale changes.

[4]On Marshall's warnings regarding constant elasticities, see Creedy (1992, p. 37).

and a rise in 'full income' (equal to the income obtained if all available time is devoted to work). With piecewise-linear budget constraints, the individual's labour supply curve can take a variety of shapes that are much more complex than the long-recognised backward-bending form, with considerable variations in the elasticity over its length.[5] Furthermore, the context gives rise to the necessary distinction between intensive and extensive margins, relating respectively to variations in positive hours and movements between non-work and work. Early reduced-form specifications of labour supply were unable to capture the complexity either of the tax and transfers system or the shape of labour supply curves. However, empirical researchers have for some time used a structural approach in which individuals select from a discrete set of hours levels. Furthermore, utility consists of a deterministic component (allowing for considerable heterogeneity) and a random component reflecting, for example, optimisation errors. Instead of generating a single deterministic number of hours worked, the approach produces, for each individual, a probability distribution over the available hours levels.[6] This kind of approach now forms the basis of a number of behavioural microsimulation models, using representative cross-sectional household surveys.

When a behavioural microsimulation model is available to examine the likely implications of tax and wage changes, it is clear that there is no need to consider elasticities at all. Such models, focussing on the supply side of the labour market, can produce comparative static analyses for a wide range of tax reforms, allowing for considerable population heterogeneity and the complexity of practical tax and transfer system. No direct use is made of elasticities in the simulation process. Yet there is frequently a demand for

---

[5]See the diagrammatic introduction in Creedy (2004).

[6]For an early influential discrete hours analysis, see Van Soest (1995). For an interesting early discrete choice model applied to New Zealand, see Chiao and Walker (1992). However they defined labour market states in terms of income ranges, rather than discrete hours levels. Furthermore they did not have hours information, so had to assume that all workers were working full time (at 40 hours).

summary information about 'the labour supply elasticity', both as a convenient description of labour supply responses and for comparison purposes. Such information cannot be obtained directly from the estimated parameters of the preference functions. It is this demand which motivates the present analysis. The question is not straightforward since the approach facilitates the use of a number of alternative elasticity formulae. Furthermore, in both deterministic and random-utility approaches it is necessary to consider aggregation over specified groups of individuals.

The elasticities discussed here are based on simulated labour supply responses. That is, they are computed by simulating the response (of a specified type of individual or group) to a given change in the gross wage rate. In the case of a group of individuals, each person is given the same proportional increase in the gross wage. Clearly, changing the net incomes of all individuals in a group by the same proportion would be very difficult, since they are unlikely to face the same marginal effective tax rate. Hence it is important to recognise that the resulting elasticities are not parameters in any sense.[7] In particular, for individuals they depend on the number of hours worked and in aggregate they depend on the distribution of hours worked. These hours levels are themselves jointly determined with the effective marginal tax rates, which can vary substantially in piecewise-linear tax and benefit systems. This aspect complicates comparisons over time and between countries.

First, Section 2 explores the relationships among various elasticities. Before presenting empirical results, the precise forms of individual and aggregate elasticities are examined closely. It is important to clarify the different possible approaches, since these do not seem to have been set out in earlier studies. Although the emphasis of this paper is on elasticities obtained from structural models using a random utility component, it is useful, in Section

---

[7]However, in early reduced-form models, elasticities can be expressed as functions of parameters but, except in the unrealistic constant-elasticity case, they depend on a range of other variables.

4

3, to begin by considering the simpler case where hours of work can vary continuously and the utility function is deterministic. Section 4 then presents the case, now more widely used in structural labour supply models, where there is a random component to each individual's utility level and a resulting probability distribution over a limited number of discrete hours levels. Section 5 presents results for New Zealand using the Treasury's behavioural microsimulation model, Taxwell-B. Brief conclusions are in Section 6.

## 2   Relationships Among Elasticities

This section shows how the elasticity of hours worked with respect to a change in the gross wage is related to other elasticities which usually attract more interest in policy discussions, in particular the response to marginal tax rate changes.

The net wage depends on the tax and transfer system. However, if the individual does not move into a different tax bracket as a result of a small gross wage rate change, there is a simple relationship between the two elasticities. In general, let $\eta_{a,b}$ denote the elasticity of $a$ with respect to $b$, then if $h$, $w$ and $w_n$ denote respectively hours worked, gross and net wage rates, it can be seen that $\eta_{h,w} = \eta_{h,w_n} \eta_{w_n,w}$. If the effective marginal tax rate is denoted, $t$, the two wage rates are related simply by:

$$w_n = w\left(1 - t\right) \tag{1}$$

Hence, if the individual remains in the same (effective) tax rate bracket when the gross wage rate changes, it is clear that:

$$\eta_{w_n,w} = 1 \tag{2}$$

Hence, the two wage rates change by the same proportion, and $\eta_{h,w} = \eta_{h,w_n}$.[8]

---

[8]Dandie and Mercante (2007, p. 14) suggest that the gross wage elasticity is expected to be less than the net wage elasticity. However, their example assumes that a change in the marginal tax rate leaves the average tax rate unchanged.

Consider instead a change in the tax rate, where the gross wage is held constant. Clearly, $\partial w_n / \partial t = -w$ and the elasticity of the net wage with respect to the tax rate is:

$$\eta_{w_n,t} = -\left(\frac{t}{1-t}\right) \tag{3}$$

The change in $h$ resulting from this change in $t$ is given by the following relationship between elasticities:

$$\eta_{h,t} = \left(\eta_{h,w_n}\right)\left(\eta_{w_n,t}\right) \tag{4}$$

Hence, using (3):

$$\eta_{h,t} = -\left(\frac{t}{1-t}\right)\eta_{h,w_n} \tag{5}$$

and, from above (using $\eta_{w_n,w} = 1$), $\eta_{h,w_n}$ can be replaced with $\eta_{h,w}$. Furthermore:

$$\eta_{h,t} = -\left(\frac{t}{1-t}\right)\eta_{h,1-t} \tag{6}$$

so that:

$$\eta_{h,1-t} = \eta_{h,w} \tag{7}$$

And since gross earnings, $z$, are simply $z = wh$, $\eta_{z,1-t} = \eta_{h,1-t}$ and:

$$\eta_{z,1-t} = \eta_{h,w} \tag{8}$$

The labour supply component of the elasticity of taxable income, $\eta_{z,1-t}$, is equal to the gross wage elasticity of hours worked.

The result may be used in a more general examination of the elasticity of taxable income, $\eta_{z,1-t}$, by supposing that a proportion, $\alpha$, of income is concealed from the tax authorities.[9] Hence:

$$z = wh\left(1-\alpha\right) \tag{9}$$

---

[9]This approach is suggested by Gemmell and Hasseldine (2014).

A change in the marginal tax rate can lead to a change in $\alpha$. Suppose also that a change in the tax rate can, following some kind of bargaining process, lead to a change in the gross wage. Then totally differentiating (9):

$$dz = \left( \frac{\partial z}{\partial w} + \frac{\partial z}{\partial h} \frac{\partial h}{\partial w} \right) dw + \frac{\partial z}{\partial h} dh - whd\alpha \qquad (10)$$

Hence:

$$\frac{t}{z} \frac{dz}{dt} = \left( \frac{w}{z} \frac{\partial z}{\partial w} + \frac{h}{z} \frac{\partial z}{\partial h} \frac{w}{h} \frac{\partial h}{\partial w} \right) \frac{t}{z} \frac{dw}{dt} + \frac{h}{z} \frac{\partial z}{\partial h} \left( \frac{t}{h} \frac{dh}{dt} \right) - \frac{\alpha}{1-\alpha} \left( \frac{t}{\alpha} \frac{d\alpha}{dt} \right) \qquad (11)$$

However, $\frac{w}{z} \frac{\partial z}{\partial w} = 1$ and $\frac{h}{z} \frac{\partial z}{\partial h} = 1$. Hence, where $\eta'$ indicates a 'partial elasticity':

$$\eta_{z,t} = \left( 1 + \eta'_{h,w} \right) \eta_{w,t} + \eta_{h,t} - \frac{\alpha}{1-\alpha} \eta_{\alpha,t} \qquad (12)$$

The elasticity of taxable income is defined in terms of changes in the net-of-tax rate, $1 - t$. Hence, using (6):

$$\eta_{z,1-t} = \left( 1 + \eta'_{h,w} \right) \eta_{w,1-t} + \eta_{h,1-t} - \frac{\alpha}{1-\alpha} \eta_{\alpha,1-t} \qquad (13)$$

It has been seen above that $\eta_{h,1-t}$ can be replaced with $\eta_{h,w}$, so that, as $\eta'_{h,w} = \eta_{h,w}$:

$$\eta_{z,1-t} = \left( 1 + \eta_{h,w} \right) \eta_{w,1-t} + \eta_{h,w} - \frac{\alpha}{1-\alpha} \eta_{\alpha,1-t} \qquad (14)$$

If there is no change in the wage rate, and no change in the proportion of income concealed, this reduces to the simple expression in (8).

# 3   Deterministic Utility: Continuous Hours

This section examines individual and aggregate elasticities of hours worked with respect to a change in the gross wage rate, in the case where, for any individual with a given wage rate and working a given number of hours which are allowed to vary continuously, there is a unique elasticity.

## 3.1 Individual Elasticities

### 3.1.1 The Intensive Margin

Let $\eta_i$ denote the labour supply elasticity for individual $i$, defined as the proportional change in hours worked divided by the proportional change in the gross wage rate. If a dot above a variable indicates proportional changes, and if $h_i$ and $w_i$ denote respectively $i$'s hours of work and gross wage rate, then from the standard definition of an elasticity:

$$\eta_i = \frac{\dot{h}_i}{\dot{w}_i} \tag{15}$$

In a microsimulation context the calculation of elasticities involves carrying out a simulation applying a small proportional (gross) wage change for each individual. Letting initial and post-change hours be $h_{i,0}$ and $h_{i,1}$ respectively, then it is possible to write:

$$h_{i,1} = h_{i,0}\left(1 + \dot{w}_i\eta_i\right) \tag{16}$$

This type of relationship only applies to wage changes which are genuinely 'small'.

### 3.1.2 The Extensive Margin

A conventional elasticity is not defined for those whose initial hours level is zero, since $\dot{h}_i$ is infinitely large when $h_{i,0} = 0$. However, when considering groups of individuals, and in the random utility context, it will be seen that a form of participation elasticity can be defined.

## 3.2 Average and Aggregate Elasticities for Groups

This section turns to the problem of obtaining 'representative' elasticities for specified groups of individuals. Some investigators report elasticity values for particular individuals, such as those with the arithmetic mean wage rate

of those in the group. However, the present approach considers some form of average or aggregate.[10]

### 3.2.1 The Intensive Margin

For a group of $N$ workers, each with a labour supply elasticity of $\eta_i$ as defined in (15) above, suppose the same proportional wage change is imposed on each person, so that $\dot{w}_i = \dot{w}$ for all $i$. Define the average value for the group, $\bar{\eta}$, using:

$$\bar{\eta} = \frac{1}{N} \sum_{i=1}^{N} \eta_i \tag{17}$$

This can be written as:

$$\bar{\eta} = \frac{1}{N\dot{w}} \sum_{i=1}^{N} \dot{h}_i \tag{18}$$

Alternatively, define the aggregate elasticity, $\eta_A$, in terms of changes in average hours, using:

$$\eta_A = \frac{1}{\dot{w}} \left( \frac{\frac{1}{N} \sum_{i=1}^{N} h_{i,1}}{\frac{1}{N} \sum_{i=1}^{N} h_{i,0}} - 1 \right) \tag{19}$$

and remembering that in this case there is just one (continuous) hours level, $h_{i,0}$, before the wage change and one level after the wage change. Using (16) this becomes:

$$\eta_A = \frac{1}{\dot{w}} \left( \frac{\frac{1}{N} \sum_{i=1}^{N} h_{i,0} \left( 1 + \dot{w}\eta_i \right)}{\frac{1}{N} \sum_{i=1}^{N} h_{i,0}} - 1 \right) \tag{20}$$

so that:

$$\eta_A = \sum_{i=1}^{N} \left( \frac{h_{i,0}}{\sum_{i=1}^{N} h_{i,0}} \right) \eta_i \tag{21}$$

In this case $\eta_A$ is an hours-share-weighted sum of the individual values. This clearly differs from the simple arithmetic mean, $\bar{\eta}$, given in (17).[11]

---

[10]For convenience, sample weights are not included in the following presentation.

[11]Dandie and Mercante (2007, p. 15) briefly refer to, 'several ways in which a "representative" elasticity could be obtained – for example from the average aggregate response of the population group ... or as the average of the elasticities of all the individuals in the population group. ... These concepts are slightly different'. However, they do not explore the relationship between the two, as here.

### 3.2.2 Non-workers

When considering groups of individuals, it is possible to define an elasticity in terms of the proportion of people who are working, that is, for whom $h_i > 0$ before the wage change. For a group of $N$ individuals, the initial proportion working may be written:

$$P_{W,0} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}\left(h_{i,0} > 0\right) \tag{22}$$

A similar proportion, $P_{W,1}$, can be defined following a common proportional gross wage change. An elasticity relating to the change in the proportion working is thus:

$$\eta_W = \frac{1}{\dot{w}} \left( \frac{P_{W,1}}{P_{W,0}} - 1 \right) \tag{23}$$

# 4 Random Utility with Discrete Hours

This section considers elasticities in the kind of structural model widely used in behavioural microsimulation, where utility is written as the sum of a deterministic component and a random component. In addition, there are only $H$ discrete hours levels available for work. The random component is usually taken to be the Type I Extreme Value distribution.[12] The crucial implication is that there is, for each individual, a probability distribution over the available work hours levels.

Letting $U_{i,j}$ denote the deterministic component of person $i$'s utility at hours level, $j$, the probabilities for each hours level are given by:

$$p_{i,j} = \frac{e^{U_{i,j}}}{\sum_{j=1}^{H} e^{U_{i,j}}} \tag{24}$$

Each individual no longer has a standard labour supply curve. For any initial gross wage rate for each individual, there are a number of elasticities,

---

[12]For a detailed introduction to behavioural microsimulation with discrete hours, random utility, models, see Creedy and Kalb (2005).

corresponding to the different hours levels before and after the wage change. The challenge is to summarise these using some kind of average. Two broad approaches to simulation are possible. In the first approach considered below, no use is made of the expression above. Instead a statistical probability distribution of hours worked for each individual after the wage change is obtained, using sets of random draws from the Type I Extreme Value distribution. A distinguishing feature of this approach is that only sets of random draws are used which produce optimal pre-change hours equal to the actual discretised hours worked: this is referred to as the calibration approach.

The second, non-calibration, approach uses probability distributions for both initial and post-change wage rates. These distributions can be obtained statistically using sets of random draws, or they can make use of the explicit formulae in (24). The latter approach can be applied to (virtually) all individuals, irrespective of whether their actual hours worked before the imposed wage change were positive or zero, since in the majority of cases the arithmetic mean hours are positive. When using the terms workers and non-workers in the titles below, these refer to the actual (that is, observed) discretised labour market state of individuals, that is, before the imposed wage change.

## 4.1 The Calibration Approach: Workers

With calibration, $K$ sets of random draws (each set consisting of $H$ draws for the hours levels available) are chosen to ensure that the individual's optimal hours in the initial situation correspond to the actual discretised hours. That is, the initial observed hours are set equal to the nearest discrete hours point, from the $H$ hours levels available.[13] In the process of generating $K$ such sets, any sets of draws which do not place the individual at the observed hours are

---

[13]For couples, a joint utility function is maximised. With $H_M$ and $H_F$ discrete hours levels for male and female partners respectively, optimisation must search over the total of $H_M H_F$ hours combinations.

rejected.[14] Suppose initial hours for individual $i$ are denoted $h_{i,0}$. A wage increase of $\dot{w}_i$ is imposed and a set of $K$ new optimal hours levels, $h_{i,1,j}$ for $j = 1, ..., K$, are obtained. For each $j$ there is an elasticity, $\eta'_{i,j}$, given by:

$$\eta'_{i,j} = \left(\frac{1}{\dot{w}_i}\right) \frac{h_{i,1,j} - h_{i,0}}{h_{i,0}} \tag{25}$$

An average value, $\eta'_{i,\cdot}$, is obtained as:

$$\eta'_{i,\cdot} = \frac{1}{K} \sum_{j=1}^{K} \eta'_{i,j} \tag{26}$$

Substituting from (25), the average in (26) can be written as:

$$\eta'_{i,\cdot} = \left(\frac{1}{\dot{w}_i}\right) \left(\frac{1}{K}\right) \sum_{j=1}^{K} \left(\frac{h_{i,1,j}}{h_{i,0}} - 1\right) \tag{27}$$

An alternative equivalent approach would be to count the frequencies at each discrete hours point and use these. For example, let the frequencies be denoted $\hat{p}_j$, for $j = 1, ..., H$. The $H$ elasticities associated with each hours level are given by:

$$\eta^D_{i,j} = \left(\frac{1}{\dot{w}_i}\right) \frac{h_j - h_{i,0}}{h_{i,0}} \tag{28}$$

for $j = 1, ..., H$. Hence $\eta'_{i,\cdot}$ can also be obtained as:

$$\eta'_{i,\cdot} = \sum_{j=1}^{H} \hat{p}_j \eta^D_{i,j} \tag{29}$$

Alternatively, consider using average hours for person $i$ after the wage increase, denoted $h_{i,1,\cdot}$, where:

$$h_{i,1,\cdot} = \frac{1}{K} \sum_{j=1}^{K} h_{i,1,j} \tag{30}$$

---

[14]In the very few cases where it is not possible, after a large number of sets of random draws, to place the individual at observed hours, the individual's labour supply is held constant.

and define an elasticity, $\eta_i''$, as:

$$\eta_i'' = \left(\frac{1}{\dot{w}_i}\right)\left(\frac{h_{i,1,\cdot}}{h_{i,0}} - 1\right) \tag{31}$$

and:

$$\eta_i'' = \left(\frac{1}{\dot{w}_i}\right)\left(\frac{\frac{1}{K}\sum_{j=1}^{K} h_{i,1,j}}{h_{i,0}} - 1\right) \tag{32}$$

Since $h_{i,0}$ is fixed, it is clear from (27) and (32) that the two approaches give identical results.

## 4.2  A Participation Elasticity

For an individual who is not working in the sample period, it is possible to define a participation elasticity in terms of the change in the probability of working zero hours as the (imputed) wage rate is increased. For such non-workers, it is necessary to impute a wage rate, using estimated wage functions.

## 4.3  Non-Calibration: Workers and Non-workers

In the non-calibration approach to simulation, it is not necessary to retain only the sets of random draws which place the individual at $h_{i,0}$; simulation can simply use the first $K$ sets of draws, giving rise to a set of $h_{i,0,j}$ values for $j = 1, ..., K$. Using similar notation to the above, define the initial average hours level for individual $i$ using:

$$h_{i,0,\cdot} = \frac{1}{K}\sum_{j=1}^{K} h_{i,0,j} \tag{33}$$

rather than a single value $h_{i,0}$. After the change in $w_i$, there is a post-change average hours level, given by the expression in (30). Although the $h_{i,0,j}$ and $h_{i,1,j}$ take only discrete integer values, the averages are of course continuous variables. Then the individual elasticity, $\eta_i^*$, can be defined both for non-workers (since $h_{i,0,\cdot} > 0$) and workers in terms of their average hours as

13

follows:

$$\eta_i^* = \left(\frac{1}{\dot{w}_i}\right) \left(\frac{h_{i,1,\cdot}}{h_{i,0,\cdot}} - 1\right) \tag{34}$$

It is also possible to define the set of $K$ values, for $j = 1, ..., K$, of an elasticity relating to each hours combination in turn:

$$\eta_{i,j}^* = \left(\frac{1}{\dot{w}_i}\right) \left(\frac{h_{i,1,j}}{h_{i,0,j}} - 1\right) \tag{35}$$

However, in this case the elasticity is not defined where $h_{i,0,j} = 0$. The average, $\eta_{i,\cdot}^*$, of these $K$ values is given by:

$$\eta_{i,\cdot}^* = \left(\frac{1}{\dot{w}_i}\right) \left(\frac{1}{K}\right) \sum_{j=1}^{K} \left(\frac{h_{i,1,j}}{h_{i,0,j}} - 1\right) \tag{36}$$

However, using:

$$h_{i,1,j} = h_{i,0,j} \left(1 + \dot{w}\eta_{i,j}^*\right) \tag{37}$$

it can be seen, by substituting (37) into (36), that (34) and (36) are equal, and the two approaches again give equivalent results: that is, $\eta_i^* = \eta_{i,\cdot}^*$.

When following the non-calibration approach, it is also possible (instead of using random draws) to use the analytical result given above for the probability distribution of hours worked in the case of the Type I Extreme Value Distribution for the random component. The expression can be used for both the initial and post-change wage rates, using the net incomes in each situation to obtain required utility levels.

## 4.4 Average and Aggregate Elasticities

The previous subsection discussed alternative expressions for individual labour supply elasticities (with respect to gross wage changes), and showed that they are equivalent. The method used therefore depends purely on which is the most convenient from a computational point of view. The present section turns to aggregation in this random utility context.

### 4.4.1 Calibration: Workers Only

In the calibration case considered above, it was shown that an elasticity $\eta'_{i,\cdot}$ is defined for each person. This is an average over available hours, conditional on being at observed discretised hours before the wage change, and can be obtained using either (26) or (31), since they produce identical results. For a specified group, again of $N$ individuals, consider the simple average of these individual values, given by:

$$\bar{\eta} = \frac{1}{N} \sum_{i=1}^{N} \eta'_{i,\cdot} \tag{38}$$

which can be expressed as:

$$\bar{\eta} = \frac{1}{NK} \sum_{i=1}^{N} \sum_{j=1}^{K} \eta'_{i,j} \tag{39}$$

From (27) this can be written:

$$\bar{\eta} = \frac{1}{NK\dot{w}} \sum_{i=1}^{N} \sum_{j=1}^{K} \left( \frac{h_{i,1,j}}{h_{i,0}} - 1 \right) \tag{40}$$

Alternatively, consider an average elasticity defined in terms of average hours. For individual $i$, the hours level before the wage change is $h_{i,0}$ and the average (that is, expected value of) hours after the wage change is equal to $h_{i,1,\cdot} = \frac{1}{K} \sum_{j=1}^{K} h_{i,1,j}$. Define the following averages over all individuals in the group:

$$\bar{h}_0 = \frac{1}{N} \sum_{i=1}^{N} h_{i,0} \tag{41}$$

and:

$$\bar{h}_1 = \frac{1}{NK} \sum_{i=1}^{N} \sum_{j=1}^{K} h_{i,1,j} \tag{42}$$

$$= \frac{1}{N} \sum_{i=1}^{N} h_{i,1,\cdot} \tag{43}$$

Again, supposing there is a common proportional wage change, $\dot{w}$, for all $i$, define the elasticity, $\bar{\eta}_A$, where:

$$\bar{\eta}_A = \left(\frac{1}{\dot{w}}\right)\left(\frac{\bar{h}_1}{\bar{h}_0} - 1\right) \tag{44}$$

which can be written:

$$\bar{\eta}_A = \frac{1}{\dot{w}}\left(\frac{\frac{1}{NK}\sum_{i=1}^{N}\sum_{j=1}^{K}h_{i,1,j}}{\frac{1}{N}\sum_{i=1}^{N}h_{i,0}} - 1\right) \tag{45}$$

The question arises of how this concept compares with the simple average in (38). From the basic definition, and for small changes:

$$h_{i,1,j} = h_{i,0}\left(1 + \dot{w}n'_{i,j}\right) \tag{46}$$

Substituting (46) into (45) and rearranging eventually gives:

$$\bar{\eta}_A = \sum_{i=1}^{N}\left(\frac{h_{i,0}}{\sum_{i=1}^{N}h_{i,0}}\right)\left(\frac{1}{K}\sum_{j=1}^{K}\eta'_{i,j}\right) \tag{47}$$

and noting that $\eta'_{i,\cdot} = \frac{1}{K}\sum_{j=1}^{K}\eta'_{i,j}$, this becomes:

$$\bar{\eta}_A = \sum_{i=1}^{N}\left(\frac{h_{i,0}}{\sum_{i=1}^{N}h_{i,0}}\right)\eta'_{i,\cdot} \tag{48}$$

Hence the aggregate, $\bar{\eta}_A$, is an hours-weighted average of the $\eta'_{i,\cdot}$, which differs from the simple average in (38) above. The discrete hours case is thus similar to the deterministic case in that two aggregate concepts apply, that of the simple average of individual elasticities and an elasticity defined in terms of changes in average hours (over all members of the group), where the latter is equivalent to an hours-weighted average of individual elasticities.

### 4.4.2  Non-Calibration

A similar result can be obtained, replacing $h_{i,0}$ with $h_{i,0,\cdot}$, for the non-calibration case.

# 5  New Zealand Examples

This section reports a number of New Zealand elasticities, using the Treasury's behavioural microsimulation model, Taxwell-B. This model uses estimates of preference functions obtained by Mercante and Mok (2014a), who used pooled cross-sectional *Household Economic Surveys* for 2006 to 2011: for a brief description of the model, see the Appendix below. The elasticities are also based on the pooled HES data for 2006 to 2011: this provides a much larger sample compared with a single year's survey, for which there are typically few sole-parent households.[15] Comparisons are made with earlier elasticities for New Zealand, obtained by Kalb (2010) for 2001, using the earlier Treasury model Taxmod-B. This version of the microsimulation model used preference functions estimated using pooled *Household Economic Survey* data for 1991 to 2001, and reported in Kalb and Scutella (2004).[16] The results are also compared with those obtained for Australia and reported by Kalb (2010), who used the Melbourne Institute Tax and Transfer Simulator (MITTS).[17]

All elasticities are uncompensated wage elasticities. The elasticities were obtained by imposing a proportional gross wage change of 0.10, as in Kalb (2010). Similarly, for comparison with Kalb (2010), wage rates of both partners were increased in the case of couples. In all cases the number of discrete hours levels for individuals is 11 (using five-hour intervals), except for married males who are assumed to have access to just 6 hours levels (using 10-hour

---

[15]The use of pooled data meant that sample weights were not used when obtaining aggregate values. The analysis excluded those aged over 65 years, the disabled, full-time students and the self-employed.

[16]These were the first such elasticities obtained for New Zealand. It has been mentioned that the earlier work by Chiao and Walker (1992), using a discrete choice model, did not produce comparable elasticities. Another earlier work on labour supply, by Maloney (2000), used reduced-form specifications: hence results are not easily compared with those obtained here.

[17]On the MITTS model, see Creedy *et al.* (2002). For Australia, Dandie and Mercante (2007) provide comparisons of various estimates obtained using different methods.

Figure 1: Uncompensated Wage Elasticities: New Zealand 1991-2001 and 2006-2011

intervals). In calibration and non-calibration calculations a total of 100 sets of random draws was used for each individual.

Figure 1 summarises wage elasticities obtained using Taxwell-B, compared with those using Taxmod-B (reported by Kalb, 2010, p. 188, line 1 of Table 8.4). It was stressed earlier that these are in no sense estimates of 'coefficients' and there is no reason to expect them to remain unchanged since they depend in a complex way on preferences, employment levels, the wage rate distribution, the tax and transfer system, and so on. The biggest changes are for single men and sole parents, with the former becoming much smaller and the latter becoming much larger. Results were obtained using non-calibration for all observations.

New Zealand participation and working-hours elasticities, obtained using Taxwell-B, are summarised in Figure 2. The wage elasticities were computed using the non-calibration method. For each person the elasticity was based on arithmetic mean hours (over the distribution of discrete hours available) after the wage change, and the average of individual elasticities is reported. The participation elasticity is the proportional change in the probability of
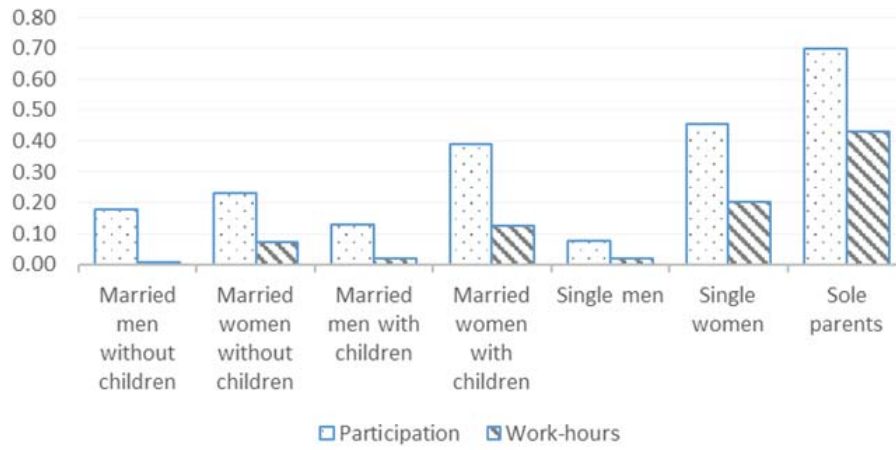
18

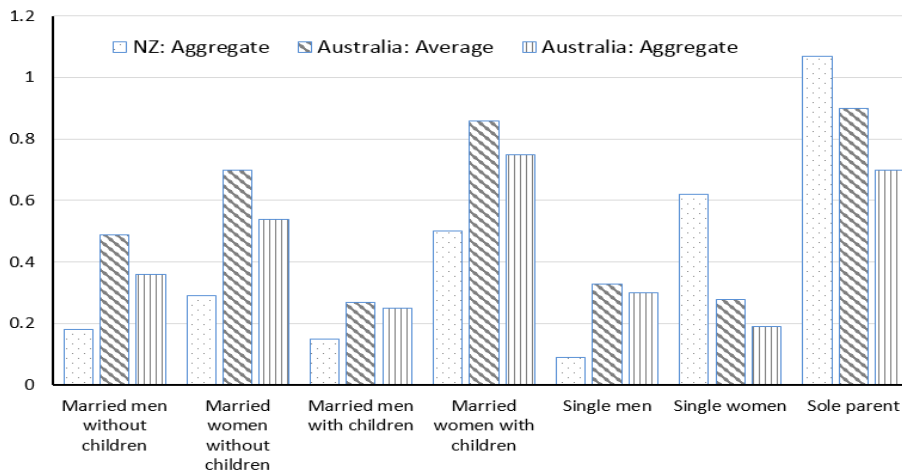Figure 2: New Zealand Participation and Working Hours Elasticities



Figure 3: Wage Elasticities: New Zealand and Australia

19

non-zero hours, divided by the proportional change in the wage rate. In all cases the participation elasticity (the extensive margin) is much higher than the hours elasticity (the intensive margin). The relative importance of the extensive margin was also stressed by Bargain *et al.* (2014) for elasticities in a range of European countries and the US.[18] Similarly, using a meta-analysis, Evers *et al.* (2006) found that labour force participation is more responsive than hours worked to wage changes.

A comparison of elasticities for Australia and New Zealand is given in Figure 3, where the Australian values are taken from Kalb (2010) and are based on preferences estimated using pooled data for the period 1999 to 2004.[19] Two sets of values are reported for Australia, showing both the arithmetic mean elasticity within each group and the aggregate value which, as shown above, is equivalent to an hours-weighted mean value. The New Zealand values are aggregate elasticities. For single women and sole parents, the elasticities are higher in NZ compared with Australia, but for other demographic groups the elasticities are lower in NZ.

Values for different age and demographic groups are reported in Table 1. These use the non-calibration approach and thus include all individuals irrespective of whether they are found to have zero hours in the sample dataset used. The values are obtained using average hours, as discussed in subsection 4.4. In the case of the earlier values obtained using Taxmod-B, the results are not available for the sole parents, when decomposed into age groups. The elasticities obtained for the different periods do not reflect the kind of reduction observed by Heim (2007), who used a 'selection corrected'

---

[18]Bargain *et al.* (2014), also based on the discrete-hours random utility model, use average hours changes, and thus appear implicity to be computing hours-weighted aggregates. They also report that results were similar when using either 1 per cent or 10 per cent wage changes.

[19]For the Australian results, wage increases of 1 per cent were imposed. Results were also reported using earlier estimates of preference functions. The fact that these differ, in some cases substantially, from later values reinforces the point made above that these elasticities are not parameters, and cannot be expected to be constant in the face of changing distributions of hours worked and preferences.

Table 1: Uncompensated Wage Elasticities: New Zealand 1991-2001 and 2006-2011

|  | Married men | Married women | Single men | Single women | Sole parents |
|---|---|---|---|---|---|
| *New Zealand: 1991-2001* | | | | | |
| age<30 | 0.22 | 0.34 | 0.55 | 0.75 | - |
| 31-49 | 0.11 | 0.37 | 0.65 | 0.72 | - |
| 50-59 | 0.4 | 0.58 | 1.21 | 1.34 | - |
| 60 and above | 0.76 | 0.83 | 1.86 | 1.77 | - |
| *New Zealand: 2006-2011* | | | | | |
| age<30 | 0.15 | 0.33 | 0.06 | 0.68 | 1.62 |
| 31-49 | 0.12 | 0.38 | 0.09 | 0.44 | 1.02 |
| 50-59 | 0.18 | 0.35 | 0.15 | 0.65 | 0.87 |
| 60 and above | 0.44 | 0.56 | 0.27 | 0.91 | 1.29 |

continuous hours regression model, for married women in the US.

More detailed comparisons between New Zealand and Australia are given in Table 2, where the New Zealand values are obtained using Taxwell-B. As mentioned above, for Australia, Kalb (2010, p. 187) was able to compare results using an average of individual elasticities and the value based on average hours changes (the method used to obtain the New Zealand results. She reports (2010, p. 184) that the elasticities, 'are clearly sensitive to the moment of averaging across the sample'. However, the precise relationship between the two concepts (one being an unweighted average of individual elasticities, while the other is an hours-weighted average) is not discussed by Kalb.[20]

---

[20]Unfortunately, limited access to the Treasury's microsimulation model meant that it was not possible to compare all alternative methods discussed above.

Table 2: Uncompensated Wage Elasticities: New Zealand and Australia

|  | Married men without children | Married women without children | Married men with children | Married women with children | Single men | Single women | Sole parents |
|---|---|---|---|---|---|---|---|
| *Education* | *New Zealand* | | | | | | |
| No quals | 0.25 | 0.35 | 0.15 | 0.43 | 0.13 | 1.34 | 1.82 |
| Vocational | 0.16 | 0.29 | 0.13 | 0.50 | 0.08 | 0.51 | 0.96 |
| School cert | 0.17 | 0.27 | 0.15 | 0.49 | 0.09 | 0.71 | 1.20 |
| University | 0.17 | 0.29 | 0.16 | 0.55 | 0.09 | 0.35 | 0.62 |
| *Education* | *Australia* | | | | | | |
| ≤ Year 12 | 0.49 | 0.51 | 0.24 | 0.75 | 0.25 | 0.56 | 1.55 |
| Vocational | 0.27 | 0.33 | 0.15 | 0.57 | 0.23 | 0.21 | 1.62 |
| Diploma | 0.21 | 0.41 | 0.12 | 0.57 | 0.1 | 0.25 | 1.70 |
| Degree | 0.14 | 0.26 | 0.10 | 0.58 | 0.12 | 0.15 | 0.83 |
| *Age* | *New Zealand* | | | | | | |
| 16-24 | 0.14 | 0.22 | 0.22 | 0.62 | 0.06 | 0.74 | 2.01 |
| 25-34 | 0.09 | 0.18 | 0.15 | 0.66 | 0.07 | 0.45 | 1.26 |
| 35-44 | 0.08 | 0.18 | 0.13 | 0.53 | 0.09 | 0.46 | 0.95 |
| 45-54 | 0.13 | 0.27 | 0.14 | 0.39 | 0.12 | 0.56 | 1.01 |
| 55-64 | 0.32 | 0.46 | 0.28 | 0.47 | 0.22 | 0.82 | 0.84 |
| *Age* | *Australia* | | | | | | |
| 16-24 | 0.36 | 0.34 | 0.39 | 1.40 | 0.21 | 0.18 | 0.75 |
| 25-34 | 0.09 | 0.21 | 0.19 | 0.81 | 0.14 | 0.07 | 1.91 |
| 35-44 | 0.08 | 0.29 | 0.13 | 0.62 | 0.26 | 0.16 | 1.39 |
| 45-54 | 0.29 | 0.52 | 0.19 | 0.43 | 0.33 | 0.41 | 1.29 |
| 55-64 | 0.64 | 0.61 | 0.27 | 0.41* | 0.34 | 1.48 | 1.40* |

Note * : These values are based on < 20 observations.

# 6    Conclusions

This paper has provided an exploration of labour supply elasticities – relating to hours and participation rate changes in response to wage rate changes – in New Zealand.  Elasticities were obtained for a range of demographic groups, and comparisons were made with the only previous estimates for New Zealand. The results rely on simulations carried out using the NZ Treasury's behavioural microsimulation model, Taxwell-B. While it is important to recognise that individual, and particularly aggregate, elasticities are not parameters and depend on a wide range of variables, there is a need to provide some summary information for those interested in the potential implications of policy changes and interdependencies between labour and other markets. In particular, it is important to know the likely incentive effects of tax policy changes.  The paper therefore began by clarifying the relationship among different elasticities, in particular the gross and net wage elasticities of hours worked and the elasticity of taxable income, which relates to changes in the net-of-tax marginal tax rate. The labour supply component of the elasticity of taxable income is shown to be equivalent, under certain conditions, to the gross wage elasticity of labour supply.

Although labour supply elasticities have been obtained for a wide range of countries, using alternative modelling strategies, the relationships between different approaches to aggregation over defined population groups are not typically set out explicitly, and indeed alternatives are sometimes implicitly assumed to be equivalent.  The paper therefore also examined the alternative measures, particularly where a discrete-hours random-utility structural specification is used, where each individual has a probability distribution over available hours of work.  An aggregate based on changes in average hours within a group is shown to be equivalent to an hours-weighted sum of individual elasticities.

Finally, labour supply elasticities at both intensive and extensive margins

23

were reported for New Zealand. In common with other studies, the participation decision – responses at the extensive margin – was found to be much more responsive to wage rate changes than were hours of work. The largest responses were obtained for sole parents and women (married with children, and single). Nevertheless, in view of the low hours of work of many sole parents, such higher elasticities do not necessarily imply that absolute hours changes are large. Furthermore, the low uncompensated elasticities for men do not necessarily imply that excess tax burdens for this group are low, since the latter are known to depend on compensated elasticities.

While it is important to recognise that elasticities are not parameters, and are determined, for each individual, jointly with marginal tax rates and hours worked, the kind of summary information presented here can provide useful information for those considering direct tax changes and examining labour supply variations in wider contexts.

# Appendix: The Microsimulation Model: Taxwell-B

Taxwell is a non-behavioural (or arithmetic) microsimulation model developed by the New Zealand Treasury. It contains the details of the social security and personal tax system and produces analyses at individual, family and household level. It utilises the Household Economic Survey (HES), a cross-sectional dataset collected by Statistics New Zealand. The model uses most of the income data from HES which includes income from current jobs and other income such as interest and dividends. The name is a tribute to Ivan Tuckwell, who played a substantial role in the development of the previous Treasury arithmetic model, Taxmod.

The Treasury's behavioural model, Taxwell-B, uses information for each sample individual, provided by Taxwell, on disposable incomes at the specified range of discrete hours labour supply levels before and after the reform, along with the individual and household characteristics. Taxwell-B thus uses estimated parameters of the deterministic component of preference functions on which the behavioural responses are based. An earlier Treasury behavioural model, called Taxmod-B, was not maintained and could not be integrated with Taxwell. An extensive programme of work was required to convert Taxmod-B to Taxwell-B and to re-estimate all the necessary imputed wages and preference functions. This was completed in 2014.

Taxwell-B assumes a 100 per cent take-up rate for welfare benefits. This may lead to some overestimation of expenditure on the different payments in both pre-reform and post-reform situations. However, as the policy changes do not expand eligibility, the simulated percentage changes reported here are not expected to be biased. All persons for whom labour supply is modelled, except sole parents, are potentially eligible for Unemployment Benefits (UB). Sole parents are eligible for DPB. The income-test rules are then applied to calculate actual benefit levels.

The budget constraints for each individual, giving net incomes at each discrete hours level, require knowledge of hourly wage rates. For workers these are directly observed. However, they are unobserved for non-workers in survey data. For these individuals, it is therefore necessary to impute their wage rates using wage equations which correct for potential sample selection bias. Wage equations were estimated separately for partnered men, partnered women, single men, single women and sole parents using pooled HES data from 2006/07 to 2010/11: see Mercante and Mok (2014b).

As mentioned above, the behavioural responses generated by Taxwell-B are based on the use of quadratic preference functions allowing for observed and unobserved heterogeneity. For couples, labour supplies are jointly determined. The parameters of the preference functions were again estimated using pooled HES dataset from 2006/07 to 2010/11: see Mercante and Mok (2014a). The specification does not include any components relating to the number of jobs available at the various discrete hours levels. It must therefore be acknowledged that the estimation of preferences may be affected by the state of the labour market following the global financial crisis of 2008.

# References

[1] Bargain, O., Orsini, K. and Peichl, A. (2014) Comparing labor supply elasticities in Europe and the US: new results. *Journal of Human Resources,* 49, pp. 723-838.

[2] Chiao, Y-S. and Walker, I. (1992) Labour market behaviour of prime-age individuals. In *Incentives and Labour Supply: Modelling Taxes and Benefits* (ed. by M. Prebble and P. Rebstock). Wellington: Victoria University of Wellington Institute of Public Studies.

[3] Creedy, J. (1992) *Demand and Exchange in Economic Analysis: A History from Cournot to Marshall.* Cheltenham: Edward Elgar Publishing.

[4] Creedy, J. (2004) Labour supply incentives in alternative tax and transfer schemes: a diagrammatic introduction. *Australian Economic Review*, 37, pp. 230-241.

[5] Creedy, J. and Kalb, G. (2005) Discrete hours labour supply modelling: specification, estimation and simulation. *Journal of Economic Surveys*, 19, pp. 697-734.

[6] Creedy, J., Duncan, A.S., Scutella, R. and Harris, M. (2002) *Microsimulation Modelling of Taxation and The Labour Market: The Melbourne Institute Tax and Transfer Simulator.* Cheltenham: Edward Elgar.

[7] Dandi, S. and Mercante, J. (2007) Australian labour supply elasticities: comparison and critical review. *Australian Treasury Working Paper*, 2007-04.

[8] Evers, M., de Mooij, R.A. and van Vuuren, D.J. (2006) What explains the variation in estimates of labour supply elasticities. *Tinbergen Institute Discussion Paper*, TI 2006-017/3.

[9] Gemmell, N. and Hasseldine, J. (2014) Taxpayers' behavioural responses and measures of tax compliance 'gaps': a critique and a new measure. *Fiscal Studies*, 35, pp. 275-296.

[10] Heim, B.T. (2007) The incredible shrinking elasticities: married female labour supply, 1978-2002. *Journal of Human Resources*, 42, pp. 882-918.

[11] Kalb, G. (2010) Modelling labour supply responses in Australia and New Zealand. In *Tax Reform in Open Economies: International and Country Perspectives* (ed. by I. Claus, N. Gemmell, M. Harding and D. White), pp. 166-193. Cheltenham: Edward Elgar Publishing.

[12] Kalb, G. and Scutella, R. (2003) New Zealand labour supply from 1991-2001: an analysis based on a discrete choice structural utility model. *New Zealand Treasury Working Paper,* WP 8/2002.

[13] Maloney, T. (2000) The impact of welfare reform on labour supply in New Zealand. *Labour Economics*, 7, pp. 427-448.

[14] Mercante, J. and Mok, P. (2014a) Estimation of labour supply in New Zealand. *New Zealand Treasury Working Paper*, WP 14/08.

[15] Mercante, Joseph and Penny Mok (2014b) Estimation of wage equations for New Zealand. *New Zealand Treasury Working Paper*, 14/09.

[16] Samuelson, P.A. (1947) *Foundations of Economic Analysis*. New York: Harvard University Press

[17] Van Soest, A. (1995) Structural models of family labour supply: a discrete choice approach. *Journal of Human Resources*, 30, pp. 63-88.

[18] Whalen, C. and Reichling, F. (2017) Estimates of the Frisch elasticity of labour supply: a review. *Eastern Economic Journal*, 43, pp. 37-42.

**About the Authors**

John Creedy is Professor of Public Finance at Victoria Business School,
Victoria University of Wellington, New Zealand
Email: john.creedy@vuw.ac.nz


Penny Mok is an Analyst at the Ministry of Business, Innovation and Employment,
New Zealand.
Email: penny.mok@mbie.govt.nz

VICTORIA
UNIVERSITY OF WELLINGTON
TE WHARE WĀNANGA
O TE ŪPOKO O TE IKA A MĀUI

Chair in Public Finance
Victoria Business School

**Working Papers in Public Finance**