Modelling New Zealand milk: From the farm to the factory

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Dairy products have long been an important dietary component, particularly for young children. Because of this the dairy industry is especially sensitive to contamination scares, and dairy is of particular importance to the New Zealand economy. This paper develops a Markov chain model for the early stages of the dairy supply-chain. Using the case of a major New Zealand Dairy company, simulations are run under various product-testing scenarios. Results point to the importance of where and when testing and interventions take place. Being strict about removing potentially contaminated product early on in the supply chain can reduce total losses and improve overall production output.

Keywords: dairy, testing, contamination, supply chain, markov-chain.

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Abstract

Dairy products have long been an important dietary component, particularly for young children. Because of this the dairy industry is especially sensitive to contamination scares. Dairy is of particular importance to the New Zealand economy. This paper develops a Markov chain model for the early stages of the dairy supply chain. Using the case of a major New Zealand Dairy company, simulations are run under various product testing scenarios. Results point to the importance of where and when testing and interventions take place. Being strict about removing potentially contaminated product early on in the supply chain can reduce total losses improve overall production output.

1 Introduction

Dairy products are an important part of the western style diet and are becoming increasingly important as part of the Asian diet as well [31]. Milk is a valuable source of essential nutrients, and often forms a large part of a young child's diet. Because of this, the dairy industry is particularly vulnerable to contamination scares. The 2008 melamine contamination of infant formula in China, the Fonterra botulism scare in 2013 and the 2015 poisoning threat to infant formula in New Zealand are all examples of this.

The 2013 Botulism scare in particular highlights the need for fast accurate testing and identification of substandard product. New Zealand is a world leader when it comes to the production and export of dairy products [21]. The dairy industry forms a large part of the country's exports [2]. The amount of time taken, to confirm the source of the contamination in the botulism scare of 2013, risked not only Fonterra's reputation but New Zealand's reputation as an exporter as well.

The aim of this paper is to develop a useful model for the early stages of the dairy supply chain, which we can use to investigate the overall effect of testing regimes and failure rates on production. We develop a model for the flow of milk from the farm to the factory and incorporate testing to investigate its effect on product output. Following this introduction we begin with an introduction to dairy production. Section 2 gives an overview of the dairy industry, with a brief look at the history of dairy production and product importance. The second half of the section covers aspects of specific importance to the New Zealand dairy industry and some aspects of testing dairy products. In Section 3 we introduce the stages of the model and develop a system of differential equations to model the value of milk in each stage over time. From the equations, we derive a set of Markov chains to reflect the stochastic nature of dairy production and contamination detection. Section 4 further develops the stochastic model and parameter estimation to incorporate more aspects of the dairy production process. Finally, we discuss the results and implications of this work, along with the limitations and potential for future research. The focus will be on the value of the milk, from which we can deduct costs associated with testing.

2 Dairy Production and Supply Chain Modelling

Milk is rich in a variety of essential nutrients [29], and the worldwide market for dairy and milk based products continues to grow [19]. Along with this growth comes increasing food safety issues, with consumer perception becoming increasingly important [8].

Aside from its value as milk, many derived dairy products are available. In particular, functional foods and health supplements made with milk proteins have proven to be of considerable value [30]. Steijns [29] discusses various components of dairy products and their role in managing a variety of health concerns. Research has also been done into how certain dairy products may be useful for cancer prevention [31, 26].

2.1 Supply chain models in Dairy

Pooley (1994) builds a, relatively simple, mixed integer linear programming (MILP) model, to study location allocation cases from the dairy industry [27]. Wouda et.al. (2002) also study a location allocation cases on the dairy industry but use a more complex model, incorporating the movement of by products such as cream and whey into their MILP model [33].

While not focussing specifically in Dairy, Zhang et.al.(2003) explicitly model quality degradation of food products In their network design model. They use a tabu search-based method with a penalty cost based on the amount of both degradation and product involved [34]. Ekşioğlu and Jin (2006) develop a general MILP approach for network planning of perishable products [7].

Dooley et.al. (2005) study the transport logistics associated with collecting different types of milk. They use a genetic algorithm and construct a simulation model taking into account the effects on milk quality and the practicalities of maintaining segregation between different milks throughout transportation [6].

Hutchison (2006) presents a multi-scale modelling perspective of Dairy, with the aim of eventually developing a multi-scale system model capable of delivering information for process troubleshooting, scheduling, process and business optimisation, and process control decision-making for the dairy industry [20].

2.2 Dairy in New Zealand

As mentioned earlier, New Zealand is a world leader in the production and export of dairy products [21]. The New Zealand dairy industry is unique among major global producers, in that the majority of its production is exported [20]. The industry is also mainly pasture based [4], and has earned a reputation for its low cost, high quality systems and technological expertise [21].

About 97% of New Zealand dairy farmers sell their milk through Fonterra Cooperative Group [21]. Cows are generally milked twice per day [17], and milk is collected from the farm in a tanker every 1-2 days [21]. Fonterra operates a national fleet of 525 tankers collecting from around 12,000 farms [22]. The frequency of collection is generally dependent on the time of year, as milk production is seasonal. The amount of milk a farmer is allowed to supply to Fonterra is limited by the number of shares they own in the cooperative. Because of this restriction, output becomes targeted [21]. The cost efficiency of New Zealand Dairy farms is examined by Jiang et.al (2014). Their results indicated that there is still room for improvement [21]. Trends in developing high capacity milking parlours and automatic miking systems, have seen an increase in cow output, along with reduced manual labour on dairy farms. As these trends continue, further labour based barriers to farm expansion may be overcome [17].

2.2.1 In the Factory

Aside from small quantities of on farm sales the first steps in production, required for all dairy products produced in New Zealand are separation, standardisation and pasteurisation [18, 32].

The Factory reception needs to have the capacity to accept what is potentially collected each day. The pumps used in the are specifically designed to run at low speeds so as not to damage whole milk fat particles [24].

The reception capacity of a processing site is about 675,000 litres. Given Fonterra operates 33 processing sites around the country we can estimate a total reception capacity of $33 \times 675,000 = 22,275,000$ litres.

Typically a factory has a bank of several separators which feed into several silos for cream and skim milk. The Hautapu site, for example, has 8 separators. This particular factory has the capability to process 4.1 million litres of milk per day. Each separator bowl has a volume of 50 litres and is capable of separating 33,000 litres every hour [3].

A typical processing plant in the paediatric supply chain has a separator capacity of 1,420,000 L, made up of three separators, feeding into various product silos. Such a facility also has a standardisation capacity of 1,600,000 litres [10].

2.2.2 The Milk Tanker

Fonterra's tanker fleet operates 24 hours a day, with a 10-12 hr day shift involving 3-6 runs per tanker [23]. Each run consists of, on average, 5 or 6 collection stops, taking about 12 minutes per collection, and a 30 minute turn around delivering to the factory [6]. There is a 1-2 hr turnover before the night shift starts with a similar pattern to the day shift [23].

Each milk tanker, truck and trailer unit can hold 28,800 litres of milk, 11,300 litres in the truck unit and 17,500 litres in the trailer [6].

Most milk tankers in New Zealand are fitted with a Nuphlo/Transport Hydraulic Solutions system, enabling them to load milk at rates up to 2000 litres per minute or 176.6 kg MS per minute. This means it takes one tanker 0.334 seconds to pump on 1 kg MS. The loading is aided by the fact that most dairy farms in New Zealand having the farm vat on a raised platform [24].

2.2.3 On the Farm

A total of 1614 million kg MS was collected by Fonterra in the 2014/2015 season ending in May 2015 [15]. This equates to an average daily production 4,421,918kg MS per day, Though this is skewed by the fact that very little production is taking place for three months of the year.

Milk is collected from suppliers on alternate days, unless daily collections are deemed appropriate based on milk volumes and vat capacity [13].

The capacity of an on-farm silo is based on each cow producing 25 litres of milk each day at the peak of the season. Fonterra currently requires their suppliers to have a minimum of 400 litres available at each collection [13]. We will estimate an average collection amount per day, during the main season, based on herd size and cow output data.

The average herd size has tripled over the last 30 seasons and is still increasing. For the 2014/2015 season, the average herd size was 419. The average output, per herd was 1,775,501 litres, containing 157,885 kg MS (Kilogram Milk solids). [5].

The number of dairy herds in New Zealand has been steadily declining since 1980, but has recently begun to increase again slightly, beginning in the 2007/08 season. The number of herds increased by 43 in the 2014/15 season to 11970 [5].

The price Fonterra pays farmers, in $/kg MS, is calculated based on the Global Dairy Trade (GDT) prices for whole milk powder (WMP), skim milk powder (SMP), anhydrous milk fat (AMF), butter and buttermilk powder (BMP). Because these prices are in US dollars, the exchange rate must be taken into account before Fonterra subtracts the lactose cost and the cash and capital cost [25]. The farm gate milk price for the 2014/2015 season was $4.40 [14]. This price takes into account fixed costs such as transport and manufacturing as well as allowing for appropriate returns on investment [11].

2.2.4 Testing

Testing for antibiotic residues costs less than 1 USD per 10 tests in Kenyan milk [28].

The Fonterra supplier's handbook lists at least 9 contamination types to be tested for along with with general quality grading and organoleptic (sensory) assessment, though only two of these tests are conducted upon tanker collection every time [13].

Fonterra milk tankers generally accept 99.99% of milk presented at the farm. Warm milk is the most common reason for rejection at this point. Temperature affects the quality of the milk and influences the growth of potentially harmful microorganisms [1]. Fonterra's milk tankers are not refrigerated, meaning if the milk is not cold enough at the time of collection it may not arrive at the factory in an acceptable condition. There is also the possibility that the milk has not been chilled fast enough following milking and some quality degradation has already taken place.

Testing of milk upon collection can also improve the quality of the milk supplied. Gorton, Dumitrashko and White identified a drop in milk rejection rates in Moldova between 1999 to 2003 from 4% to 1%, following the introduction of a collection and testing system holding individual supplies accountable for the quality of the milk supplied.[16].

3 Developing a Single-Event Model

In this section we develop a model for the value of milk at each stage between the farm and the first stage of processing at the factory. Figure 3.1 shows an overview of the flow of dairy products from the farm to the consumer. In this paper we will be focusing on the left side of this figure up to the first stage of processing.

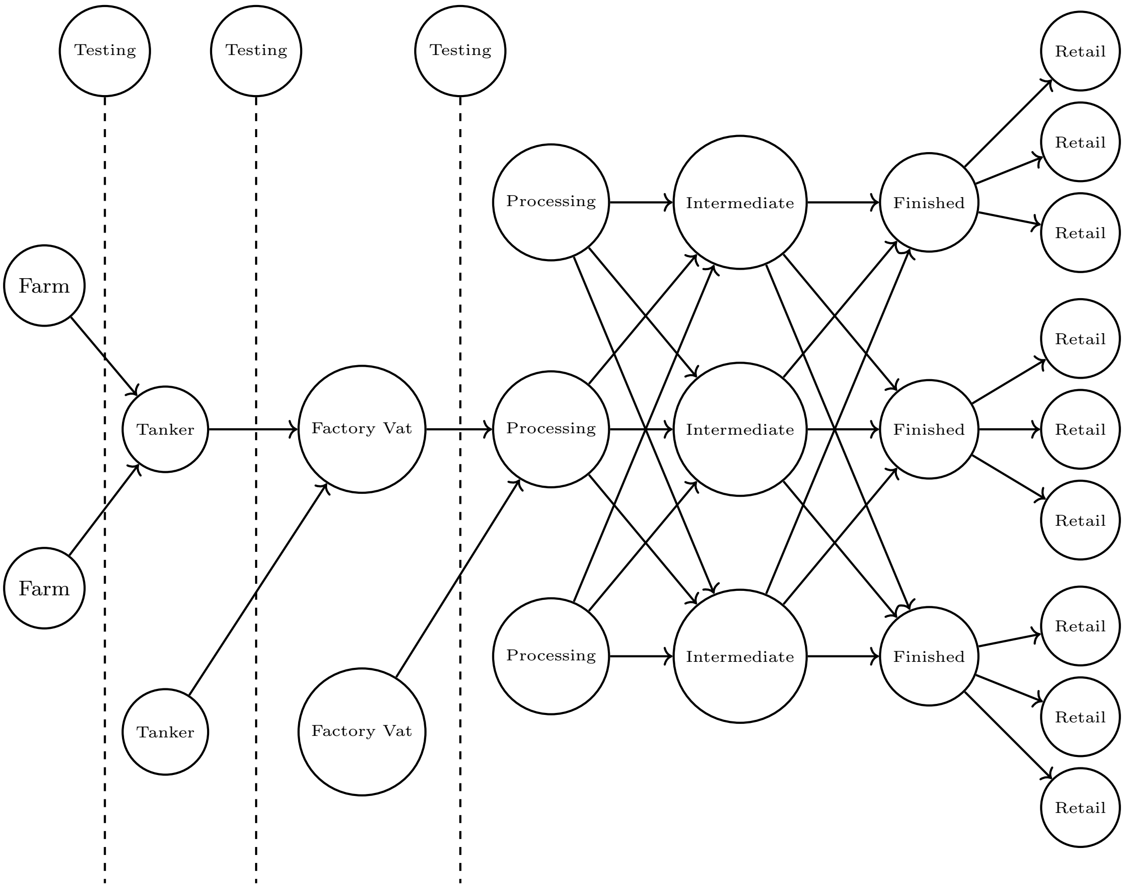

Figure 3.1: The overall picture of the dairy product chain

There are three possible points for testing, each before mixing together milk from different sources. These are between the farm vat and the milk tanker, between the tanker and the vat at the factory, and between the vat and entering processing.

Table 3.1 summarises the parameters we will use in this model. We make the assumption that any costs associated with the care and milking of the cows, is the responsibility of the farmer and does not influence our model.

Parameter	Description	Units
V	Amount of Milk collected from an on-farm vat	\$ or KgMS
Φ	Frequency of collection attempts	Collections per day
$\mathcal{X}$	Frequency of delivery to factory	Deliveries per day
Ψ	Frequency with which milk enters processing	Silos per day
Ω	Frequency of process exit	Units per day
E_T	Cost of testing milk at collection site	\$
E_F	Cost of testing milk upon delivery	\$
D_F	Cost of disposing of rejected milk	\$
E_P	Cost of testing prior to processing	\$
D_p	Cost of disposing of rejected silo milk	\$
α	Probability of acceptance by tanker	N/A Probability
β	Probability of passing all tests upon delivery	N/A Probability
γ	Probability of passing pre-processing tests	N/A Probability

Table 3.1: A Summary of each parameter used in out model and its units. Frequencies are represented by the capital Greek letters $\Phi, \mathcal{X}, \Psi$ and Ω. Probabilities are represented by lowercase Greek letters α, β, and γ. Testing costs are represented by an E with a subscript for the associated stage. The subscript T is associated with costs entering the tanker stage, F the factory stage and P the processing stage. Similar notation is used for the disposal costs using a D . The vat value, given by V uses units appropriate to the particular model simulation.

3.1 The deterministic model

Initially we will develop a set of differential equations to model the basic situation. The resulting deterministic model will be a useful starting point when we construct the stochastic model. It will also assist in validating the simulation results.

3.1.1 Tanker Collection

For the purposes of our model, the milk becomes the responsibility of the factory when it is collected by the tanker. This is also when the first test can be applied, before mixing with any previously acquired milk already in the tanker.

In transferring the milk from the farm's vat to the tanker, the value of that milk is transferred to the tanker. The tanker gains the value of the milk, but loses the cost of any testing performed. If a test is failed before the milk is added to the tanker, the milk is rejected and no value is gained, the tanker still loses the cost of testing.

After collecting milk from multiple farms, the tanker will deposit it's load at a factory, along with all the value associated with it.

We can represent the change and value of the tanker with a differential equation, as given in equation 3.1 below

$$\frac{dT(t)}{dt} = (\alpha V - E_T)\Phi - \mathcal{X}T(t) \tag{3.1}$$

In any given period of time the tanker will make a certain number of collections and certain number of deliveries. In equation 3.1 the frequency of collection, i.e. the number of collections attempts per day, is represented by Φ while delivery frequency is denoted by $\mathcal{X}$.

The probability that the milk being collected passes all tests on site is denoted by α, meaning $1-\alpha$ is the probability that a test is failed and the milk is rejected. $T(t)$ is the value contained by the tanker at time t. To keep things simple we assume that all vats contain the same amount of milk, and therefore the same value, V represents the the value of the farm vat that is transferred to the milk tanker.

The tanker will make its delivery to the factory, regardless of the results of any tests. What happens to the milk after this is at a different stage of the model and does not affect the tankers.

The cost of the test is incurred independent of its results. This cost is assumed to be constant and is represented by E_T. If there is no testing prior to collection by the tanker, $\alpha = 1$ as the milk cannot be rejected, and $E_T = 0$ as there is no cost of testing.

Figure 3.2 is a flow diagram of the value entering and leaving the tanker stage.

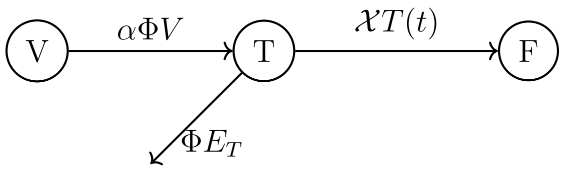

Figure 3.2: The flow of value into and out of the tanker stage.

If milk is rejected, the co-operative does not pay the farmer for that milk, it is considered as if the milk was never supplied. The responsibility of disposing of milk rejected at this stage lies with the farmer[13].

3.1.2 Delivery to the Factory

The next production stage is the reception silo at the factory where the tanker deposits its load.

The value in storage at the factory is increased by $T(t)$ with each successful delivery, while the costs of testing E_F results in some value loss. β is the probability that milk passes all testing upon arrival at the factory.

Because at this stage the milk is the responsibility of the factory, there is also some disposal cost (D_F) associated with any milk that that is rejected upon arrival at the factory.

Milk leaves the Factory reception stage at a rate of Ψ units per day.

$$\frac{dF(t)}{dt} = [\beta T(t) - E_F - (1-\beta)D_F]\mathcal{X} - \Psi F(t) \tag{3.2}$$

Because milk is being processed almost continuously, milk leaves factory storage in amounts dependent on how much is in the factory at time t.

Figure 3.3 is a flow diagram of the value entering and leaving the factory stage.

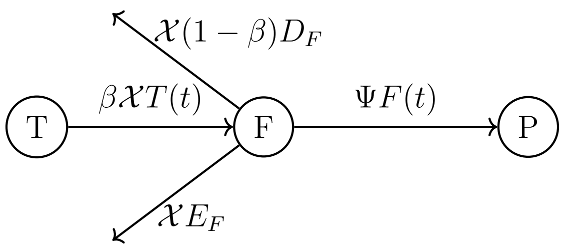

Figure 3.3: The movement of product into and out of the factory reception stage.

3.1.3 Processing

Here we look at how the value contained in the processing stage is changing over time. Processing of milk begins with separation, followed by standardisation. After these steps every dairy product undergoes pasteurisation [18, 32]. The milk comes in from the factory storage vats at the same rate it leaves them in equation 3.2, the value increases by this amount ($\Psi F(t)$) minus the value of testing (E_P) conducted. There will also be a disposal cost (D_P) for rejected milk. Value leaves this processing stage at the rate $\Omega P(t)$, to go on to the next stage in production.

$$\frac{dP(t)}{dt} = \Psi[\gamma F(t) - E_P - (1-\gamma)D_P] - \Omega P(t) \tag{3.3}$$

The probability of accepting milk into this stage is denoted byγ.

Figure 3.4 shows a flow diagram for the value entering and leaving the first production stage.

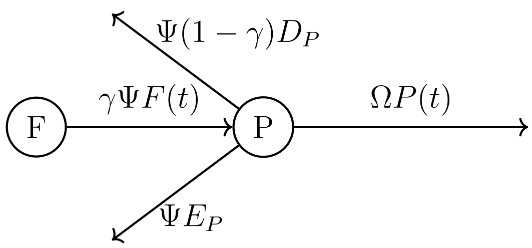

Figure 3.4: The rate of value flow into and out of the first processing stage.

3.1.4 Equlibrium

Now that we have a system of differential equations, we can solve for the value contained in each stage at equilibrium. The equilibrium point of the system occurs when all three first derivatives are zero.

Equation 3.1 set equal to zero gives:

$$\begin{aligned}\Phi(\alpha V - E_T) - \mathcal{X}T(t) &= 0\\ \implies \mathcal{X}T(t) &= \Phi(\alpha V - E_T)\\ \implies T(t) &= \frac{\Phi}{\mathcal{X}}(\alpha V - E_T) \qquad (3.4)\end{aligned}$$

Setting equation 3.2 equal to zero we get:

$$\begin{aligned} 0 &= \mathcal{X}[\beta T(t) - E_F - (1-\beta)D_F] - \Psi F(t) \\ \Psi F(t) &= \mathcal{X}[\beta T(t) - E_F - (1-\beta)D_F] \\ \implies F(t) &= \frac{\mathcal{X}}{\Psi}[\beta T(t) - E_F - (1-\beta)D_F] \end{aligned} \tag{3.5}$$

Substituting in the solution from equation 3.4 gives

$$\begin{aligned} F(t) &= \frac{\mathcal{X}}{\Psi}[\beta\frac{\Phi}{\mathcal{X}}(\alpha V - E_T) - E_F - (1-\beta)D_F] \\ &= \frac{\beta\mathcal{X}}{\Psi}\left[\frac{\Phi}{\mathcal{X}}(\alpha V - E_T)\right] - \frac{\mathcal{X}}{\Psi}[E_F + (1-\beta)D_F] \\ F(t) &= \frac{\beta\Phi}{\Psi}\left[\alpha V - E_T\right] - \frac{\mathcal{X}}{\Psi}[E_F + (1-\beta)D_F] \end{aligned} \tag{3.6}$$

equation 3.3 gives

$$\begin{aligned} 0 &= \Psi[\gamma F(t) - E_P - (1-\gamma)D_P] - \Omega P(t) \\ \Omega P(t) &= \Psi[\gamma F(t) - E_P - (1-\gamma)D_P] \\ P(t) &= \frac{\Psi}{\Omega}[\gamma F(t) - E_P - (1-\gamma)D_P] \end{aligned} \tag{3.7}$$

Substituting in F(t) at equilibrium, as given in equation 3.6 gives us

$$\begin{aligned} P(t) &= \frac{\Psi}{\Omega}\left(\gamma\left[\frac{\beta\Phi}{\Psi}\left[\alpha V - E_T\right] - \frac{\mathcal{X}}{\Psi}[E_F + (1-\beta)D_F]\right] - E_P - (1-\gamma)D_P\right) \\ P(t) &= \frac{\gamma\beta\Phi\left(\alpha V - E_T\right) - \gamma\mathcal{X}\left[E_F + (1-\beta)D_F\right] - \Psi\left[E_P + (1-\gamma)D_P\right]}{\Omega} \end{aligned} \tag{3.8}$$

This can be interpreted as all the material that makes it through each stage, minus the costs associated with material testing and rejection, all divided by the rate it is leaving the processing stage.

3.2 Discrete Time Markov Chain Model for Dairy

Using Markov chains, we can derive realistic stochastic models that are based on our deterministic models. Let p_{ij}= the probability of transition from state $I(t) = i$ to state $I(t + \Delta t) = j$ during the time interval Δt.

Discrete time Markov chain (DTMC) models are defined on discrete time steps, where one unit changes state, or in out case location in in the supply chain, at each step. The main issue with this type of model is that we need to be very careful choosing the size of the time step. Too small a time step and the computational burden becomes to great, too large a time step and we start to underestimate the changes.

We use discrete time Markov chains because the events where milk moves between stages are clearly defined. Beginning with each equation separately, we can derive the probability of each event happening in a discrete time period. Figure 3.5 shows the path milk takes from the farm to the factory, and where the event decisions occur.

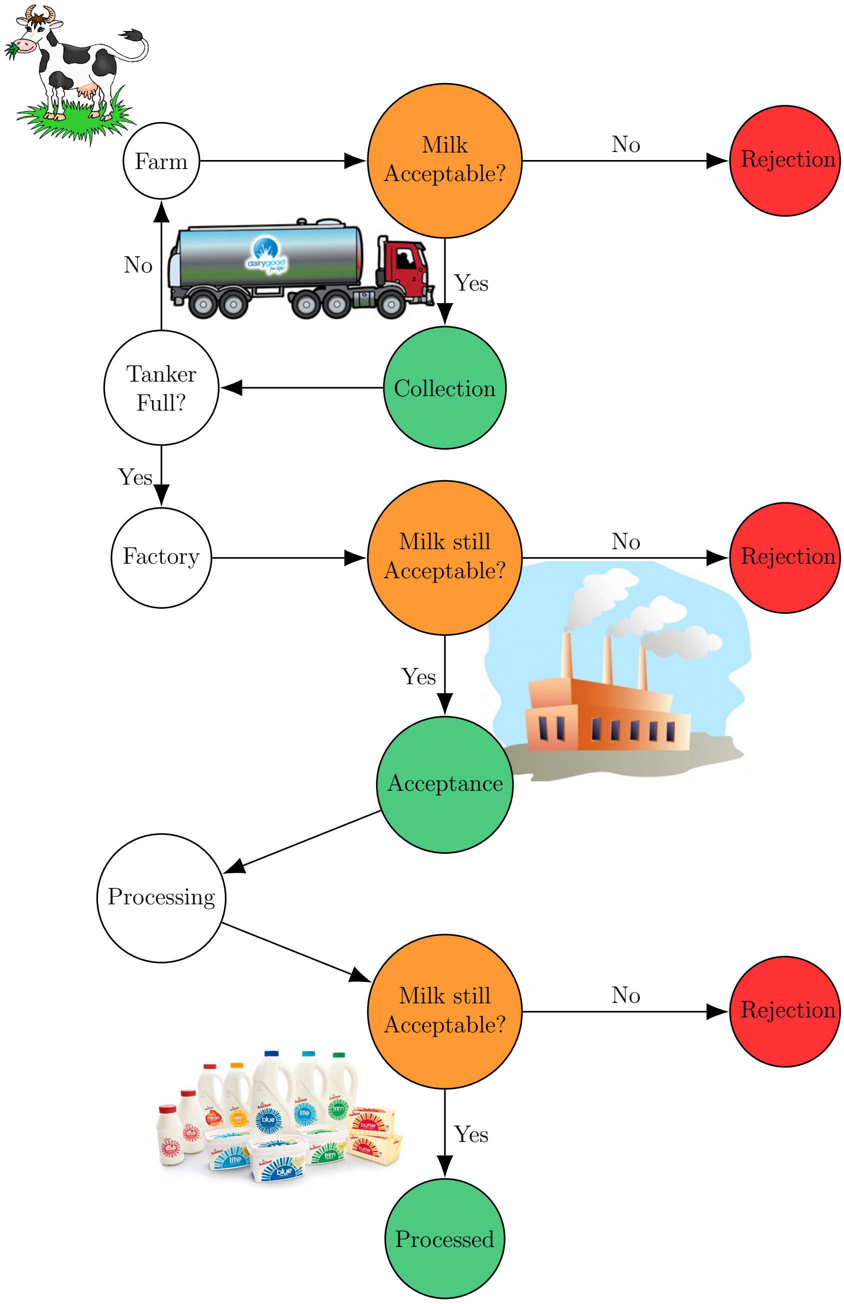

Figure 3.5: Flow chart, showing the path milk takes from the farm to the factory and the decisions that are made along the way.

The state of the Markov chain at a given time is the amount of milk in each stage. This may be dollar value or some measure of volume, depending on the units we are using for a particular situation. Generally if the scenario does not include any costs, we will measure milk in kilograms of milk solids (kg MS).

Because we are using three stages, the Tanker, Factory reception and Processing, the state of the Markov chain is described by a vector of three values.

The total number of possible states depends on the amount of product that is allowed to move between stages in each time step, and the maximum capacity of each stage. For example if the maximum capacity vector is given by $(T_{\text{Max}}, F_{\text{Max}}, P_{\text{Max}}) = (130, 200, 880)$ using a constant movement value of 1, The total number of states is given by $130 \times 200 \times 880 = 22880000$. If however we increase the movement each time step to 10, we have $\frac{130}{10} \times \frac{200}{10} \times \frac{880}{10} = 22880$ possible states.

The transition probabilities for each stage are described below in section 3.3. One strict requirement we must meet, is that each set of transition probabilities adds to 1. This is achieved via several modelling restrictions and techniques.

First, the probability that no movement of material into or out of a given stage is defined as 1 minus the sum of all the probabilities that some product movement does occur. For example the probability that milk in the tanker stage is only being transported in a given time step is given by 1 minus the probability that milk is being collected, rejected or delivered by a tanker.

This definition is all well and good as long as the probability of any movement adds to less than 1, this we can ensure by choosing an appropriate time step. The frequency with product enters a stage, even when multiplied by the probability of acceptance (which is always less than or equal to 1), will still likely result in a value greater than 1. By choosing a value of Δt that is small enough we can ensure the resulting probability is between 0 and 1.

There is a bit of a balancing act required in choosing the size of the time step. The smaller the time step used the more steps required to cover a certain period of time. Because each time step is another iteration of the Markov chain, small step size can greatly increase the running time of a simulation. Too small a time step can also make it more difficult to see what is happening in plots depending on the nature of the model. We therefore choose the time step size to be as large as realistically possible, within the restrictions required by our transition probabilities.

3.3 The Simplified Single Event Model

We start with a simple model where only one event can take place at any point in time. We may lose some realism, but it provides a good place to start building the model from. The transition probabilities are given in equation 3.9. The value of milk in the tanker, factory reception and processing stages at time t are represented by i, u and g respectively. This is the state of the Markov chain. The variables j, v and h denote the value change that takes place in each stage over a time step Δt.

$$p_{(i,u,g),(i+j,u+v,g+h)}(\Delta t) = \begin{cases} \alpha\Phi\Delta t & (j,v,h) = (V - E_T, 0, 0) \quad \text{Collection} \\ (1-\alpha)\Phi\Delta t & (j,v,h) = (-E_T, 0, 0) \quad \text{Tanker Rejection} \\ \beta\mathcal{X}\Delta t & (j,v,h) = (-T(t), T(t) - E_F, 0) \quad \text{Delivery/Acceptance} \\ (1-\beta)\mathcal{X}\Delta t & (j,v,h) = (-T(t), -E_F - D_F, 0) \quad \text{Factory Rejection I} \\ \gamma\Psi\Delta t & (j,v,h) = (0, -F(t), F(t) - E_P) \quad \text{Passing on/Acceptance} \\ (1-\gamma)\Psi\Delta t & (j,v,h) = (0, -F(t), -D_P) \quad \text{Rejection I} \\ \Omega\Delta t & (j,v,h) = (0, 0, -P(t)) \quad \text{Passing on} \\ 1 - [\Phi + \mathcal{X} + \Omega + \Psi]\Delta t & (j,v,h) = (0,0,0) \quad \text{Transporting/Holding/Producing} \end{cases} \tag{3.9}$$

The Milk Tanker: There are four possible activities that a tanker can undertake, in a given time step, Milk collection, Milk rejection, Milk delivery, or Transporting.

Milk collection can only occur if no test results come back with an unsatisfactory result. The value contained in the tanker stage increases by the value of the milk collected, V, minus the cost of the testing, E_T.

Milk rejection occurs if a test is failed. In this case the value contained in the milk tanker is decreased by the cost of the test, E_T.

Milk delivery occurs when the tanker empties its load at the factory. The value contained in the tanker is reduced to 0, that is, it decreases by the total value currently contained by the tanker, $T(t)$.

The Factory Reception Stage: Delivery by the tanker implies either acceptance or rejection at the Factory reception stage.

In the case of rejection, the milk is rejected due to tests done upon the tanker's arrival at the factory. The cost of the test is lost E_F, along with the disposal cost D_F.

Other possible events in this stage are passing on for processing, or holding.

Initial Processing: Product being passed on by the factory reception stage leads to acceptance or rejection in the processing stage.

Similar to the factory reception stage other possible activities include passing on product for further processing, but instead of just holding product, this is time spent in production.

Transporting, Holding and Producing: Milk being transported by a tanker, held at the factory reception stage, or in the middle of processing results in no change in the value of milk in any compartment for that time step. The effect of each of these events is identical, therefore in this model they are represented as same event even though they occur in different stages.

3.4 Parameter estimation

The initial parameter values, which we will discuss in this section, are compiled in Table 3.3. Initially we will use a scenario where there is no risk of milk contamination, no testing is implemented and therefore the probability of acceptance at each stage is 1. This scenario gives us a starting point from which to change parameter values as necessary. In this case we are left with only five parameters to estimate; Farm vat collection amount V, Frequency of Collection Φ, Frequency of Delivery $\mathcal{X}$, Frequency of process entry Ψ and Frequency of process exit Ω.

Because milk price in New Zealand is measured in $/kg MS, it is handy to have a conversion estimate. The kg MS per litre of milk varies throughout the year. If we choose to cover only the main season, from August to April, when most farms are regularly producing milk, we should use the estimate that best applies to that time period for conversion. 1 Litre = 0.0899 kg MS. This is given in Table 3.2 along with values for the daily production per farm or per cow for each month of the year in the 2014/2015 season.

Using the information in Table 3.2 we can estimate daily production values for the main season when most farms are producing, as well as just the peak season.

	Production per day						kg MS
	per cow				per herd		per litre
Month	**Litres**	**Milkfat kg**	**Protein kg**	**kg MS**	**Litres**	**kg MS**	
June	17.26	0.83	0.66	1.49	7231.94	624.31	0.0863
July	18.34	0.87	0.72	1.59	7684.46	666.21	0.0866
Aug	22.01	1.04	0.84	1.88	9222.19	787.72	0.0854
Sep	23.50	1.08	0.89	1.97	9846.50	825.43	0.0838
Oct	23.66	1.08	0.90	1.98	9913.54	829.62	0.0836
Nov	21.33	1.01	0.82	1.83	8937.27	766.77	0.0857
Dec	20.12	0.96	0.78	1.74	8430.28	729.06	0.0864
Jan	17.41	0.86	0.67	1.53	7294.79	641.07	0.0878
Feb	15.30	0.80	0.61	1.41	6410.70	590.79	0.0921
Mar	13.19	0.74	0.56	1.30	5526.61	554.70	0.1003
Apr	12.24	0.72	0.56	1.28	5128.56	536.32	0.1045
May	13.05	0.74	0.59	1.33	5467.95	557.27	0.1019
Peak (Aug - Oct)							
Ave	23.05	1.06	0.87	1.94	9660.74	807.59	0.0842
Main Season (Aug - Apr)							
Ave	18.75	0.92	0.73	1.65	7856.71	693.49	0.0899
Full Season (Aug - July)							
Ave	18.11	0.89	0.71	1.61	7591.23	674.10	0.0903

Table 3.2: Milk production per cow per day, then per herd per day, by month from June 2014 to May 2015. The average kg MS is also given for each month. Average production values are calculated for peak production season between August and October and the whole of the main production season from August to April. May to July is the off season, generally only farms with special winter milk contracts are producing during these months, which may make the data during this period less reliable [5].

Given there were 11970 herds supplying Fonterra in the 2014/2015 season [5], the average daily production from August to April can be estimated as $11970 \times 693.49 = 8,301,075$kg MS, equivalent to 92,336,763 litres. The average daily production in just the peak season from August to October was $11970 \times 807.59 = 9,666,859$kg MS, the equivalent of 114,808,222 litres of Milk.

- V Based off the information in Table 3.2, during the main milking season each farm is producing an average of 693 kg MS per day, during the peak months of the year this jumps to 808 kg MS. Using a price of \$4.40 per kg MS, as detailed in section 2.2.3, we can estimate the average value of milk produced by and collected from each farm per day as $V = \$4.40 \times 693 = \$3049.2$.

- Φ Given a milk tanker's capacity of 28,800 litres, we can estimate the number of farms a tanker can collect milk from in one run as between $\frac{28800}{7856.71} = 3.66$ and, $\frac{28800}{9660.74} = 2.98$ or approximately 3 farms per collection run. If 3 farms are each collected from in one tanker run then we need $\frac{11970}{3} = 3990$ tanker runs every day. Remembering that Fonterra has a fleet of 525 tankers, $\frac{3990}{525} = 7.6$ so each tanker needs to make 3 or 4 runs per shift, which fits in the situation as described in section 2.2.2. If each tanker makes six collections in each run

Parameter	Description	Initial values
V	Amount of Milk collected from an on-farm vat	693 kg MS
Φ	Frequency of collection a by tanker	11970
$\mathcal{X}$	Frequency of delivery to factory	3990
Ψ	Frequency with which milk enters processing	311
Ω	Frequency of production	346
E_T	Costs of testing milk at collection site	0
E_F	Cost of testing milk upon delivery	0
D_F	Cost of disposing of unwanted milk	0
E_P	Cost of testing prior to processing	0
D_P	Cost of disposing of unwanted milk at factory level	0
α	Probability of acceptance by tanker	1
β	Probability of passing tests upon arrival at factory	1
γ	Probability of passing pre-processing tests	1

Table 3.3: Parameter values in the no risk scenario. All frequencies are the average number of occurrences per day.

then each tanker will make $7.6 \times 3 = 22.8$ collections. $\Phi = 11970$ collections each day.

$\mathcal{X}$ Each run involves delivering to the factory once every run, so the frequency with which that a tanker delivers to the factory is 7.6. There will be $\mathcal{X} = 3990$ deliveries each day.

Ψ Fonterra has the capacity to process about 70 Million litres of milk per day during the peak season[12]. Milk reception silos range in size from 225,000 to 500,000 litres. A typical paediatric processing site has 3 silos of 225,000 capacity, or a total reception capacity of 675,000 litres. Given that Fonterra operates 33 processing sites [9] and $\frac{70000000}{33\times225000} \approx 9.43$, each site would need to process 9 or 10 silos of raw milk each day, or a total of 311 silos each day. So we set $\Psi = 311$.

Ω The typical paediatric processing site has a bank of three separators feeding into 2 cream silos, 3 skim milk silos and 2 excess silos. This gives a total capacity of $3 \times 50 + 2 \times 95,000 + 3 \times 350,000 + 2 \times 90,000 = 1,420,150$ Litres. It will be one silo capacity that empties each time which we estimate as $\frac{1,420,000}{7} = 202,857$ Litres. We can then estimate the rate number of times this volume will need to be separated each day as $\Omega = \frac{70000000}{202,857} = 346$.

3.5 Single-Event Model Simulations

Before we start analysing how testing affects the model, we need to check that the model gives realistic output in simulations. Initially, as we have not yet included any costs we can run our simulations using Kg MS as our units. This is simpler to deal with as it is not subject to exchange rate and purchasing power fluctuations.

Figure 3.6 shows a single simulation of the cumulative milk produced though our model over 10 hours, along with a cumulative plot of the deterministic model.

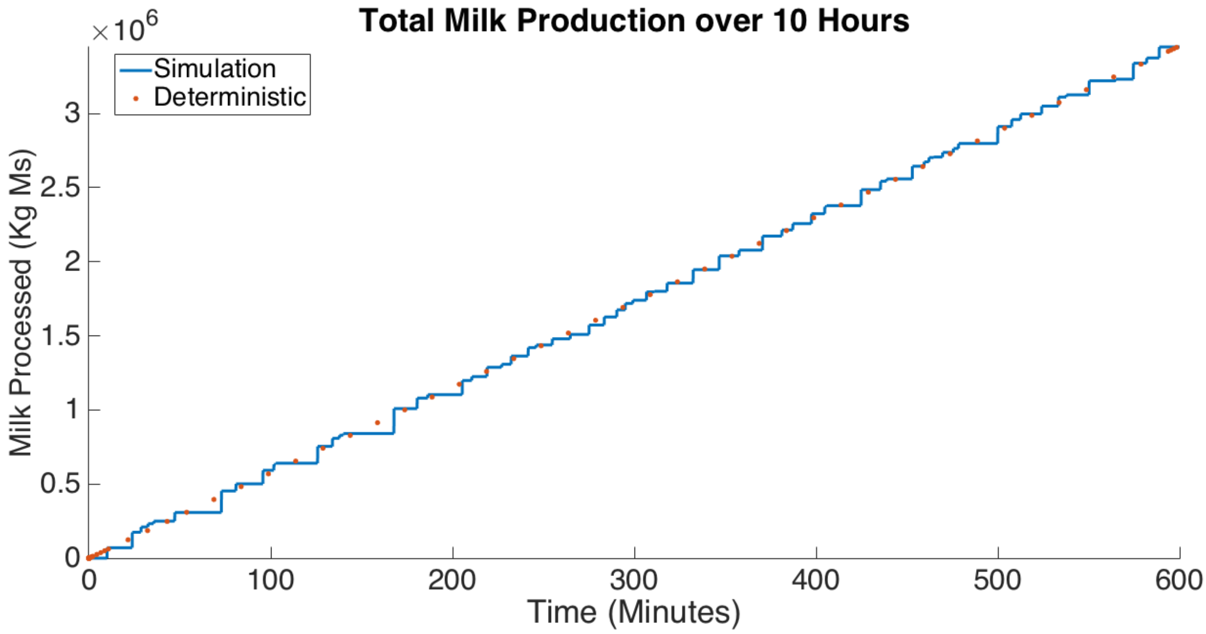

Figure 3.6: The amount of milk produced after the three stages of processing in our model. The red dotted line shows deterministic model, where the milk moves though each stage smoothly. The solid blue line shows a stochastic simulation using a time step of 5 seconds.

After 10 hours The stochastic model, in the case, has produced 3,449,800 kg MS, the deterministic model reaches 3,456,300. These values equate to 8,279,500kg MS per day and 8,160,000kg MS per day respectively. Given the collection rates seen in section 2.2.3 and table 3.2 and the absence of any product rejection, these values are what we would expect production to average during the main milking season, from August to April.

For each of the plots in this section, including Figure 3.6, The initial values in each stage at time $t = 0$ are estimated by equations 3.4, 3.6 and 3.7. These equilibrium values are where we would expect each stage to settle regardless of what initial values are chosen, using these initial values allows us to estimate average values and produce realistic plots without requiring a 'burn-in' period we would need to remove.

The interaction of all three stages is shown in figure 3.7. The effect the tanker value has on the factory stage can be seen in the little bumps as the tankers feed into the factory. The processing stage shows less frequent but larger bumps as the the factory storage stage empties into it.

Zooming in on an hour near the middle of figure 3.7, as is shown in figure 3.8 allows us to see the interaction between the Tanker stage and the factory reception stage more clearly. The emptying of the tanker stage coincides with an increase in the factory stage as expected.

In this simulation the amount of milk in the tanker stage averages 1936.3 Kg MS. The Factory reception and Processing stages each average 22941 Kg MS and 23414 Kg MS respectively.

Using the current parameter values and equation 3.4, the deterministic solu-

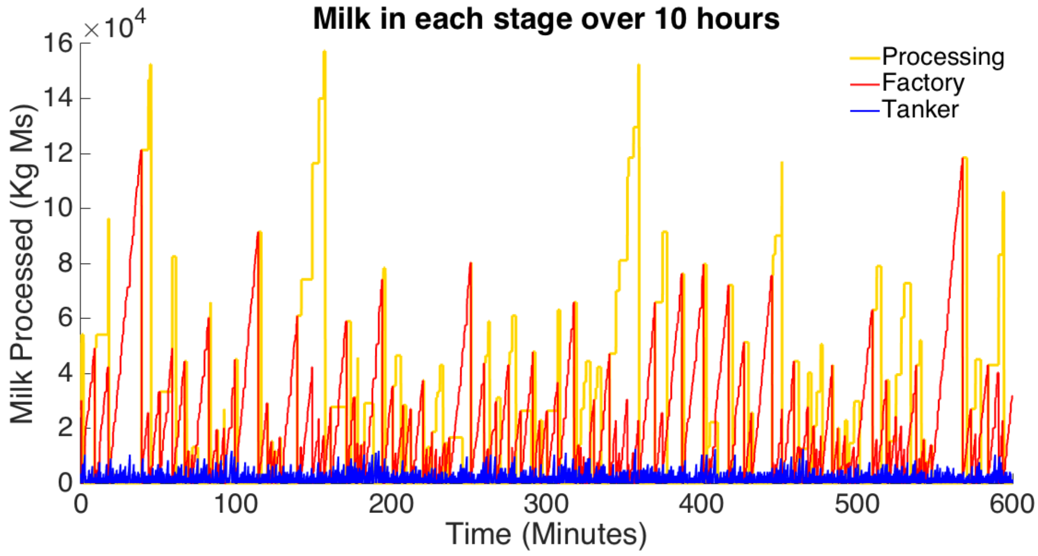

Figure 3.7: A simulation of the milk in each stage over a 10 hour period, using a time step of 5 seconds. The parameter values are those given in Table 3.3.

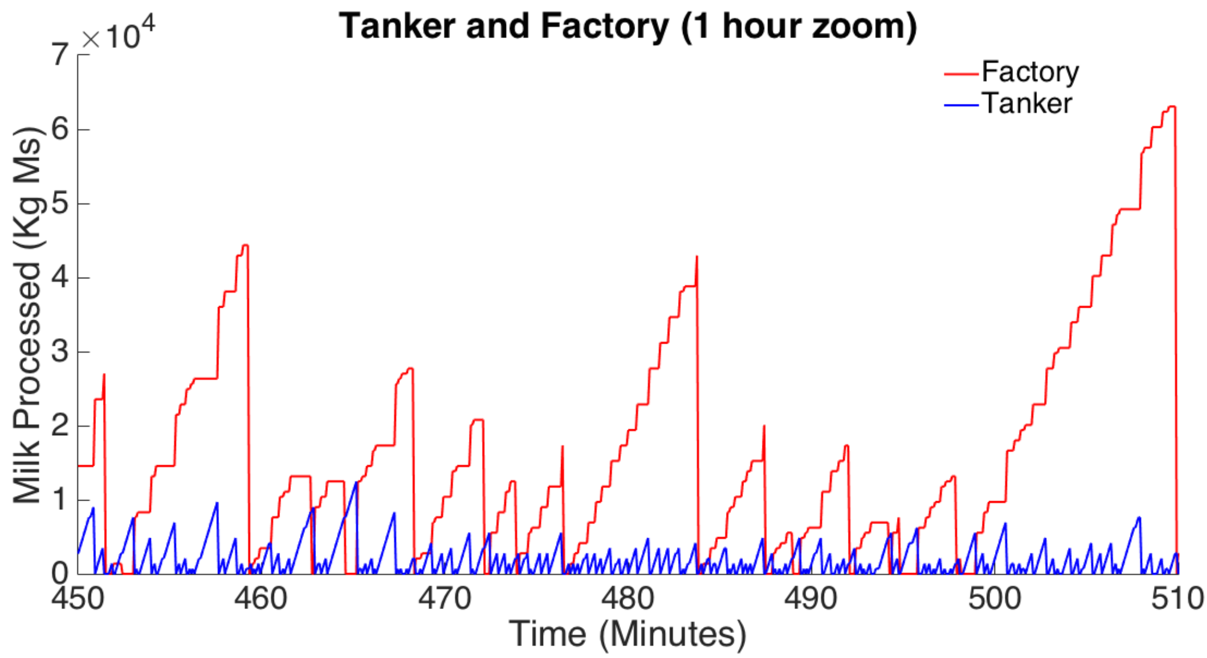

Figure 3.8: An hour from the middle of Figure 3.7 focusing on the simulation of the Milk Tanker and Factory Reception stages.

tion for the tanker stage, when in equilibrium, is 2079kg MS. The values given by equations 3.6 and 3.7 are 26673kg MS and 23975kg MS respectively. In the Tanker case the difference is 142.7kg MS or 6.86% of the expected value, over just this 10 hour simulation. In the case of the Factory receptions stage the difference is 3732kg MS or 13.99% while the difference in the processing stage is just 561kg

MS or 2.34%. Keeping in mind that this is just a 10 hour simulation there is still a large difference between how well each stage matches the expected equilibrium value, though they are all at least in the right order of magnitude. The tanker stage may match the equilibrium value so well because the value in the stage is regularly changing by small amounts. The other two stages change value less often, but change dramatically when they do change.

Over 60 runs, equivalent to 600 hours or 25 days, The tanker average in 2074.98kg MS, this is much closer to the predicted equilibrium value of 2079kg MS, a difference of just 4.02kg MS or 0.19%. The Factory reception and Processing stages average 26861.03kg MS and 23467.68kg MS. The difference between the Factory simulation average and the deterministic equilibrium has drastically decreased as well to just 188.03kg MS or 0.70%. In the case of the processing stage, the average level is 507.32kg MS below that of the deterministic equilibrium or 2.11%. Interestingly this is very similar to the difference after a single simulation and may be due to the large volume of product that is potentially moving in and out of this each time step.

Over 120 runs, or essentially 1200 hours or 50 days the Tanker average is 2076.24kg MS, the Factory Reception average is 26851.49kg MS and the processor average is now 23689.76kg MS. The percentage differences from the deterministic equilibrium are now 0.13%, 0.67% and 0.12%. All three stages now average closer to the deterministic equilibrium value as we would expect them to after more and more runs.

Stage	Deterministic Equilibrium	25 day Average	50 day average
Tanker	2,079 kg MS	2074.89 kg MS	2076.24 kg MS
Factory Reception	26,673 kg MS	26861.03 kg MS	26851.49 kg MS
Processing	23,975 kg MS	23467.68 kg MS	23689.76 kg MS

Table 3.4: The average value in each stage after simulations of 25 and 50 days along with the equilibrium values given by the deterministic model given in section 3.1.4.

All three stages spend a significant amount of time empty in this model, to the point where the lower quartile for each stage is 0 in every simulation. This is likely due to how a stage is totally emptied each time it passes product on the the next stage.

In the next section we will adjust the model to change values more smoothly in each stage and see how this affects the simulations relative to the equilibrium values.

4 Developing The Multi-Event Model

In the previous section, each stage was essentially only dealing with the events for one unit, i.e. in the tanker stage we were only dealing with one tanker with the capacity of 525 tankers. This may have been a simple way of dealing with the different numbers of units at each stage but, because the volume of milk in each stage was changing so much with each time step, we had very large and sudden peaks and troughs. This was particularly noticeable in the factory reception and processing stages, and is something we will attempt to improve in this section. We will start by standardising the volume of milk transfered at each transition.

4.1 Event Restrictions

We will also present the transition probabilities individually for three stages in this section.This means that, though the transition probabilities in each stage of the supply chain will be influenced by the events that take place in the previous production stage, events occurring at different stages can take place simultaneously during the same time step. We need to be aware of, and account for, resulting exclusions and event implications.

4.2 Capacities and Transition Values

The capacity of each stage, and of the individual compartments in that stage, affects the movement of product through the supply chain. A milk tanker can only carry a certain volume of milk which limits how much milk it can collect and how much it can deliver to the factory within a given time frame. The same applies to the Factory reception and Processing stages.

The amount the moves in a given time step will affect the probability of movement. The larger the movement the less frequently it will occur. The length of the time step will also be affected, a time step that is too long for a given movement amount will not allow a realistic flow of product, resulting in a production rate below what we would expect.

While allowing only a set amount of material to move in each time step removes the effect of whole stages emptying at once, it is not quite realistic to have the same amount moving at each stage. For example, the amount collected by a tanker in one step is smaller than the amount delivered to the factory in a given time step.

In this section we modify the existing transition probabilities and transition quantities to reflect the capacities of each stage.

4.3 Transition probabilities

4.3.1 The Milk Tanker

The value in each tanker is still incorporated into one pool, so the value collected or delivered by one tanker is still added to, or deducted from the tanker value pool, but tankers will add to or remove material from that pool individually.

The probability that a milk tanker will collect milk varies with the amount already contained in the tanker stage. If for some time t, $T(t) \geq N_T$ then all of the milk tankers are full and milk collection must be impossible for that time step. N_T is the capacity of the entire milk tanker fleet

So the probability of an attempted milk collection in time step Δt is given by

$$\begin{aligned} &\Phi \frac{N_T - T(t)}{N_T} \Delta t \\ &= \left(1 - \frac{T(t)}{N_T}\right) \Phi \Delta t \end{aligned} \tag{4.1}$$

As $T(t)$ approaches N_T the probability of a collection attempt approaches 0. Remember that Φ is the number of attempted collections per day.

Collection: If α is the probability a silo of milk is accepted, then the probability that a milk tanker will collect milk in a time step Δt is given by:

$$\alpha \left(1 - \frac{T(t)}{N_T}\right) \Phi \Delta t, \qquad \text{for } T(t) \leq N_T \tag{4.2}$$

The upper limit on the value of $T(t)$ is required to prevent a negative value. The change in the tanker stage in this case is $V - E_T$, the amount provided by the farm vat minus the cost of testing it.

Rejection: The probability of the milk tanker rejecting the milk supplied at a farm is given by

$$(1-\alpha) \left(1 - \frac{T(t)}{N_T}\right) \Phi \Delta t, \qquad \text{for } T(t) \leq N_T \tag{4.3}$$

This results in a change of $-E_T$, the cost of testing.

Delivery: The probability that a tanker will deliver to the factory is dependent on the amount milk in the tanker stage available to deliver. Delivery is also not possible if the factory reception stage is full.

The capacity of the factory reception stage is given by N_F Thus if $F(t) \geq N_F$ delivery to the factory is not possible.

If we set $F(t) \leq N_F$, the probability of a tanker delivering milk to the factory in time step Δt is be given by

$$= \frac{\mathcal{X} T(t)}{N_T} \left(1 - \frac{F(t)}{N_F}\right) \Delta t \tag{4.4}$$

Where $\mathcal{X}$ is the number of deliveries to the factory attempted each day. The change in tanker value upon delivery is $-C_T$.

$T(t)$ logically contributes to the probability of a delivery occurring in a given time step. Because of this if there is no milk in any tankers, i.e. $T(t) = 0$, the probability of a delivery in that time step is 0. The probability of delivery also increases proportionally with the value of milk contained in the tankers.

Time step: To ensure that the probabilities in this stage all add to less than one we need to choose Δt, such that

$$\left[\left(1 - \frac{T(t)}{N_T}\right) \Phi + \frac{\mathcal{X} T(t)}{N_T} \left(1 - \frac{F(t)}{N_F}\right)\right] \Delta t < 1 \tag{4.5}$$

Because $T(t)$ and $F(t)$ change with time, we need to ensure 4.5 holds for all values these can take. The lefthand side of 4.5 is maximised when $(T(t), F(t)) = (N_T, 0)$, for $\mathcal{X} > \Phi$.

If $\mathcal{X} < \Phi$, the left hand side is maximised at T(t)=0, but this is not the case for any scenario presented in the paper.

So we now require

$$\begin{aligned} \left[\left(1 - \frac{N_T}{N_T}\right) \Phi + \frac{\mathcal{X} N_T}{N_T} (1 - 0)\right] \Delta t < 1 \\ \implies \mathcal{X} \Delta t < 1 \\ \implies \Delta t < \frac{1}{\mathcal{X}} \end{aligned} \tag{4.6}$$

Summary equation Equation 4.7 summarises the new transition probabilities we have just described for the milk tanker stage.

$$p_{ij}(\Delta t) = \begin{cases} \alpha\left(1-\dfrac{T(t)}{N_T}\right)\Phi\Delta t & j = i+V-E_T \quad \text{Collection} \\ (1-\alpha)\left(1-\dfrac{T(t)}{N_T}\right)\Phi\Delta t & j = i-E_T \quad \text{Rejection} \\ \dfrac{\mathcal{X}T(t)}{N_T}\left(1-\dfrac{F(t)}{N_F}\right)\Delta t & j = i-C_T \quad \text{Delivery} \\ 1-\left[\left(1-\dfrac{T(t)}{N_T}\right)\Phi+\mathcal{X}T(t)\left(1-\dfrac{F(t)}{N_F}\right)\right]\Delta t & j = i \quad \text{Transporting} \\ 0 & \text{Otherwise} \end{cases} \tag{4.7}$$

4.3.2 The Factory Reception

Fonterra has 33 processing sites around the country [9], as with the tankers we will be treating these as part of the factory reception pool, but each site receives and passes product on individually.

Each milk tanker has both a trailer and a truck compartment. Because these two compartments can be kept separate they can be accepted and rejected separately.

Figure 4.1 shows a probability tree for how the probability of each acceptance and rejection combination is calculated. We end up with three main possibilities; total acceptance, partial acceptance or total rejection.

The probability that a tanker compartment is contaminated is not totally independent of the other compartment's status. There is the possibility that some milk from one of the farms collected from, ends up in each tanker. For example if each tanker is collecting from 6 farms, there is a 1 in 6 chance that an overlapping collection is from the contaminated batch.

In the case of total acceptance no contamination is detected in either tanker compartment.

Partial acceptance implies just one tanker compartment is accepted by the factory.

Total acceptance requires both tanker compartments to pass all tests. Therefore the probability of a milk delivery being fully accepted, is simply the probability of a delivery attempt, multiplied by the probability that both compartments pass. The probability of a delivery attempt is given in equation 4.4. If β is the probability of a tanker compartment passing all tests conducted at this stage, the probability of total acceptance is given by

$$= \frac{\beta^2 \mathcal{X}T(t)}{N_T}\left(1-\frac{F(t)}{N_F}\right)\Delta t \tag{4.8}$$

The resulting value change is $C_T - E_F$. Where E_F is the cost of conducting tests at the factory reception stage.

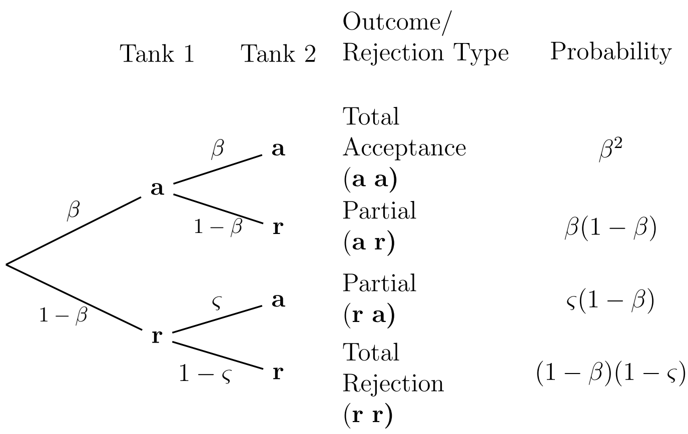

Figure 4.1: Probability tree showing the possible outcomes when milk is delivered to the factory. **r** denotes the rejection of a tanker compartment, while **a** denotes acceptance.

Partial acceptance: A partial acceptance occurs with probability

$$\frac{(1-\beta)(\beta+\varsigma)\mathcal{X}T(t)}{N_T}\left(1-\frac{F(t)}{N_F}\right)\Delta t \tag{4.9}$$

This is the sum of the probabilities that the rejection occurs in either tank. The order of rejection has no effect in this model. Because only one tanker compartment is rejected, half of the milk that was delivered is added to that currently contained in the factory reception stage. Half of the disposal cost is also incurred due to the compartment that needs to be disposed of. The full testing costs still apply. Thus the value change in the factory stage in the case of of a partial rejection is given by $\frac{C_T-D_F}{2}-E_F$.

Total rejection: A total rejection requires both tanks to be rejected due to the results of testing on arrival at the factory. The probability of this occurring is given by

$$\frac{(1-\beta)(1-\varsigma)\mathcal{X}T(t)}{N_T}\left(1-\frac{F(t)}{N_F}\right)\Delta t \tag{4.10}$$

The value change in this case is simply the cost of testing and disposing of both tanker compartments

$$-E_F-D_F$$

Passing on: material to the initial processing stage is the other possible change that can occur in given time step. The probability that milk will leave the factory reception stage and move on for processing is dependent on the value of milk contained in the factory reception stage.

The probability that milk will be passed on for processing is given by

$$\frac{\Psi F(t)}{N_F}\Delta t \tag{4.11}$$

Where N_F is the total capacity of the factory reception stage. The change in value when milk is passed on for processing is $-C_F$.

Time step Similar to the tanker stage we need to define an upper bound for Δt that ensures the transition probabilities in this stage always add to less than one. We require

$$\left[\frac{\mathcal{X}T(t)}{N_T}\left(1-\frac{F(t)}{N_F}\right)+\frac{\Psi F(t)}{N_F}\right]\Delta t < 1 \tag{4.12}$$

In our all the scenarios presented in this paper $\mathcal{X} > \Phi$, meaning the left side of the equation is maximised when $(T(t), F(t)) = (N_T, 0)$ Thus the upper bound for Δt in the factory reception stage is given by

$$\frac{\mathcal{X}N_T}{N_T}\Delta t < 1 \tag{4.13}$$

$$\implies \Delta t < \frac{1}{\mathcal{X}} \tag{4.14}$$

Summary equation The Factory reception transition probabilities just described are summarised in equation 4.15 below

$$p_{uv}(\Delta t) = \begin{cases} \frac{\beta^2\mathcal{X}T(t)}{N_T}\left(1-\frac{F(t)}{N_F}\right)\Delta t & v = u + C_T - E_F \quad \text{Acceptance} \\ \frac{(1-\beta)(\varsigma+\beta)\mathcal{X}T(t)}{N_T}\left(1-\frac{F(t)}{N_F}\right)\Delta t & v = u + \frac{C_T - D_F}{2} - E_F \quad \text{Partial Rejection} \\ \frac{(1-\beta)(1-\varsigma)\mathcal{X}T(t)}{N_T}\left(1-\frac{F(t)}{N_F}\right)\Delta t & v = u - E_F - D_F \quad \text{Rejection} \\ \frac{\Psi F(t)}{N_F}\Delta t & v = u - C_F \quad \text{Passing on} \\ 1 - \left[\frac{\mathcal{X}T(t)}{N_T}\left(1-\frac{F(t)}{N_F}\right)+\frac{\Psi F(t)}{N_F}\right]\Delta t & v = u \quad \text{Holding} \\ 0 & \text{Otherwise} \end{cases} \tag{4.15}$$

4.3.3 The Processing Stage

This model represents the first stages of processing that every Fonterra dairy product undergoes, separation and standardisation.

Acceptance: The probability that all of the milk in a reception silo is accepted for processing is now

$$\frac{\gamma\Psi F(t)}{N_F}\Delta t \tag{4.16}$$

The value change, now limited by the capacity of a reception silo, becomes $C_F - E_P$. Where E_P is the cost associated with testing at this stage.

Rejection: The probability of a rejection upon entry to the processing stage is given by

$$\frac{(1-\gamma)\Psi F(t)}{N_F}\Delta t \tag{4.17}$$

The whole reception silo is lost and the value change becomes

$$-E_P - D_P, \tag{4.18}$$

where D_P is the disposal cost at this stage.

Passing on: The probability that material is passed on from this processing stage is given by

$$\frac{\Omega P(t)}{N_P}\Delta t \tag{4.19}$$

Material is passed on at a constant rate which we will denote as Q.

Because the the probability of passing on contains $P(t)$, this value is now proportional to the amount contained in this stage.

Time step: In order to define the upper bound for Δt in this stage, we require

$$\left(\frac{\Psi F(t)}{N_F} + \frac{\Omega P(t)}{N_P}\right)\Delta t < 1 \tag{4.20}$$

This is maximised when $(F(t), P(t)) = (N_F, N_P)$. Giving us

$$\Delta t < \frac{1}{\Psi + \Omega} \tag{4.21}$$

Summary equation: Equation 4.22 summarises the transition probabilities we have just described.

$$p_{gh}(\Delta t) = \begin{cases} \frac{\gamma \Psi F(t)}{N_F}\Delta t & h = g + C_F - E_P \quad \text{Acceptance} \\ \frac{(1-\gamma)\Psi F(t)}{N_F}\Delta t & h = g - E_P - D_P \quad \text{Rejection} \\ \frac{\Omega P(t)}{N_P}\Delta t & h = g - Q \quad \text{Passing on} \\ 1 - \left[\frac{\Psi F(t)}{N_F} + \frac{\Omega P(t)}{N_P}\right]\Delta t & h = g \quad \text{Producing} \\ 0 & \quad \text{Otherwise} \end{cases} \tag{4.22}$$

4.3.4 Transition Probability Restrictions and Implications

The three sets of transition probabilities in equations 4.22, 4.15 and 4.7 are not independent. What happens in one stage affects what happens in the other stages. For example, a tanker delivering milk to the factory must coincide with the factory either receiving milk or rejecting it.

4.4 General Parameter Values

Some parameters, such as collection frequency, will remain the same throughout the following model simulations, regardless of scenario. We will therefore discuss the estimates for these parameters before continuing with individual testing scenarios.

As defined in section 2.2.3, we are using a milk price of \$4.40.

Milk Tanker Parameters

V is the average amount of milk collected from a farm vat. The estimate for V used throughout the following simulations is 693 kg MS, this is equivalent to $V = \$3,050$.

Φ The frequency of collection attempts can be estimated as the number of on farm vats that are collected from each day. Referring back to section 3.4 there will be $\Phi = 11970$ collection attempts each day.

C_T is defined as the average capacity of one milk tanker. As discussed in section 3.4 each tanker has a capacity of about 28,800 L =2433 kg MS, therefore $C_T = \$10,700$

N_T is the capacity of the entire fleet of tankers. In this case, using a fleet of 525 tankers we have a capacity of 1277325kg MS, which gives us $N_T = \$5,620,230$.

$\mathcal{X}$ The frequency with which a tanker delivers milk to factory is given by $\mathcal{X} = 3990$.

Factory Reception Parameters

C_F Each processing site has multiple reception silos, as mentioned in section 3.4, a typical paediatric site has silos of 225,000 Litre capacity. This equates to $20,228$kg MS, giving $C_F = \$89,000$.

N_F The typical paediatric processing site has three reception silos, with 33 processing sites around the country this gives a total reception capacity of $N_F = \$8,811,000$.

Ψ is the rate at which milk moves into the processing stage from the reception silos. Each separator is capable of separating 33,000 L of raw milk each hour. This gives $33 \times 3 \times 33,000 \times 24 = 78,408,000$ litres per day, which is equivalent to 6,923,463 kg MS day. Dividing this by the capacity of one reception silo C_F, gives us $\Psi = 343$.

Initial Processing Parameters

C_P As described in section 3.4, the total capacity of separators and silos for separated product is 1,420,150 litres which equates to 127,670kg MS. This gives us $C_P = \$561,750$.

N_P is the total capacity of the processing stage. Again there are 33 processing sites, giving $N_P = \$18,537,750 = 4,213,110$kg MS.

Q is the unit of milk leaving the processing stage each time step. We need to choose $Q \leq C_P$, small enough for the frequency of movement to be realistic but large enough that enough product can move for a given time step size. Because the stage is continuous we may vary the value of Q. For our initial simulations we use $Q = \$187,000$.

Ω is the rate which product leaves the initial processing stage and moves on to further processing depending on the end product. Again this is continuous so it only matters that $\Omega \times Q > \$308,000,000$. We use $\Omega = 7100$

4.5 The Best of all Possible Worlds Scenario

The initial set of simulations, using the model outlined in section 4.3, involves no testing cost or possible contamination. A scenario that would exist in a perfect world. The parameter values for this scenario are summarised in Table 4.1.

A simulation of the total product produced over 24 hours is given in Figure 4.2, along with the deterministic solution discussed in section 3.1. This particular simulation reaches a value of \$26.5 million worth of milk after 24 hours. Given that Fonterra can produce over \$26 million of product per day, this model appears sensible.

Over 500 simulations, each 24 hours in length, the average value of milk produced was \$26.52 million. The maximum produced in any of these simulations was \$28.05 million and the minimum \$25.24 million.

It is possible using this model to see the interaction between the states, as shown in Figure 4.3. The changes are not large due to the size restriction we have placed on the transitions, but a decreasing trend in the Tanker stage is reflected by an increasing trend in the factory reception stage.

The dashed lines in Figure 4.3 show the deterministic solution for each stage. These are the values we would expect the simulations to average to given enough run time in equilibrium. We use these as our initial values for the simulations to remove the need for a burn in period. These values are given in table 4.2 along with the average value after 500 simulations of 24 hours each.

The processing stage shows the most variation. While this stage has the smallest standard deviation of the three, this is over half the average value of this stage. This is possibly due, in part, to the large about of time the processing stage spends below Q, being essentially empty, with the occasional spike in product levels. This may also explain the slightly larger difference in

Parameter	**Description**	**Initial values**
V	Amount of milk collected from an on-farm vat	\$3,050
Φ	Frequency of collection a by tanker	11970
E_T	Costs of testing milk at collection site	0
α	Probability of acceptance by tanker	1
$\mathcal{X}$	Frequency of delivery to factory	3990
E_F	Cost of testing milk upon delivery	0
β	Probability of passing tests upon arrival at factory	1
D_F	Cost of disposing of unwanted milk	0
Ψ	Frequency with which milk enters processing	343
E_P	Cost of testing prior to processing	0
γ	Probability of passing pre-processing tests	1
D_P	Cost of disposing of unwanted milk at factory level	0
Ω	Frequency of production	7100
C_T	Capacity of one Milk Tanker	\$10,700
N_T	Capacity of tanker stage	\$5,620,230
C_F	Capacity of one reception silo	\$89,000
N_F	Capacity of factory reception stage	\$8,811,100 MS
C_P	Capacity of one separator unit	\$561,750
Q	Process exit amount	\$22,000
N_P	Capacity of Processing stage	\$18,537,700

Table 4.1: Parameter values in the no risk scenario. All frequencies are the average number of occurrences per day.

value we see between the stochastic simulation average and the deterministic equilibrium. Often as soon as the processing stage makes it to a value higher than Q, it will pass on product, dropping its value again.

Stage	Initial Value	500 Run Average	Min	Max	Std Dev
Tanker	\$1,540,200	\$1,542,586	\$7,1125	\$2,089,800	\$110,680
Factory Reception	\$3,824,300	\$3,825,362	\$992,620	\$5,280,300	\$306,480
Processing	\$184,990	\$179,830	\$99	\$728,990	\$77,447

Table 4.2: Simulation results for each production stage after 500 simulation runs in the perfect world scenario.

4.6 Effects of Testing

By introducing testing, we are now able to reject product. Initially we run the simulations using parameter values defined by real world information.

4.6.1 Initial Testing parameters

E_T This is the estimated cost of testing milk when its collected by the tanker. Based on information about testing in dairy mentioned in section 2 we

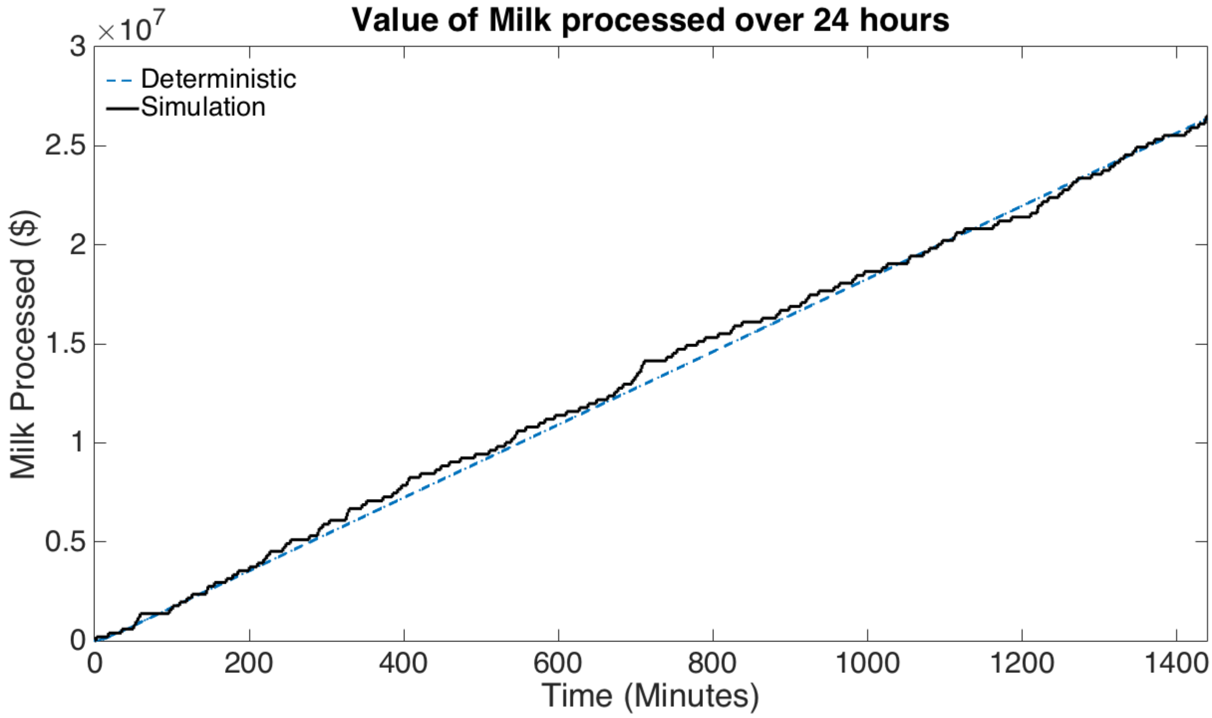

Figure 4.2: This plot shows the total value of the milk produced over a 24 hour period. Using the transition probabilities summarised in section 4.4, the parameter values summarised in table 4.1. The step size used is $\Delta t = 8$ Seconds. The dashed blue line shows the deterministic solution, the solid black line is the simulation.

will estimate $E_T = \$0.30$ NZD.

α is the probability that the milk passes all testing and is accepted by the tanker. The vat acceptance rates mentioned in section 2 suggest we set $\alpha = 0.9999$.

E_F is the cost of testing milk as it arrives at the factory. Using on the earlier test cost estimate along with the test costs discussed in section 2, we estimate $E_F = \$1.50$.

β is the probability that a tanker load is accepted by factory. This is the stage with the greatest rate of rejection, with 1% of milk being discarded upon arrival at the factory. This means $\beta = 0.99$.

ς is the probability the second tank of a tanker will be accepted, given that the first tank was rejected. Based on the number of farms visited by each tanker, there is about a 1 in 4 chance that the contaminated load spans both tanks. Taking this into account along with the possibility there is a second unrelated contamination we can estimate $\varsigma = 0.75\beta = 0.7425$

D_F Depending on the reason for rejection, most rejected milk can be used as calf feed or sprayed on crops as fertiliser. Fonterra does contract tankers from outside their own fleet to transport this rejected milk, but the associated costs can generally recouped in the price paid for this

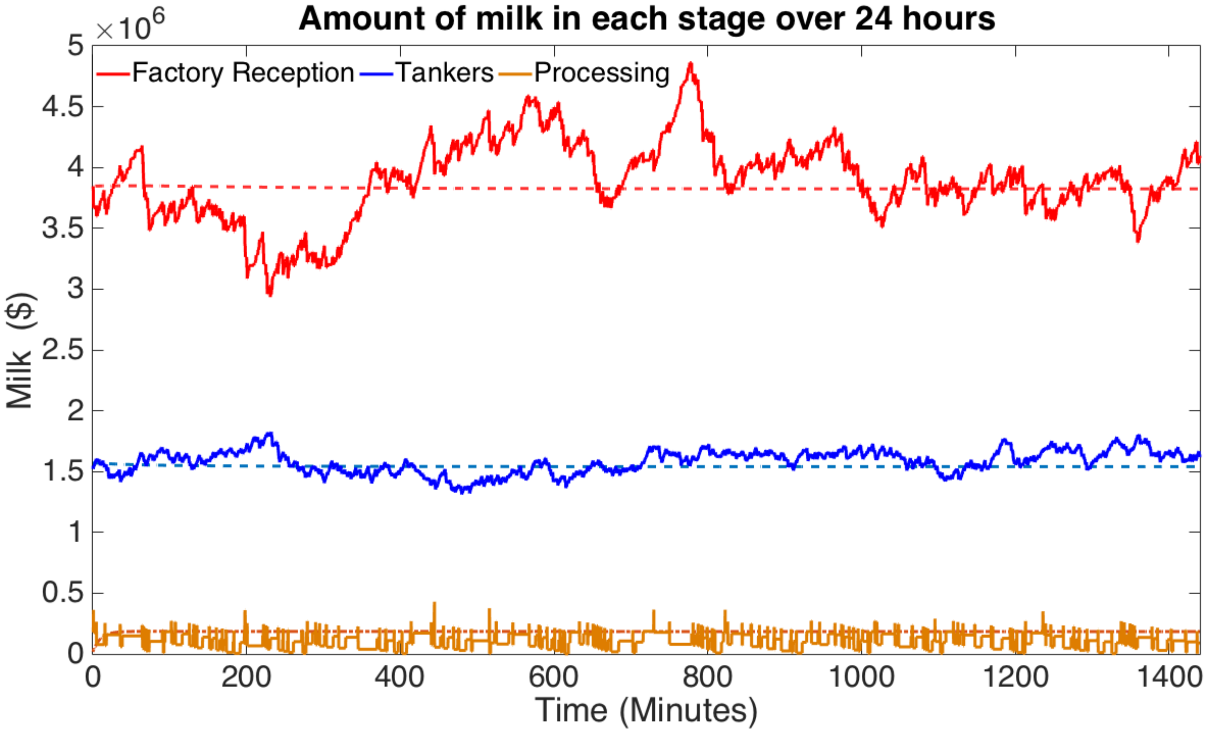

Figure 4.3: This plot shows value in the milk tanker factory reception and processing stages, over two hours of simulation along with the deterministic solution as dashed lines. The transition probabilities are those developed in Section 4.3 and the parameters values are summarised in Table 4.1. The step size used is $\Delta t = 8$ Seconds.

rejected product. Because of this we set $D_F = 0$.

- E_P Assuming the range of tests conducted pre-processing is similar to those conducted before acceptance into the factory, we set $E_P = \$1.50$
- γ The rate of rejection before entry into the processing stage is the lowest of the three stages. Once the milk is inside the factory the environment is much more controlled, the potential for contamination or spoilage is greatly reduced. We set the chance of rejection at 0.00001%, implying $\gamma = 0.99999$.
- D_P As discussed in reference to D_F, disposal cost is negligible so we can set $D_P = 0$.

These parameters are summarised in Table 4.3.

4.6.2 Simulations for initial testing scenario

The total value of milk produced over 24 hours is shown in Figure 4.4, the value reached after 24 hours is \$26 million.

The average production over 500 simulations is \$26.16 Million with a minimum of \$24.68 Million and a maximum of \$27.67 Million. This drop in production is not huge but it is consistent.

Parameter	Description	Initial values
V	Amount of milk collected from an on-farm vat	\$3,050
Φ	Frequency of collection a by tanker	11970
E_T	Costs of testing milk at collection site	\$0.30
α	Probability of acceptance by tanker	0.9999
$\mathcal{X}$	Frequency of delivery to factory	3990
E_F	Cost of testing milk upon delivery	\$1.50
β	Probability of passing tests upon arrival at factory	0.99
ς	Conditional 2nd acceptance probability	0.7425
D_F	Cost of disposing of unwanted milk	0
Ψ	Frequency with which milk enters processing	343
E_P	Cost of testing prior to processing	\$1.50
γ	Probability of passing pre-processing tests	0.99999
D_P	Cost of disposing of unwanted milk at factory level	0
Ω	Frequency of production	7100
C_T	Capacity of one Milk Tanker	\$10,700
N_T	Capacity of tanker stage	\$5,620.230
C_F	Capacity of one reception silo	\$89,000
N_F	Capacity of factory reception stage	\$8,811,100
C_P	Capacity of one separator unit	\$561,750
Q	Process exit amount	\$22,000
N_P	Capacity of Processing stage	\$18,537,700

Table 4.3: Parameter values in the testing only scenario. All frequencies are the average number of occurrences per day. Capacities and movement amounts are measured in NZ dollars of milk, for better compatibility with the introduced costs.

The average values in each stage for a given point in time are given in Table 4.4. There is a slight drop in the level maintained by the tankers and factory receptions but the most significant change is in the processing stage. The equilibrium value maintained in the processing stage has basically been halved. Figure 4.5 plots a simulation for each of the three stages in the initial testing scenario.

Stage	Initial Value	500 Run Average	Min	Max	Std Dev
Tanker	\$1,532,700	\$1,535,000	\$1,019,000	\$2,075,600	\$110,680
Factory Reception	\$3,790,700	\$3,790,800	\$2,128,300	\$5,175,200	\$306,169
Processing	\$97,430	\$99,100	\$267	\$551,280	\$58,170

Table 4.4: Simulation results for each production stage over 500 runs using the initial testing parameters given in Table 4.3.

4.6.3 Simulations with various rejection probabilities

In this section we investigate how changing the acceptance probabilities affects the overall output and the equilibrium values in each stage. We start by

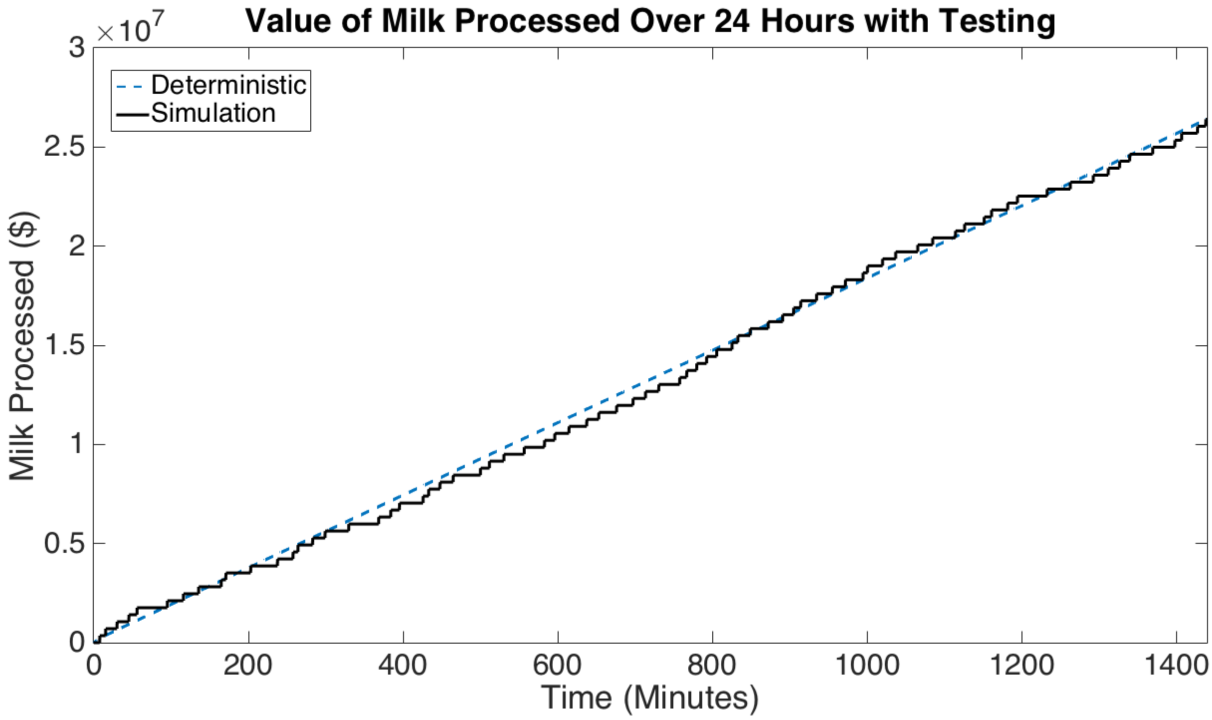

Figure 4.4: This plot shows the total value of the milk produced over a 24 hour period, along with the deterministic solution. Using the transition probabilities described earlier and summarised in section 4.3. The parameters values are summarised in table 4.3. The step size is $\Delta t = 8$ Seconds.

varying the acceptance probability of one stage while holding those of the remaining stages constant.

Milk Tanker acceptance Figure 4.6 show simulation for the total output when we vary the probability of the milk tanker accepting milk presented for collection by farmers. We vary the value for α between 1 and 0.75.

Note that there is a lot of variability in a single simulation and, as seen in Figure 4.6, at certain times the amount of milk processed may be the same for multiple values of α. Table 4.5 gives the 500 simulation average for each of these acceptance values.

The output in this case varies between \$26.5 million, in a good run with acceptance probability $\alpha = 1$, and \$22.2 million for a run where $\alpha = 0.75$.

Factory Acceptance Now we will vary the acceptance rate upon arrival at the factory reception. Again keeping the other variables the same as in Table 4.3. Figure 4.7 shows a simulation for each of acceptance probabilities upon entry to the factory reception stage.

As noted for Figure 4.6, due to the variability in a single simulation, at certain times the amount of milk processed may be the same for multiple values of β.

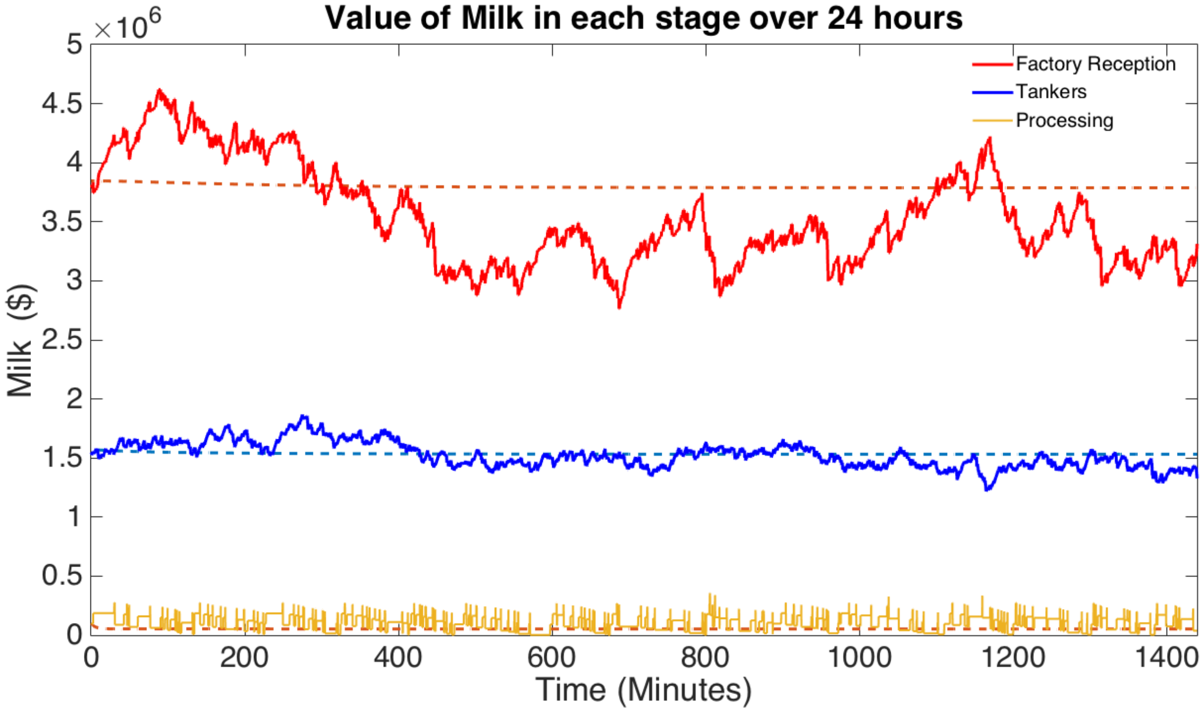

Figure 4.5: This plot shows value of milk in each stage, over 24 hours of simulation, starting at the expected equilibrium values. The parameters values are summarised in Table 4.3. The step size is $\Delta t = 8$ Seconds.

Tanker Acceptance Probability (α)	**500 Run Average**	**Lower Quartile**	**Upper Quartile**
1	\$26,167,284	\$25,806,000	\$26,554,000
0.99	\$26,050,970	\$25,619,000	\$26,367,000
0.95	\$25,553,176	\$25,245,000	\$25,993,000
0.9	\$24,811,160	\$24,497,000	\$25,245,000
0.85	\$24,112,154	\$23,849,000	\$24,310,000
0.8	\$23,306,932	\$23,001,000	\$23,375,000
0.75	\$22,515,922	\$22,253,000	\$22,814,000

Table 4.5: Simulation results for the average daily production over 500 simulations. Values are given for simulation using various milk tanker acceptance probabilities

As highlighted in Table 4.6 the value of milk produced in a 24 hour period varies between \$26.7 million worth of milk for a high simulation with 100% acceptance of tanker deliveries, and \$19.8 million for a low producing simulation with only 75% acceptance.

Processing Acceptance Still keeping the other variables the same as in Table 4.3, simulations for various values of γ are shown in Figure 4.8.

Again different values of γ may at times produce similar production values,

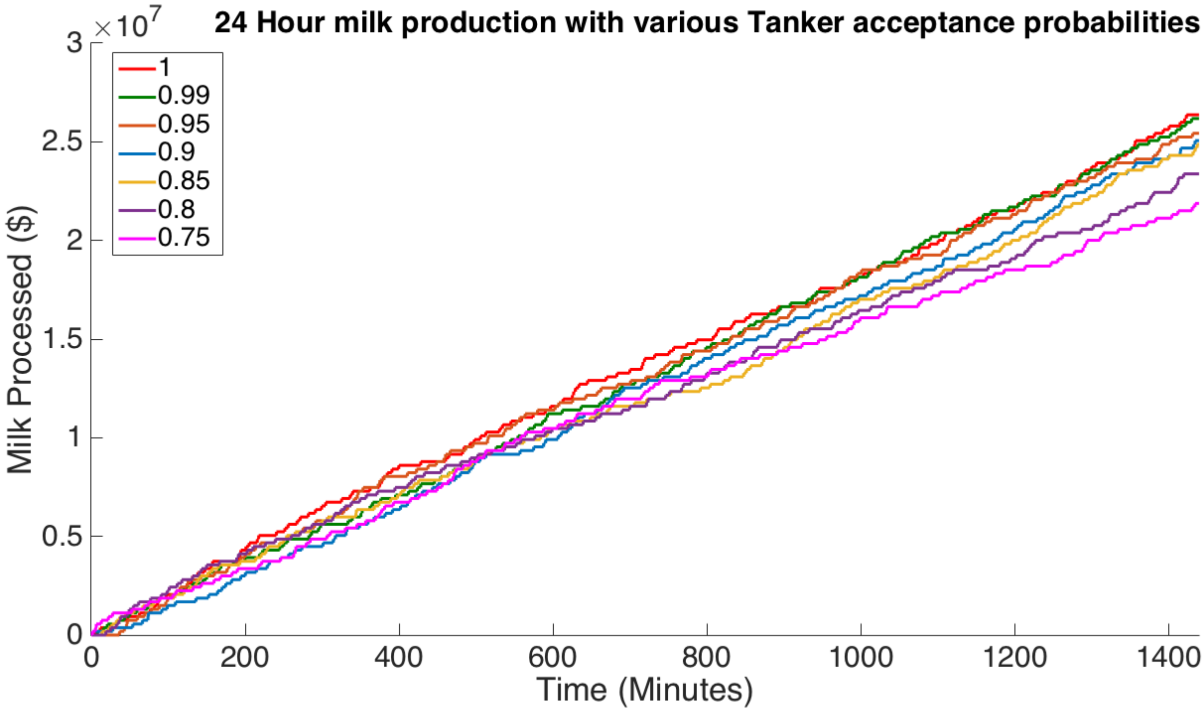

Figure 4.6: The total value of the milk produced over a 24 hour period for seven different vat acceptance probabilities. Using the transition probabilities described in section 4.3. The value α is varied between 0.75 and 1, while β and γ remain the same as in Table 4.3, along with the rest of the parameters. The step size is $\Delta t = 8$ Seconds.

Factory Acceptance Probability	500 Run Average	Lower Quartile	Upper Quartile
1	$26,413,657	$25,993,000	$26,741,000
0.99	$26,181,496	$25,806,000	$26,180,000
0.95	$25,177,680	$24,824,000	$25,619,000
0.9	$23,933,756	$23,562,000	$24,310,000
0.85	$22,592,592	$22,253,000	$23,001,000
0.8	$21,352,034	$20,944,000	$21,692,000
0.75	$20,144,014	$19,822,000	$20,383,000

Table 4.6: Simulation results for the average daily production over 500 simulations. Values are given for simulations using various values for β between 0.75 and 1, while holding all other parameters constant.

due to the variability of single simulations.

Table 4.7 Shows the average 24 hour output for various values of γ after 500 simulations. Production value in this case varies between $26.5 million for a good run with $\gamma = 1$ and $21 million for a simulation with $\gamma = 0.75$.

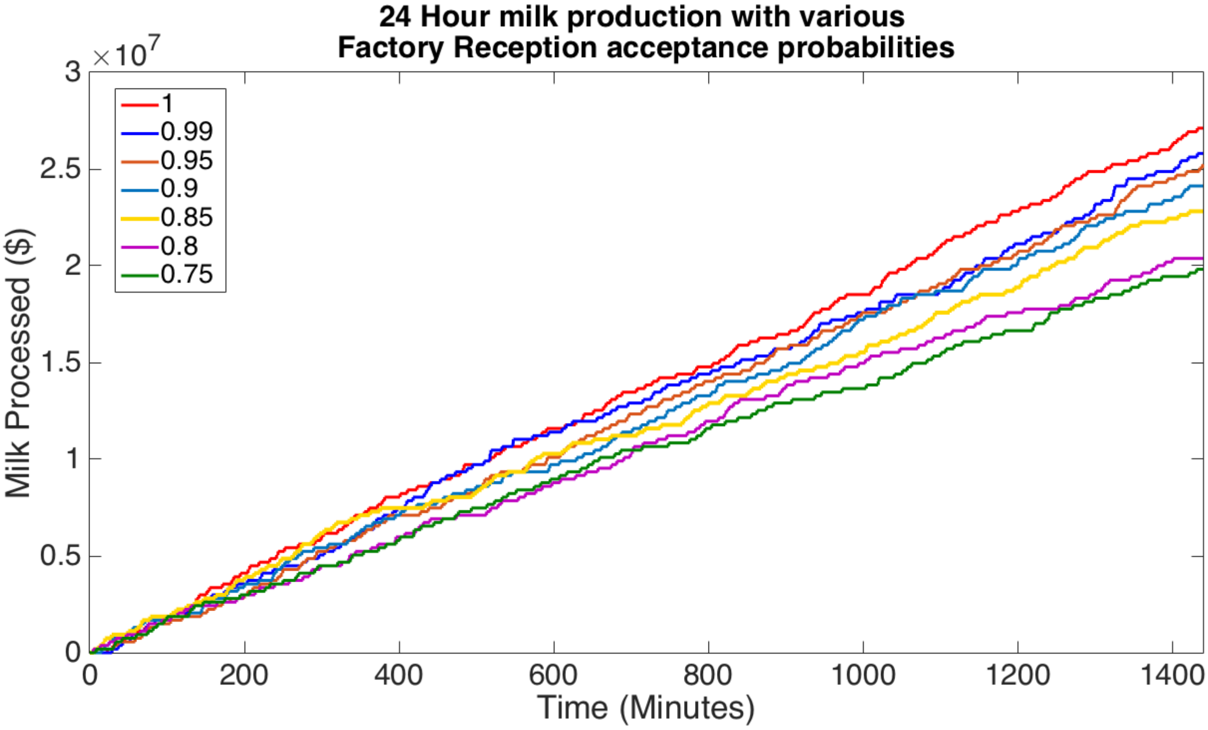

Figure 4.7: The total value of the milk produced over a 24 hour period for seven different milk tanker acceptance probabilities. Using the transition probabilities described in section 4.3. The value β is varied between 0.75 and 1, while α and γ remain the same as in Table 4.3, along with the rest of the parameters. The step size is $\Delta t = 8$ Seconds.

Factory Acceptance Probability	**500 Run Average**	**Lower Quartile**	**Upper Quartile**
1	\$26,205,806	\$25,993,000	\$26,554,000
0.99	\$26,050,596	\$25,619,000	\$26,367,000
0.95	\$25,307,084	\$24,871,000	\$25,806,000
0.9	\$24,470,820	\$24,123,000	\$24,871,000
0.85	\$23,463,638	\$23,001,000	\$23,936,000
0.8	\$22,578,380	\$22,253,000	\$23,188,000
0.75	\$21,516,968	\$21,084,000	\$22,066,000

Table 4.7: Simulation results for various values of γ. The average over 500 simulation runs for selected values of γ between 0.75 and 1.

Comparing acceptance probability effects One of the aims of this model and simulations is to see at which stage rejection of product has the most impact. Table 4.8 shows the average 24 hour production outcomes for various rejection probabilities at each stage. For each column only the stated parameter is changed while the other two remain constant.

The largest impact on production occurs when we alter the value of β, the probability of the factory reception accepting tanker deliveries. The greatest

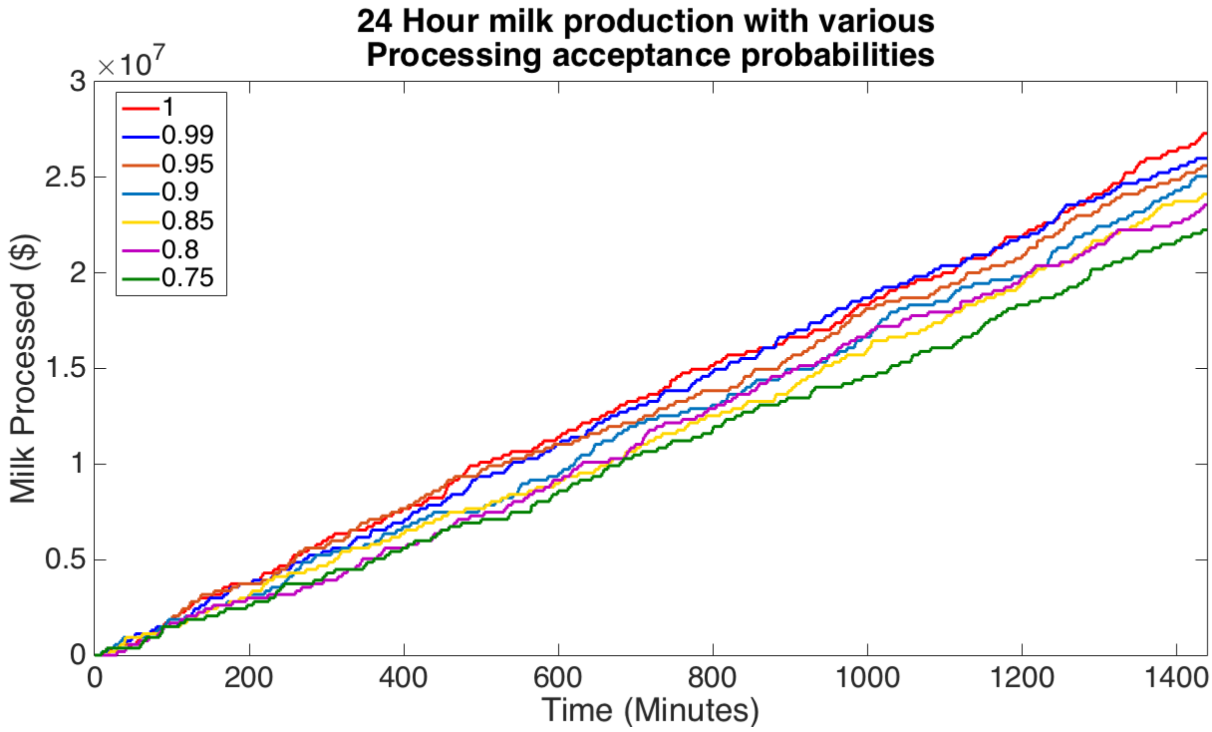

Figure 4.8: The total value of the milk produced over a 24 hour period for seven different milk tanker acceptance probabilities. The value of γ is varied between 0.75 and 1, while α and β remain the same as in Table 4.3, along with the rest of the parameters. The step size is $\Delta t = 8$ Seconds.

output value is achieved when $\beta = 1$, but reducing the value of β also results in the largest reduction in output. A 95% rejection rate upon delivery produces a lower output than this rejection rate occurring at either collection or process entry. The second largest impact is caused by the value of γ.

This suggests that delivery of milk to the factory by the tanker is where we most want to control rejection rates. Interestingly this is currently the stage with the highest rate of rejection. Each 5% increase in acceptance probability results in an average production value increase of $1.25 million per day.

A 5% increase in γ results in an average of $0.94 million per day, while increasing vat acceptance, α, by 5% produces and extra $0.73 million per day on average.

Increasing acceptance rates

One possible method for increasing acceptance rates at the factory reception could be to increase testing when the tanker collects milk from the farm. If we were able to remove all contamination before the milk even enters the tanker the we could theoretically reduce need to reject material later. A glance at Table 4.8 suggests decreasing the rates at which a tanker accepts milk from the farm (α) to 75% could result in about $2 million per day extra production if it were to result in an increase in factory acceptance rates (β), from 75%

Acceptance	Parameter Varied		
Probability	α	β	γ
1	$26,167,284	$26,413,657	$26,205,806
0.99	$26,050,970	$26,181,496	$26,050,596
0.95	$25,553,176	$25,177,680	$25,307,084
0.9	$24,811,160	$23,933,756	$24,470,820
0.85	$24,112,154	$22,592,592	$23,463,638
0.8	$23,306,932	$21,352,034	$22,578,380
0.75	$22,515,922	$20,144,014	$21,516,968

Table 4.8: Simulation results for the average daily production over 500 simulations. Values are given for simulations using various acceptance probabilities for each stage, while keeping all other parameters, aside from the probability being varied the same as in Table 4.3

to 99%. We also see an increase in output when we increase the rejection rates for processing entry, possibly reducing testing at the factory reception and increasing rejection rates at the processing stage potentially results in a $1.5 million increase in output. There are obviously costs and other issues involved in implementing this proposition such as appropriates test not being available at this point, or analysis taking too long to wait for, but it bears further investigation. If the necessary testing could be implemented at a cost less than the increase in output it would be a worthwhile investment.

Another possible solution could be investing in refrigerated milk tankers. Because, in reality, Fonterra's milk tankers are currently unrefrigerated, accidents and unforeseen delays can result in good milk begin spoilt during transport. We do not have access to data on how often this occurs, but if the loss reduction is large enough this could be a good option.

A third solution is to improve our knowledge of where contaminated milk has come from and exactly which products may be influenced. This may allow us to only discard the substandard product and retain as much unaffected product as possible for further production stages. This may or may not include extra testing costs along with the costs of tracking the product though the supply chain, but could effectively reduce rejection rates without requiring more intensive testing regimes such as in the first option.

All three of these suggestions could be valid methods of reducing product rejection rates and increasing overall output. Further investigation of intensifying testing at one or two stages to reduce rejection rates at others could easily be done using the model described in this paper. The effect of reducing losses by knowing more about out supply chain would require a little more work but again this model provides a good starting point. Investigation of refrigerated tankers is plausible but obtaining appropriate data may be difficult.

5 Conclusions

We have developed a working model for the flow of milk from the farm through the first stage of processing. This model allows for testing costs and resulting rejection of substandard product. Through various simulations we have seen the effect of different rejection rates at each stage of production.

The results of our simulations suggest the probability of accepting tanker deliveries to factory reception has the biggest impact on overall production compared with the other two possible rejection points in our model. This result suggests further investigation into managing product rejection and in particular reducing tanker delivery rejection if possible.

The model developed here is a good basis into which other aspects of production may be incorporated to investigate their potential impacts on overall dairy production.

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TE WHARE WĀNANGA O TE ŪPOKO O TE IKA A MĀUI

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