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Trading Dynamics with Adverse Selection and Search: Market Freeze, Intervention and Recovery

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We study trading dynamics in an asset market where the quality of assets is private information and finding a counterparty takes time. When trading ceases in equilibrium as a response to an adverse shock to asset quality, a government can resurrect trading by buying up lemons which involves a financial loss. The optimal policy is centred around an announcement effect where trading starts already before the intervention for two reasons. First, delaying the intervention allows selling pressure to build up thereby improving the average quality of assets for sale. Second, intervening at a higher price increases the return from buying an asset of unknown quality. It is optimal to intervene immediately at the lowest price when the market is sufficiently important. For less important markets, when the shock to quality and search frictions are small, it is optimal to rely on the announcement effect. Here delaying the intervention and fostering the effect by intervening at the highest price tend to be complements.

Key words: Adverse Selection, Search, Trading Dynamics, Government Asset Purchases, Announcement Effect

JEL Codes: G1, E6

1. INTRODUCTION

This paper uses asymmetric information and search to understand trading dynamics in financial markets and studies the optimal design of government asset purchase programs when such frictions are present. During the recent financial crisis, there was a stunning difference in market performance. Markets for transparent assets and with centralized trading functioned well. To the contrary, in over-the-counter (OTC) markets – where trading takes place on a decentralized basis and where assets are opaque in the sense that they vary widely in their characteristics – trading came to a halt. Most prominently, collateralized debt obligations, asset backed securities and commercial paper were traded only sporadically or not at all (see Gorton and Metrick, 2012). This market freeze is commonly linked to a reassessment by market participants of the average quality of the assets traded in these markets.¹ Among other measures, governments reacted to this situation by purchasing distressed assets in these markets. These asset purchase programs, however, had to be set up with little guidance for how to design them.

Our goal is to provide guidance for how to design asset purchase programs that seek to restore trading in asset markets. We start by exploring the reason for why asset markets are fragile when trading is decentralized. In many financial markets, assets are traded bilaterally, where it is hard for sellers to find a counterparty and where the buyer often cannot observe directly the quality of the asset or infer it from past trades. We capture

1. For example, Moody's Investor Service (2010) reports large spikes in impairment probabilities for structured debt products across all ratings and products for 2007 to 2009.

these features by using a model of bilateral trade with search and adverse selection. With a lemons problem à la Akerlof (1970), a deterioration of the average asset quality can bring trading to a halt. Furthermore, in our model such quality shocks lead more readily to a market freeze when search frictions are larger, as selling off lemons is more difficult for investors.

After a quality shock, trading can be restored, if the government reduces the adverse selection problem (Mankiw, 1986). By buying lemons, it can raise the average quality of assets in the market. It thus acts as a one-time market-maker that can resurrect the functioning of the market by buying a sufficient amount of lemons in response to the market freeze. Since our set-up is dynamic, we can study the transitional equilibrium dynamics starting from a market freeze due to a deterioration in asset quality to a possible recovery induced by a public intervention. The government is constrained here by these equilibrium dynamics when deciding on the optimal timing of its intervention, the optimal quantity of lemons it will buy and the optimal price it will pay for these lemons.

The dynamics of trading in our economy are non-trivial and are driven by two fundamental effects that determine the incentives to trade. As is standard in a dynamic model of trade, the incentives to buy an asset today depend also on how easy it is to sell the asset in the future. In our economy, these considerations are also important due to a lemons problem. If a buyer obtains a lemon in a trade, he would like to sell it again as quickly as possible. Hence, more frequent future trade reduces the cost of acquiring a lemon. We call this effect *the resale effect* which summarizes all *future trading behaviour* in the value of a lemon.

The second effect is novel in our analysis. It concerns how the average quality of assets that are for sale in the market changes over time. While lemons are constantly offered for sale, investors will sell good assets only if they are hit with a random shock. Whenever trading stops in the market, selling pressure will build up slowly and improve the average quality of assets. Similarly, when trading starts again, such pressure will dissipate slowly over time due to trading frictions. These dynamics will determine the average quality of assets – or, equivalently, the severity of the lemons problem – that a buyer for the asset faces. We call this effect *the quality effect* as it summarizes all *past trading behaviour* in the current quality of assets for sale.

The interplay of these two dynamic effects is important for the optimal design of the intervention as it can cause what we call an *announcement effect*. After a shock, merely announcing to intervene at a later point in time with a specific price and quantity of lemons to be bought can cause the market to recover prior to the actual intervention.² The intervention increases the resale effect, since investors can lay off lemons either during the intervention or after it when there is trade again in the market. At the same time, when there is no trade before the intervention, the average quality of assets in

2. There are some empirical studies that provide evidence for the existence of announcement effects in the context of asset purchase programs that were conducted during the crisis, while our paper seems to be the first theoretical study on what generates this effect and how policy should optimally use this effect. One example is Hancock and Passmore (2014) report that “Within minutes of the Federal Reserve’s announcement [of the MBS purchase program], the Fannie Mae option adjusted current coupon mortgage-backed security spreads (OAS) over swap yields plummeted from about 65 basis points to almost zero”, even though no MBS had (yet) been purchased by the Federal Reserve. Other empirical contributions are Gagnon et al. (2011), Kettemann and Krogstrup (2014) and the papers mentioned in Kozicki et al. (2011). All these studies have in common that they mostly look at how asset prices react to policy announcements. We view such price impact as indirect evidence for an increase in private demand for these asset or in trading in general.

the market improves as selling pressure builds up. Hence, even if the intervention does not take place immediately, but is delayed, these two effects will decrease the severity of the lemons problem for investors already before the intervention. Consequently, the announcement effect will be stronger when the quantity of lemons bought and the price at which they are purchased increase as well as when the intervention is delayed.

We show that this announcement effect makes the design of the optimal intervention very stark. In terms of the quantity, it is always optimal to buy the minimum quantity of lemons that is necessary for there to be trading after the intervention. Were the quantity higher, one could intervene earlier with a lower quantity without affecting trading, but save on the net present value of costs for intervening. In other words, it is never optimal to rely on or foster the announcement effect by increasing the number of lemons that are purchased. In terms of the price, it is only optimal to intervene at extreme prices – the lowest one that makes lemons indifferent to sell to the government or the highest one that just prevents investors with good assets to also sell to the government. The reason is that once it becomes possible to foster the announcement effect through a higher price, one can increase the price further and delay the intervention, thereby again saving costs without affecting trading.

The main variable for the intervention is then its optimal timing which is governed by how important the market is and by how strong the announcement effect is. In general, when the market is very important, it is optimal to intervene immediately. As the market becomes less important, it becomes optimal to delay the intervention and eventually to not intervene at all. How strong the announcement effect is depends on two key variables, the size of the quality shock and the severity of the trading frictions. Hence, these variables determine whether it is optimal to make use of the announcement effect when choosing the optimal time for the intervention.

When the quality shock is large, an announcement effect can only arise through a sufficiently high price at which the intervention takes place. This is due to the fact that the quality effect alone is not strong enough to raise the quality of assets for sale sufficiently for supporting trading before the intervention. Whether such a policy is optimal depends then on the trade-off between the additional benefit of having the announcement effect at the high price and the extra cost of paying such a high price. For large shocks, we point out three situations where it is not optimal to increase the price and, thus, not to use the announcement effect. First, the magnitude of the announcement effect is limited when the intervention is conducted early because a sufficient delay is needed for an announcement effect to emerge. In a very important market, an early intervention is however optimal. As a result, it is not optimal to set a high price in such a situation since there is no room for a sufficiently strong announcement effect to emerge. Second, the quality effect is small when there are only few good assets. Consequently, it is not optimal to use the announcement effect when the quality shock is sufficiently large. Third, the announcement effect is also limited whenever search frictions are high because it takes a long time to turn around lemons in the market.

To the contrary, when the shock to quality is small, an announcement effect can arise simply from delaying the intervention sufficiently and without increasing the price. Whether to delay the intervention or not depends on the comparison between two costs. First, with delay more assets are being misallocated as the market does not function. When search frictions are large, this is again very costly (in terms of welfare) as the market requires more time to reallocate assets after the intervention. Second, delaying the intervention saves on costs (in terms of government spending) as the net present value is smaller. How much these cost savings matter depends on the importance of the

market. Consequently, if trading frictions are small or if the market is not too important, it is optimal to delay the intervention and rely on the announcement effect.

Finally, the announcement effect can be important when interventions cannot be carried out immediately because of operational delay. Continuous trading can still be achieved when the intervention is announced immediately with a sufficiently high price and quantity. With this increase in price and quantity, the resale effect needs to become strong enough to overcome the low quality of assets in the market before the intervention. Such a policy is optimal whenever the market is sufficiently important, since the additional gains from a continuously functioning market outweighs the extra costs associated with the larger intervention.

Our model is a simplified version of Duffie, Garleanu and Pedersen (2005) that incorporates adverse selection in an asset market with random matching. Traders meet randomly to trade an asset, but adverse selection makes trading difficult. Only the current owner of the asset can observe its quality, while the potential buyer only learns the quality after he has bought the asset.³ In equilibrium, this leads to optimal pooling contracts where investors that sell lemons earn informational rents.⁴

As such we combine two strands of literature on market microstructure that focus on different frictions in asset markets, search and adverse selection.⁵ While it is well known that adverse selection can cause a breakdown of trading in a market, we show here that search frictions worsen this problem and can make markets more fragile. Furthermore, the interaction of adverse selection and search introduces a second effect – the quality effect – that drives the dynamics of trading beyond the strategic complementarity associated with future trading behaviour.⁶

While we have worked on this paper, a literature has emerged that studies interventions in asset markets in response to shocks. Closest to our question is the contribution by Tirole (2012) that uses a static framework to analyze a similar government policy. As a consequence, Tirole (2012) can neither address the issue of optimal timing nor look at the interaction of this decision with the quantity and price of the intervention (i.e., the announcement effect). Also in a static setting, Philippon and Skreta (2012) concentrate on the restrictions that a private market imposes on the effectiveness of government intervention. Such restrictions arise naturally in our framework, since the government needs to take into account that investors face a functioning asset market again after the intervention.⁷

One contribution that also studies the timing of policy is Fuchs and Skrzypacz (2015). In their paper, a government has an incentive to subsidize early trade and

3. A key difference to Duffie, Garleanu and Pedersen (2005) is that investors follow a cyclical trading pattern like in Vayanos and Wang (2007) instead of random valuation shocks. This difference is immaterial for our results, but allows for analytical solutions of most of the trading dynamics.

4. This distinguishes us from Guerreri, Shimer and Wright (2010) that use competitive search to obtain a separating equilibrium in asset markets with adverse selection. Chang (2011) builds on this work to show that liquidity in the form of endogenous market tightness is disturbed downwards in equilibrium when there is a lemons problem for trading assets. Other papers with dynamic adverse selection also study pooling equilibria, but simply require that transactions have to take place at a single price (see for example Eisfeldt, 2004; Kurlat, 2010).

5. Starting with Kyle (1985) and Glosten (1989), there is a large number of contributions that use models where some traders are privately informed about the asset quality to shed light on pricing and transaction costs in financial markets.

6. Garleanu (2009) has pointed that this complementarity can be important for understanding trade size and portfolio choice in asset markets. For simplicity, we abstract from such considerations when modelling adverse selection and the quality effect.

7. More recently, Guerreri and Shimer (2011) have also looked at interventions in dynamic asset markets where trading is competitive. They do not, however, analyze the design of the policy.

tax late trade when adverse selection impedes trading and market participants extract information through delayed trading. The reason is that such a policy improves the quality of assets at the start of trading thereby leading to more trade early on which is welfare improving. Thus, the ideal intervention tries to always achieve early trade. Delay in our model is optimal for very different reasons. First, we take into account the costs of intervention explicitly from which Fuchs and Skrzypacz (2015) abstract. Second, the dynamics of adverse selection are very different in our work, since the quality effect gives rise to an announcement effect which is absent in their work.

Another difference with some contribution to the literature on asset purchases is that our intervention involves buying lemons at price higher than their fundamental value and holding them permanently. Consequently, there will be losses that private intermediaries will be unwilling to absorb. This feature distinguishes our paper from the work on for-profit dealers in OTC markets who alleviate temporary selling pressure by holding inventories (see for example Weill, 2007; Lagos and Rocheteau, 2009; Lagos, Rocheteau and Weill, 2011), but where a lack of deep pockets or the expectation of negative profits can prevent market-making in response to a liquidity shock.⁸

More broadly, our work is also part of the burgeoning literature on dynamic lemons markets. Focusing only on contributions that involve search, a key difference from other work is that the dynamics in our model arise within a closed model without new assets arriving or assets leaving the market. This creates intricate endogenous dynamics through the quality effect that are not present in other models. One example that also studies trading dynamics with bilateral trade and adverse selection is Moreno and Wooders (2010). The dynamics in their set-up are limited, however, for two reasons. First, each asset is traded only once so that there is no forward-looking dimension as in our model. Second, new assets arrive in the economy at a fixed, exogenous rate that is independent from past trading in the market. This causes very different quality dynamics over time compared to our model where the number of good assets is fixed and the amount of good assets that are for sale in the market depends on past trading behaviour. Camargo and Lester (2014) make an interesting contribution, since they study how quickly a market clears when there is asymmetric information and the market has to work through a certain amount of lemons before it can function again. In our paper, however, the lemons problem does not diminish over time making an intervention necessary for a recovery. A recent contribution by Zhu (2012) shares with our paper that the degree of adverse selection in a given market is endogenous. Different to our approach where search is random, Zhu (2012) generates adverse selection endogenously in a sequential search model when sellers visit multiple buyers and infer the quality of the assets from the frequency of their meetings.

2. THE ENVIRONMENT

We employ a basic model of asset pricing under search frictions and introduce adverse selection. Time is continuous. There is a measure of $1 + S$ traders that trade S assets. These assets are of two types. A fraction π of the assets yields a dividend δ (good assets), whereas the rest does not yield a dividend (lemons). The return on these assets is private

8. Bolton, Santos and Scheinkman (2011) also study the timing of an intervention in the context of liquidity shortages. The main difference is that, while an early intervention can prevent a market freeze, the cost of the intervention stems from precluding the supply of private liquidity in the secondary market for assets.

information for the owner of the asset; i.e., only the trader who owns the asset can observe its return, but not other traders.

Traders are risk-neutral and discount time at a rate r . We assume that each investor can either hold one unit of an asset or no asset.⁹ A trader who owns a good asset is subject to a random preference shock that can reduce his valuation from δ to $\delta - x > 0$. Conditional on holding a good asset, the preference shock arrives according to a Poisson process with rate $\kappa \in \mathbb{R}_+$. Once a trader experiences this shock, his valuation of the asset will remain low until the asset is sold. This captures the idea that some traders who own an asset might have a need for selling it – or in other words, have a need for liquidity. The higher κ , the more likely an investor will face such needs. Traders therefore go through a trading cycle depending on their asset holdings and their valuation of the asset. There are four different stages that occur sequentially: (i) buyers (b) do not own an asset; (ii) owners (o) have a good asset and a high valuation; (iii) traders (ℓ) who own a lemon; and (iv) sellers (s) who have a good assets, but have experienced a transition to low valuation. We denote the measure of traders of the different types at time t as $\mu_b(t)$, $\mu_o(t)$, $\mu_\ell(t)$ and $\mu_s(t)$ respectively.¹⁰

There is no centralized market mechanism to trade assets. Instead, traders with an asset and buyers are matched according to the matching function $M(t) = \lambda\mu_b(t)[\mu_o(t) + \mu_s(t) + \mu_\ell(t)]$, where $M(t)$ is the total number of matches, and λ is a parameter capturing the matching rate.¹¹ We assume throughout that in pairwise meetings the buyer makes a take-it-or-leave-it offer to the seller to buy one unit of the asset at price $p(t)$ ¹² and that traders cannot dispose of an asset to become a buyer again.¹³

We can then describe the economy by a flow diagram as shown in Figure 1. Denote the probability of conducting a trade given a match as γ . A buyer becomes an owner by buying a good asset (with probability $\lambda\gamma\mu_s$) or a lemon by buying a bad asset (with probability $\lambda\gamma\mu_\ell$). He turns from an owner into a seller when receiving a negative preference shock (with probability κ). Finally, if there is trade, good sellers and lemons sell their assets and become buyers (with probability $\lambda\gamma\mu_b$). If there is no trade, traders remain in their respective states – except for owners of good assets that experience preference shocks. A classic adverse selection problem arises here, because lemons will choose in equilibrium to transit immediately from buying to selling the asset, while

9. This is a restriction on total asset holdings. Traders are still allowed to use lotteries and to employ mixed strategies to trade assets. In this regard, assets are not really indivisible.

10. The preference shock thus only affects traders that own a good asset, but neither traders with lemons or traders who do not own an asset. We like to think about assets in our model as a *class of assets* where individual assets have slightly different characteristics. The preference shock captures the valuation of an investor towards the specific piece of asset he is holding. When an owner's preference switches from high to low, he has an incentive to sell the asset currently held, but still wants to re-enter the market as a buyer immediately afterwards because his valuation to other assets remains high. Our interpretation of the shock is a need for investors to unravel or rebalance their positions periodically for hedging purposes or portfolio considerations. Consequently, lemon holders do not have such considerations as they try to sell off bad assets. Neither do traders that do not hold assets in the first place. Note that this assumption is different from other papers like Duffie, Garleanu and Pedersen (2005) which impose shocks on traders independently of asset holdings. Our set-up significantly simplifies the analysis, but is immaterial for the nature of our results as we show in the Online Appendix.

11. The interpretation is that traders are matched according to a Poisson process with a fixed arrival rate. As a result, matches with traders seeking the opposite side of a trade occur at a rate λ which is proportional to the measure of traders in that group.

12. This is a simplifying assumption merely to avoid the issue of formulating a bargaining procedure in the presence of imperfect information.

13. By restricting the number of assets relative to the measure of traders in the economy, we can easily dispense with this assumption (see the Online Appendix).

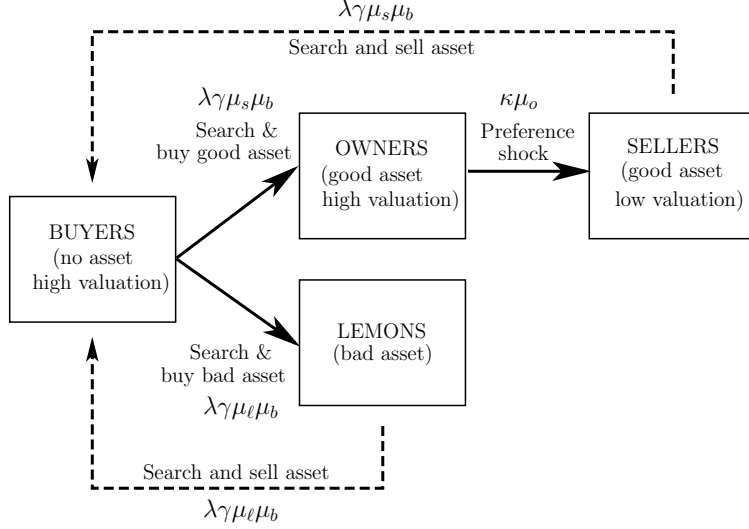


FIGURE 1
Flow Diagram

owners have first to experience a preference shock in order to have an incentive to sell their assets.

3. TRADING DYNAMICS

3.1. Trading Incentives with Pooling

What are the incentives for buyers to purchase an asset of unknown quality at a price that pools sellers of good assets and lemons?¹⁴ To allow for mixed strategy equilibria, a buyer makes a take-it-or-leave-it offer with probability $\gamma(t)$, if in a meeting with another trader at time t . When making his offer, a buyer needs to take into account whether their price induces sellers with good assets to accept the offer. Denoting the first random time a seller meets a buyer by τ , we obtain for the seller’s value function

$$v_s(t) = E_t \left[\int_t^\tau e^{-r(s-t)} (\delta - x) ds + e^{-r(\tau-t)} \max\{p(\tau) + v_b(\tau), v_s(\tau)\} \right]. \quad (1)$$

The first expression on the right-hand side is the flow value from owning the asset. The second term gives the discounted value of meeting a buyer at random time $\tau > t$. In such a meeting, the seller either accepts the offer or rejects it. If he rejects the offer, he stays a seller ($v_s(\tau)$). If he accepts the offer, he receives the price $p(\tau)$ and becomes a buyer with value $v_b(\tau)$. Differentiating this expression with respect to time t and rearranging yields the following differential equation

$$rv_s(t) = (\delta - x) + \gamma(t)\lambda\mu_b(t) \max\{p(t) + v_b(t) - v_s(t), 0\} + \dot{v}_s(t). \quad (2)$$

We can derive similar value functions for the other types of traders denoted by $v_o(t)$ for owners, $v_\ell(t)$ for lemons and $v_b(t)$ for buyers. Notice that there are no gains from trading

14. We show in the Online Appendix that pooling always dominates any separating contract with lotteries.

between owners and buyers, as they have the same valuation of a good asset. We thus have

$$rv_o(t) = \delta + \kappa(v_s(t) - v_o(t)) + \dot{v}_o(t) \quad (3)$$

$$rv_\ell(t) = \gamma(t)\lambda\mu_b(t) \max\{p(t) + v_b(t) - v_\ell(t), 0\} + \dot{v}_\ell(t) \quad (4)$$

$$rv_b(t) = \gamma(t)\lambda(\mu_s(t) + \mu_\ell(t)) \cdot \max\{\max_{p(t)} \tilde{\pi}(p(t), t)v_o + (1 - \tilde{\pi}(p(t), t))v_\ell(t) - p(t) - v_b(t), 0\} + \dot{v}_b(t). \quad (5)$$

An owner enjoys the full value of the dividend flow until he receives a liquidity shock and turns into a seller which occurs with probability κ . Sellers of lemons – which we will simply call lemons from now on – are willing to sell their bad assets for a reservation price $p(t) \geq v_\ell(t) - v_b(t)$. Upon selling the asset at price $p(t)$, they become buyers again. Finally, the value function of a buyer takes into account that he can choose not to buy the asset in a meeting. If he makes an offer, the buyer will choose a price that maximizes his expected payoff given the composition of traders that are willing to sell. This is reflected in the probability of obtaining a good asset, $\tilde{\pi}(p(t), t)$ which is a function of the price he offers.

Upon acquiring a lemon, a buyer will immediately try to sell it again since it offers no dividend flow. To the contrary, when acquiring a good asset, he has the highest valuation of the asset and will sell it only after receiving a preference shock that lowers his valuation which occurs with frequency κ . This implies that the measure of different types of traders evolves according to the following flow equations

$$\dot{\mu}_b(t) = -\gamma(t)(\mu_s(t) + \mu_\ell(t)) + \gamma(t)(\mu_s(t) + \mu_\ell(t)) = 0 \quad (6)$$

$$\dot{\mu}_o(t) = -\kappa\mu_o(t) + \gamma(t)\lambda\mu_s(t) \quad (7)$$

$$\dot{\mu}_s(t) = \kappa\mu_o(t) - \gamma(t)\lambda\mu_s(t) \quad (8)$$

$$\dot{\mu}_\ell(t) = -\gamma(t)\lambda\mu_\ell(t) + \gamma(t)\lambda\mu_\ell(t) = 0. \quad (9)$$

Due to the trading structure, the number of buyers stays constant and we normalize it to $\mu_b(t) = 1$. Similarly, all lemons are constantly for sale and, hence, $\mu_\ell(t) = (1 - \pi)S$.

For a buyer to induce a seller to accept his take-it-or-leave-it offer, he needs to offer a price that compensates the seller for switching to become a buyer, or $p(t) \geq v_s(t) - v_b(t)$. Since lemons do not derive any flow utility from their asset, we have that $v_s(t) \geq v_\ell(t)$ and, consequently, they will accept the buyer's offer whenever sellers do. For the buyer, the probability of buying a good asset is thus given by

$$\tilde{\pi}(t) = \begin{cases} \frac{\mu_s(t)}{\mu_s(t) + \mu_\ell(t)} & \text{if } p(t) \geq v_s(t) - v_b(t) \\ 0 & \text{if } p(t) < v_s(t) - v_b(t). \end{cases} \quad (10)$$

This formulates the basic adverse selection problem. While lemons are always for sale, good assets are sold only if their current owner has experienced a preference shock. As a consequence, there are fewer good assets for sale than in the population (i.e., $\tilde{\pi}(t) \leq \pi$). Also, if the buyer offers a price that is too low, good sellers will reject the offer and he will acquire a lemon for sure. Any offer by the buyer will thus be given by $p(t) = v_s(t) - v_b(t)$. For the analysis below, it is convenient to define the buyer's expected surplus from making an offer to buy the asset

$$\Gamma(t) = \tilde{\pi}(t)v_o + (1 - \tilde{\pi}(t))v_\ell(t) - v_s, \quad (11)$$

where we have taken into account that any offer will set $p(t) = v_s(t) - v_b(t)$. The function Γ will be central in all of our analysis, since it summarizes whether there will be trade

or not. Note that changes in Γ over time can arise only from two sources, the average quality of assets for sale, $\tilde{\pi}(t)$, and the value of a lemon, $v_\ell(t)$. We define then equilibrium as follows.

Definition 1. *An equilibrium is given by measurable functions $\gamma(t)$ and $\tilde{\pi}(t)$ such that*

1 for all t , the strategy $\gamma(t)$ is optimal taking as given $\gamma(\tau)$ for all $\tau > t$; i.e.,

$$\gamma(t) = \begin{cases} 0 & \text{if } \Gamma(t) < 0 \\ \in [0, 1] & \text{if } \Gamma(t) = 0 \\ 1 & \text{if } \Gamma(t) > 0. \end{cases} \quad (12)$$

2 The function $\tilde{\pi}(t)$ is generated by $\gamma(t)$ and the law of motion for $\mu_s(t)$.

3.2. Steady State Equilibria

In steady state, the measure of traders with good assets are given by

$$\mu_s = \frac{\kappa}{\gamma\lambda + \kappa} S\pi \quad (13)$$

$$\mu_o = \frac{\gamma\lambda}{\gamma\lambda + \kappa} S\pi. \quad (14)$$

With pooling, this implies that the probability of obtaining a good asset $\tilde{\pi}$ is given by

$$\tilde{\pi} = \begin{cases} \frac{\kappa\pi}{\kappa + (1-\pi)\gamma\lambda} & \text{if } p \geq v_s - v_b \\ 0 & \text{if } p < v_s - v_b. \end{cases} \quad (15)$$

The value functions are then given by

$$v_s = \frac{\delta - x}{r} \quad (16)$$

$$v_o = \frac{1}{r + \kappa} (\delta + \kappa v_s) \quad (17)$$

$$v_\ell = \frac{\gamma\lambda}{\gamma\lambda + r} v_s. \quad (18)$$

In the pooling equilibrium, the value of lemons depends on the trading strategy γ of buyers. Lemons earn an informational rent and extract some surplus from buyers despite the take-it-or-leave-it-offer; in other words, if $\gamma > 0$, then $v_\ell > 0$.

To characterize steady state equilibria, we only need to consider the optimal strategy of buyers. Buyers trade if and only if they have a positive expected surplus from trading

$$\Gamma = \tilde{\pi}v_o + (1 - \tilde{\pi})v_\ell - v_s \geq 0. \quad (19)$$

Using the value function, we can determine two thresholds for the asset quality, below which a no-trade equilibrium exists ($\underline{\pi}$) and above which a trade equilibrium exists ($\bar{\pi}$). First, set $\gamma = 1$ and define $\delta/(\delta - x) = \xi$. Using the expression for $\tilde{\pi}$ in (15), there is trade in a steady state equilibrium whenever

$$\pi \geq \frac{(\kappa + \lambda)(r + \kappa)}{\kappa(\xi r + \kappa) + \lambda(\xi\kappa + r)} \equiv \bar{\pi}. \quad (20)$$

Similarly, we get no trade ($\gamma = 0$) in a steady state equilibrium if

$$\pi \leq \frac{r + \kappa}{\xi r + \kappa} \equiv \underline{\pi}. \quad (21)$$

Comparing the two thresholds, we obtain that $\underline{\pi} \geq \bar{\pi}$ if and only if $\kappa \geq r$. Finally, for any given π between these thresholds, buyers are indifferent between making an offer or not whenever

$$\pi = \frac{(\kappa + \gamma\lambda)(r + \kappa)}{\kappa(\xi r + \kappa) + \gamma\lambda(\xi\kappa + r)}. \quad (22)$$

Differentiating this expression with respect to γ , we get (up to a positive factor)

$$\frac{\partial \pi}{\partial \gamma} = (\xi - 1)(r + \kappa)\lambda\kappa(r - \kappa), \quad (23)$$

which depends on r relative to κ . In particular, π increases with γ if and only if $r > \kappa$. This gives the following result.

Proposition 2. *For any given $\pi \in (0, 1)$, a steady state equilibrium exists.*

If $\pi \geq \bar{\pi}$, we have that $\gamma = 1$ is a steady state equilibrium in pure strategies, i.e. all buyers trade.

If $\pi \leq \underline{\pi}$, we have that $\gamma = 0$ is a steady state equilibrium in pure strategies, i.e. buyers do not trade.

If $\kappa < r$, the steady state equilibrium is unique, with the equilibrium for $\pi \in (\underline{\pi}, \bar{\pi})$ being in mixed strategies.

If $\kappa > r$, for $\pi \in (\bar{\pi}, \underline{\pi})$, there are three steady state equilibria including a mixed strategy one.

Figure 2 depicts steady state equilibria. When the average quality of the assets π is too low, there cannot be any trading in equilibrium – a situation which we call *market freeze*. This is associated with welfare losses as good assets cannot be allocated between traders that have different valuations for the asset. Similarly, for high average quality π , trade ($\gamma = 1$) is the unique equilibrium. For intermediate values of π , there can be multiple equilibria with partial trade ($\gamma \in (0, 1)$).

The structure of equilibria arises from the interplay between two effects and can be best understood by rewriting the expected surplus for buyers as

$$\frac{\Gamma}{(1 - \tilde{\pi})v_s} = \left(\frac{\pi}{1 - \pi} \right) \left(\frac{\kappa}{\kappa + \lambda\gamma} \right) \left(\frac{1 - \underline{\pi}}{\underline{\pi}} \right) - \frac{r}{r + \lambda\gamma}. \quad (24)$$

The first term of the surplus function now captures a *quality effect* and describes how the average quality of assets for sale affects the trade surplus. If the trading volume expressed by $\lambda\gamma$ is large, there are relatively few good assets for sale at any point in time. This lowers the expected quality of the asset purchased by a buyer and, hence, his expected surplus. The second term is independent of the average quality and captures a strategic complementarity due to a *resale effect*. When a buyer decides to purchase an asset, it matters how easy it is to turn around a lemon in the future. If future buyers are more willing to purchase assets, the trading volume $\gamma\lambda$ is high and it becomes easier for a buyer to turn around a lemon in the market, which increases the value of acquiring a lemon. Hence, the quality and resale effect both depend on the trading volume $\lambda\gamma$ but work in opposite directions.

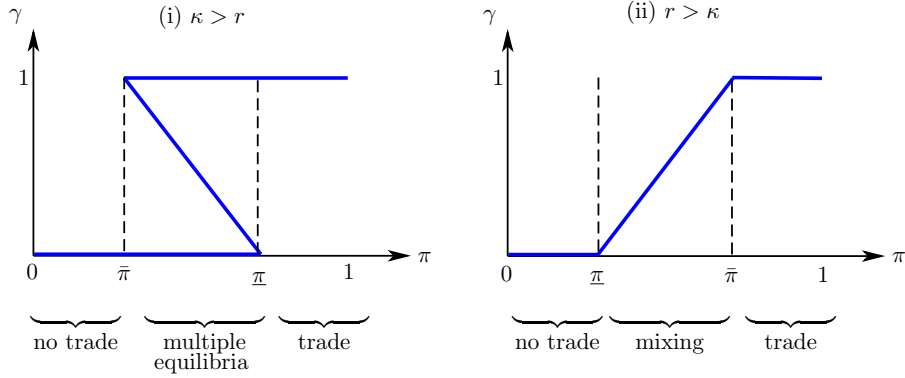


FIGURE 2
Steady State Equilibria

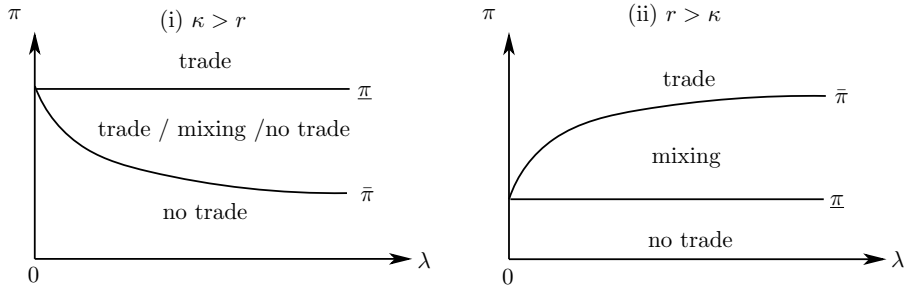


FIGURE 3
Role of Search Frictions

How do search frictions (as captured by λ) influence trading in equilibrium and the severity of the adverse selection problem? The resale effect is standard. With more search frictions it becomes more difficult to sell assets in the market again. What distinguishes our environment, however, is the quality effect. As search frictions increase, fewer good assets exit the market, improving the quality of assets for sale and thus the buyer’s incentive to make an offer. Looking at the case where $\gamma = 1$ in equilibrium¹⁵, the resale effect dominates for $\kappa > r$, so that larger search frictions increase the quality threshold that is necessary for trade, whereas for $r > \kappa$ larger search frictions lower it (see Figure 3).¹⁶

3.3. Equilibrium Transition after Market Freeze

We investigate next how shocks to asset quality can freeze the market. Suppose that the average quality of the asset drops unexpectedly at $t = 0$ to a level $\pi(0)$.¹⁷ If the drop in

15. Equilibria with no trade are independent of λ .

16. The latter case mirrors somewhat the results of Moreno and Wooders (2010) which compare trading with frictions to Walrasian trading which can be viewed as the case $\lambda \rightarrow \infty$ in our setting.

17. The assumption of unanticipated shocks greatly facilitates our analysis, but is immaterial for our analysis as shown in the Online Appendix.

asset quality is sufficiently large – specifically, if $\pi(0) < \min\{\underline{\pi}, \bar{\pi}\}$ – there is a unique steady state equilibrium of no trade, as illustrated in Figure 2. As we show next, there also exists a transition path with no trade converging to this new steady state. Moreover, this path is unique whenever $\pi(0) \leq r\underline{\pi}/(r + (1 - \underline{\pi})\lambda) = \tilde{\pi}$, which is the steady state average quality in the market, $\tilde{\pi}$, at the threshold level for full trade, $\bar{\pi}$.

Proposition 3. *For $\pi(0) < \underline{\pi}$, there exists an equilibrium with no trade for any t that converges to the steady state with no trade. This equilibrium is unique, if $\pi(0) \leq \tilde{\pi}$.*

Proof. See Appendix.

This implies that for a large enough shock to the asset quality, the market will instantaneously move from an equilibrium with trading to one without – our definition of a market freeze. Note that even a small shock to π can permanently freeze the market when $\kappa > r$ as shown in Figure 2. Furthermore, the threshold of quality for $\tilde{\pi}$ for which such a market freeze happens increases with search frictions. Hence, a smaller drop in asset quality $\pi - \pi(0)$ is required to freeze trading, implying that markets with larger search frictions (lower λ) are more fragile to asset quality shocks.

For the subsequent analysis, it is useful to introduce the new variable

$$\alpha(t) = \frac{\tilde{\pi}(t)/(1 - \tilde{\pi}(t))}{\tilde{\pi}/(1 - \tilde{\pi})} \quad (25)$$

which captures the asset quality in the market at time t by relating the ratio of good to bad assets that are for sale at t to the same ratio in steady state for $\bar{\pi}$, the threshold value for full trade. Indeed, equation (11) implies that $\Gamma(t) \geq 0$ if and only if $\alpha(t) \geq 1$. Consequently, the value of $\alpha(t)$ captures how much the average quality of assets for sale in the market differs from the threshold that is necessary for trading. Thus, when $\alpha(t) < 1$, it is a measure of the severity of the adverse selection problem.

When there is no trading until some time T , for any given $\pi(0)$ the dynamics of this function is described for $t \in [0, T)$ by the differential equation¹⁸

$$\dot{\alpha}(t) = \kappa(\alpha - \alpha(t)). \quad (26)$$

where

$$\alpha = \frac{\pi(0)/[1 - \pi(0)]}{\tilde{\pi}/(1 - \tilde{\pi})}. \quad (27)$$

Solving the differential equation we obtain

$$\alpha(t) = \alpha \left(1 - \frac{\lambda}{\lambda + \kappa} e^{-\kappa t} \right) \quad (28)$$

which reflects that with no trading the dynamics are driven solely by the inflow of good assets into the market due to the liquidity shock for traders. Hence, $\alpha(t)$ is increasing and converges monotonically to α when there is never any trading in the market.¹⁹

18. In general, the dynamics of $\alpha(t)$ is endogenous and cannot be described analytically.

19. This implies that for $\alpha < 1$, we have a unique equilibrium without trade (see Proposition 3).

3.4. Trading Dynamics with Intervention

We turn next to the question whether an intervention in the market can resurrect trading. Here, a large (strategic) player – called market-maker-of-last-resort (MMLR) – will purchase bad assets in response to an unanticipated quality shock that causes the market to freeze.²⁰ More formally, an *intervention* is defined by an announcement at time $t = 0$ to permanently purchase an amount Q of lemons at a price P at some time $T \geq 0$.²¹ We assume further that the MMLR, like other traders, does not have information on the quality of an asset, but knows the average quality $\pi(0)$ of assets after the unanticipated shock has occurred and trading has ceased in the market. The MMLR can commit to its policy, and meeting the MMLR is frictionless; i.e., at time T every trader with a lemon has an equal chance to trade with the MMLR. Finally, we assume that sellers of lemons that trade with the MMLR permanently exit the economy.²²

Asset purchases will increase the average quality of the assets that are for sale. We only consider interventions that purchase bad assets and that raise the average quality of assets sufficiently so that there is full trade in steady state. We call such interventions *feasible* and they imply restrictions on the quantity of lemons bought by the MMLR and the price paid for them which we discuss next.

First, to achieve full trading in steady state, the MMLR needs to purchase a sufficient number of lemons so that the fraction of good assets is above the threshold required for full trade

$$\frac{\pi(0)S}{S-Q} \geq \bar{\pi} \quad (29)$$

or expressed equivalently in our measure for the severity of the adverse selection problem

$$\frac{Q}{S(1-\pi(0))} \geq \frac{Q_{\min}}{S(1-\pi(0))} = 1 - \alpha(0). \quad (30)$$

Hence, the minimum quantity for a feasible intervention Q_{\min} is independent of time and depends only on the initial shock $\pi(0)$.

Second, the intervention needs to induce lemons to sell their assets at the time of the intervention T . Hence, lemons need to obtain a price that is high enough to compensate them for the opportunity cost of remaining in the market. Since this value is given by $v_\ell(T)$, we require that $P \geq v_\ell(T)$. Similarly, the price cannot be too high as otherwise the intervention would attract also good sellers; i.e, $P \leq P_{\max} = v_s$. We summarize these results in the following proposition.

Proposition 4. *An intervention (T, Q, P) is feasible if and only if*

(i) $\frac{Q}{S(1-\pi(0))} \in [1 - \alpha(0), 1]$

and

(ii) $P \in [v_\ell(T), v_s]$.

20. In the Online Appendix we discuss how a MMLR can use a different policy – a guaranteed price floor – to respond to a self-fulfilling freeze by eliminating equilibria with less trade when multiple equilibria co-exist.

21. We rule out purchasing good assets. This assumption is innocuous, if we assume that the MMLR does not enjoy the dividend flow from good assets (or sufficiently less so than the traders).

22. This keeps the number of buyers constant at $\mu_b = 1$. If we allowed lemons to become buyers, the intervention would become more powerful as it permanently increases liquidity in the market. While our results are largely robust to this change, we have chosen to abstract from this effect in order to concentrate on the primary channel of the intervention which is removing bad assets from the market. See the Online Appendix for a robustness analysis.

Beyond a minimum intervention at (Q_{\min}, P_{\min}) , the MMLR can provide additional value to traders by increasing the price and the quantity of assets purchased. We call this the *option value* of the intervention and denote it by V_I . To assess this option value, we look at the value of acquiring a lemon just an instant before the intervention

$$\begin{aligned} v_\ell(T^-) &= \frac{Q}{S(1-\pi(0))}P + \left(1 - \frac{Q}{S(1-\pi(0))}\right)v_\ell(T) \\ &= \frac{Q}{S(1-\pi(0))}(P - v_\ell(T)) + v_\ell(T) \\ &= V_I + v_\ell(T), \end{aligned} \quad (31)$$

where $V_I \in [0, v_s - v_\ell(T)]$. Hence, the chance of transacting with the MMLR at a higher price gives rise to a positive expected transfer to lemons beyond the value of the lemon in the market as given by $v_\ell(T)$.²³

This implies that the surplus function $\Gamma(t)$ has a jump at the time of intervention T for two reasons. The intervention itself removes bad assets and discretely increases the average quality $\tilde{\pi}$. In addition, whenever the option value is strictly positive, there is a downward jump in the value function of the lemon v_ℓ when the intervention takes place. More generally, the dynamics of the surplus function $\Gamma(t)$ depend only on the dynamics of $\tilde{\pi}(t)$ – or, equivalently, how the number of sellers $\mu_s(t)$ changes over time – and $v_\ell(t)$. The value function of a lemon for any $t < T$ is given by

$$v_\ell(t) = E_t \left[e^{-r(\tau_m - t)} v_s \mathbf{1}_{\{\tau_m < T\}} + e^{-r(T-t)} v_\ell(T^-) \mathbf{1}_{\{\tau_m \geq T\}} \right], \quad (32)$$

where τ_m is the random time of the next trade opportunity where buyers are willing to buy an asset. Solving this expression for any given trading strategy $\gamma(t)$, we obtain

$$v_\ell(t) = \lambda v_s \int_t^T \gamma(s) e^{\int_t^s -(r+\lambda\gamma(\nu))d\nu} ds + (v_\ell(T) + V_I) e^{\int_t^T -(r+\lambda\gamma(s))ds}. \quad (33)$$

The option value V_I of an intervention positively influences market trading through its effects on $v_\ell(t)$, but is discounted by the rate of time preference r and the chance of selling a lemon prior to the intervention on the market as expressed by the additional discount factor $\lambda\gamma(t)$. Furthermore, if there is continuous trading after the intervention, we have that $v_\ell(T) = \frac{\lambda}{\lambda+r} v_s \equiv P_{\min}$. These insights allow us to characterize the transitional dynamics as follows.

Proposition 5. *For any feasible intervention, full trading is an equilibrium after the intervention at T . Trading before the intervention can be characterized by two breaking points $\tau_1(T) \in (0, T)$ and $\tau_2(T) \in [\tau_1, T)$ such that*

- (i) *there is no trade ($\gamma(t) = 0$) in the interval $[0, \tau_1)$,*
- (ii) *there is partial trade ($\gamma(t) \in (0, 1)$) in the interval $[\tau_1, \tau_2)$,*
- (iii) *there is full trade ($\gamma(t) = 1$) in the interval $[\tau_2, T)$.*

Proof. See Appendix.

23. The reason is that, as long as $P = v_\ell(T)$, the chance to transact with the MMLR or afterwards in the market are perfect substitutes from the perspective of an individual lemon. Only when the MMLR increases the price above $v_\ell(T)$ do lemons obtain an additional transfer through the intervention.

The dynamics after the intervention has taken place at T are entirely driven by how the average quality of assets for sale evolves over time. First, the floor for the average quality of assets that are for sale at t , $\tilde{\pi}(t)$ is given by initial average quality in the market after the shock, $\tilde{\pi}(0)$. When there is continuous trading, the measure of sellers with good assets remains constant. When there is no trading at any point in time, the average quality increases because more and more owners become sellers over time due to preference shocks – or in other words, selling pressure builds up over time. When selling pressure has built up, it cannot dissipate completely before $T = \infty$. This implies that $\mu_s(t) \geq \mu_s(0)$ for all $t < T$ so that the average quality can never fall below the floor. Second, the MMLR removes only lemons from the market which causes a discrete jump in the average quality at time T . Since $Q \geq Q_{\min}$, this jump is sufficient to raise the average asset quality above the threshold necessary for trading in steady state. Importantly, this is independent of how much trading there was before the intervention. With trading after the intervention, any built up selling pressure will dissipate over time and the average quality of assets for sale needs to decrease monotonically to a new steady state level where $\tilde{\pi} \geq \tilde{\pi}$. As a result, after the intervention, there is trade and the economy converges monotonically to the new steady state with trading.

For the dynamics before the intervention, it is possible that trading starts already before the intervention to which we refer as the *announcement effect*. This result can be understood best by looking at how the intervention influences the quality effect and the resale effect. We analyze this in detail in the next section and simply point out here that in equilibrium the trading volume $\lambda\gamma(t)$ has to be consistent with the evolution of the quality of assets that are for sale in the market. The key insight to prove the proposition is that once the surplus function $\Gamma(t)$ becomes strictly positive it has to stay so. The intuition for this result is that, at the point when $\Gamma(t)$ becomes strictly positive, the decrease in asset quality due to trading is largest while the time of the intervention – and, thus, the point when there is continuous trading – gets closer which increases the value of buying a lemon. As a result, $\Gamma(t)$ has to stay strictly positive afterwards.

3.5. The Announcement Effect

When does an announcement effect occur and how is it related to a feasible intervention (T, P, Q) ? The announcement effect is driven by the quality effect and the option value of the intervention. When there is no trading, selling pressure builds up in the market which in turn increases the average quality of assets that are for sale; in other words $\alpha(t)$ increases towards 1, the critical level for trading. Since the intervention resurrects trading, the value of a lemon also increases over time as buyers anticipate that they will be able to sell lemons in the future again. A positive option value ($V_I \geq 0$) – through a higher price and a higher quantity – provides here an additional benefit from holding a lemon that can be sold to the MMLR during the intervention. Consequently, if the quality for assets improves sufficiently and if we are sufficiently close to the intervention, there will be an announcement effect. By Proposition 5, some trading will then take place continuously over time from the moment when the surplus from trading is positive ($\Gamma(t) \geq 0$). This yields the following proposition that fully characterizes when an announcement effect happens as a function of the time of intervention T and the option value V_I .

Proposition 6. *For any feasible intervention (T, P, Q) there is an announcement effect if and only if*

$$V_I \geq (1 - \alpha(T^-)) \frac{r}{r + \lambda} v_s. \quad (34)$$

Suppose $\alpha > 1$. There is always an announcement effect provided one intervenes sufficiently late.

Suppose $\alpha \leq 1$. If $P = P_{\min}$, there is no announcement effect independent of the time of intervention.

Proof. See Appendix.

There are two particular cases that are of interest later on when we look at the optimal design of the intervention. First, when $\alpha > 1$ (i.e., when the shock to quality is small), without trading the average quality of assets will increase sufficiently over time to make it optimal for traders to start trading again before the intervention, even without a positive option value. Hence, as long as the intervention is delayed sufficiently, there will be an announcement effect. To the contrary, when $\alpha \leq 1$ (i.e., when the shock to quality is large) and when the intervention does not provide an option value, the quality effect can never be strong enough to make it optimal to start trading again before the intervention. The average quality of assets will only increase sufficiently after the intervention to induce trading in the market.

When the announcement effect starts to arise depends again on the quality and the resale effect which correspond to the two terms on the right-hand side of inequality (34). The first term, $1 - \alpha(T^-)$, expresses how severe the adverse selection problem is in the market and is decreasing over time, as more good asset flow into the market when there is no trade. The second term $\frac{r}{r+\lambda} v_s$ describes the difference in the value of acquiring a good asset and a lemon when there is trade again. For there to be trading before the intervention, the option value V_I needs to be large enough to compensate buyers for the lack of quality in the market and the risk of acquiring a lemon at the price of a good asset.

Increasing the option value V_I therefore decreases the critical time for the announcement effect to occur. Similarly, the critical time increases with the size of the shock to quality, since $\alpha(t)$ is decreasing in $\pi(0)$. With a larger shock it takes longer for the quality effect to become strong enough for trading to start again. How search frictions influence the critical time is, however, less clear as there are two effects. On the one hand, less trading frictions increase the resale effect since it is easier to turn around lemons which is reflected in the second term of inequality (34). On the other hand, for the quality effect less search frictions imply that the adverse selection problem is more severe in the first place, but also that the inflow of good assets into the market is larger when there is no trade.²⁴ In order to evaluate the overall effect of search frictions, we totally differentiate the right-hand side of inequality (34) in Proposition 6 and use the

24. Note that the effect of search frictions on the quality effect is slightly different from the one we have described in the context of the steady state (see equation (24)). In a steady state equilibrium with trading, a higher λ increases the outflow of good assets from the market and, hence, lowers the average quality of asset in the market. Without trading before an announcement effect emerges, there is, however, no outflow of good assets from the market.

definition of $\alpha(T)$ and α to obtain

$$\frac{dT}{d\lambda} = \frac{1}{\lambda(\lambda + \kappa)} \left[1 - \left(\frac{\bar{\pi}}{1 - \bar{\pi}} \right) \left(\frac{\pi(0)}{1 - \pi(0)} \right) \left(\frac{\kappa + \lambda}{r + \lambda} \right) e^{\kappa T} \right]. \quad (35)$$

Hence, for $\kappa \geq r$, the critical time for the emergence of the announcement effect decreases when there are less search frictions. This is also the case for $r > \kappa$ provided the shock to quality is sufficiently large.

4. OPTIMAL INTERVENTIONS AND THE ANNOUNCEMENT EFFECT

4.1. Objective Function

For the remainder of the paper, we provide guidance for policy makers when and how to intervene in markets in response to an adverse shock to quality that brings trading to a halt. We first look at the optimal quantity, time and price for the intervention, before discussing in detail what role the announcement effect plays for optimal policy.

In order to study the optimal intervention, we need to adopt a social welfare function that takes into account the costs of the intervention against the benefits of the market allocating assets among traders with different valuations. Our welfare function is akin to one that is commonly used in the public finance literature on regulation and given by

$$W(T, Q, P) = \int_{t=0}^{\infty} \left(\mu_o(t)\delta + \mu_s(t)(\delta - x) \right) e^{-rt} dt - \theta PQ e^{-rT}. \quad (36)$$

The first term describes the surplus from allocating good assets to traders with high valuation. Without trading, the number of owners, $\mu_o(t)$, declines at the expense of having more sellers, $\mu_s(t)$, in the market. An intervention that resurrects trading in the market can achieve a larger surplus as more assets are again allocated away from sellers to owners. Note here that whenever lemons are sold, there is only a zero sum transfer between different traders which does not enter the welfare function.

The second term expresses the costs of financing the intervention. There is a direct transfer of PQ to the lemons that sell to the MMLR at time T and permanently leave the market. Due to linear utility, these transfers are also zero sum and, consequently, do not enter the welfare function either. The parameter $\theta \in (0, \infty)$ expresses then the additional social costs of carrying out the transfer. It can also be understood as capturing the importance of the market relative to the costs of the intervention.²⁵ A more important market is thus captured by a smaller θ .

The main trade-off is with the timing of the intervention. A later intervention has a lower net present value of costs, but increases the costs of asset misallocation among traders. The announcement effect matters here, since delaying the intervention might not cause a 1-1 increase in the misallocation of assets whenever trading starts already before the actual intervention. Similarly, increasing the price or quantity to offer a positive option value ($V_I > 0$) is costly. This could be optimal, however, since a large enough option value can lead to an announcement effect in the first place and increasing it further can foster the effect.

25. These costs are commonly interpreted as the distortions from having to tax the economy to provide this transfer to traders. Our welfare function then also implies that there is no role for an intervention when the market is functioning. There is a positive cost of financing the intervention, but no benefit as the intervention does not affect the allocation of good assets when there is trade.

4.2. *Optimal Interventions*

We will now examine the optimal choice of quantity, price and time. We first show that it is never optimal to buy more than the minimum number of lemons Q_{\min} . This is due to a basic difference when increasing the quantity and the price of an intervention. Increasing the quantity Q involves a deadweight cost. It simply increases the probability for lemons to transact with the MMLR at T instead of them selling later on the market. Independent of the price, the MMLR thus provides a transfer of utility equal to P_{\min} to some lemons that would otherwise be provided for by future buyers. As a consequence, increasing the price has a larger impact on the option value than increasing the quantity which only increases the option value by the difference $P - P_{\min}$. Hence, it is always better to first increase the price in order to provide a larger option value V_I .²⁶ Once paying a price P_{\max} however, the MMLR has no choice other than increasing the quantity to achieve a higher option value. Consequently, the cost function for delivering option value V_I exhibits a kink at this point. This allows us to show that providing a higher option value can never be part of an optimal intervention as the MMLR would save costs by intervening earlier, but buying less lemons without affecting the incentives to trade. In other words, it is never optimal to generate an announcement effect (or foster it) by buying more than the minimum amount of lemons necessary to resurrect trading.

Proposition 7. *Any optimal intervention features $Q^* = Q_{\min}$.*

Proof. See Appendix.

Based on this result, we can then establish that it is never optimal to set $P \in (P_{\min}, P_{\max})$. If there is no announcement effect, an intervention taking place at $P > P_{\min}$ is clearly dominated by a minimal intervention. Setting P_{\min} instead saves costs without affecting trading. If there is an announcement effect at $P > P_{\min}$, the MMLR could increase P and simultaneously delay the intervention further. As costs increase linearly with the price, it turns out that such a change in policy can always save costs while fostering the announcement effect sufficiently to keep trading constant in the market. This establishes a “bang-bang” result where an optimal intervention either offers the highest or the lowest price.

Proposition 8. *Any optimal intervention features $P^* \in \{P_{\min}, P_{\max}\}$.*

Proof. See Appendix.

One cannot derive the optimal time for the intervention, since there are no closed form solutions for the breaking points τ_1 and τ_2 as a function of the policy (T, P, Q) . In general, however, the importance of the market and the size of the shock determine when to intervene. Our next result establishes that when markets are important enough,

²⁶ This is somewhat an artefact of the MMLR not being able to sell back any additional amount of lemons $Q - Q_{\min}$ to the market immediately after the intervention at a price equal to $v_\ell - \epsilon$. For more details, refer to the Online Appendix.

it is optimal to intervene immediately, and, conversely, when a market is not important, one should not intervene at all. Furthermore, the optimal policy treats delaying the intervention and increasing the price as complements: as markets become less important, increasing the price and delaying the intervention go hand-in-hand. An immediate consequence of this last result is that – when one holds the price constant – it is always optimal to intervene later in less important markets.²⁷

Proposition 9. *For $\theta \leq \underline{\theta}$ intervening immediately ($T = 0$) is optimal.
 For $\bar{\theta} \leq \theta < \infty$, never intervening ($T = \infty$) is optimal.
 For $\theta_2 > \theta_1$, if $P^*(\theta_2) \geq P^*(\theta_1)$, then $T^*(\theta_2) \geq T^*(\theta_1)$.*

Proof. See Appendix.

More general results on the optimal timing cannot be obtained analytically except for the special case where $\alpha < 1$ and where the intervention is restricted to be at $P = P_{\min}$. In this case, there can never be an announcement effect and the optimal time of intervention is then given by

$$T^* = -\frac{1}{\kappa} \ln \left(\frac{\kappa \bar{\theta} - \theta}{\lambda \underline{\theta}} \right) \quad (37)$$

where the bounds $\underline{\theta}$ and $\bar{\theta}$ both increase in $\pi(0)$.²⁸ This case is instructive to build some intuition for the optimal timing of the intervention. Less important markets and larger shocks imply later interventions, a result we will confirm later numerically for the general case when the announcement effect is present. When the market becomes less important, the cost of the intervention increases relative to gains from trading. Hence it is optimal to intervene later. With a large shock to quality the number of good assets is small, while the flow of assets that get misallocated is proportional to the number of good assets. This implies that fewer assets get misallocated and at a slower pace without trading. At the same time, the intervention needs to remove a large amount of lemons which is very costly. Hence, it is again optimal to intervene later.

4.3. The Optimal Use of the Announcement Effect

As we have pointed out, the announcement effect arises through a combination of delaying the intervention sufficiently and increasing its price. To characterize the optimal use of the effect, we first give sufficient conditions for when to use the effect by delaying the intervention and for not using it by setting a minimum price. Then, we provide conditions when to combine a delay with a price increase to foster the effect. Finally, we briefly discuss a special case that is of interest for policy design.

When the shock to quality is small ($\alpha > 1$), an announcement effect can arise from delaying the intervention sufficiently and without increasing the price (see Proposition

27. One can also derive the effects of changes in the gains from trade, x , on the optimal policy. Qualitatively, increases in the gains from trade x generate results similar to increases in market importance (lower θ). Specifically, for sufficiently large gains from trade (i.e., x close to δ), it is optimal to intervene in the market. For sufficiently small gains from trade (i.e., x close to 0), it is optimal not to intervene. For the special case with $\alpha < 1$ and $P = P_{\min}$, one can show further that as the gains from trade go up, the optimal time for the intervention decreases, while the bounds $\underline{\theta}$ and $\bar{\theta}$ increase.

28. For details, see the Online Appendix.

6). Whether to delay the intervention or not depends on two different costs. First, with delay, more assets are being misallocated as investors switch their valuations from high to low. When search frictions are large, this is very costly (in terms of welfare) as the market requires more time to reallocate these misplaced assets to investors with high valuations after the intervention. Second, delaying the intervention saves on intervention costs as the net present value is smaller. How much these cost savings matter depends on the importance of the market. Consequently, if trading frictions are small (λ sufficiently large) or if the market is not too important (θ sufficiently large), it is optimal to delay the intervention and rely on the announcement effect.

Proposition 10. *For a small shock ($\alpha > 1$), it is optimal to delay the intervention and rely on the announcement effect when the market is not important and trading frictions are small.*

Proof. See Appendix.

When the quality shock is large ($\alpha < 1$), an announcement effect can only arise through a sufficiently high price $P > P_{\min}$ at which the intervention takes place. Whether such a policy is optimal depends on the trade-off between the additional benefit of having the announcement effect and the extra cost of paying a high price. There are three situations where it is not optimal to increase the price. First, the magnitude of the announcement effect is limited when the intervention is conducted early because a sufficient delay is needed for an announcement effect to emerge. In a very important market, an early intervention is however optimal (Proposition 9). As a result, it is suboptimal to set a high price since there is no room for an announcement effect to arise. Second, the announcement effect is limited when the quality effect is small after a large shock to quality. The average quality in the market needs to be sufficiently high in order for buyers to purchase an asset already before the intervention. Third, the announcement effect is limited when search frictions are high because buyers cannot turn around lemons quickly. In addition, it is then cheap to conduct a minimum intervention, since $P_{\min} = \frac{\lambda}{\lambda+r}v_s$ is decreasing in the search friction.

Proposition 11. *For a large shock ($\alpha < 1$), it is optimal not to use the announcement effect and intervene at the minimum price when the market is important, trading frictions are large or the remaining number of good assets is sufficiently small.*

Proof. See Appendix.

Conditional on using the announcement effect, when is it optimal to increase the price to foster the effect? By how much should the price be increased? When $\alpha < 1$, Proposition 6 implies that there is an announcement effect only when $P > P_{\min}$. Moreover, by Proposition 8, whenever $P \in (P_{\min}, P_{\max})$, it is optimal to delay the intervention and increase the price to P_{\max} in order to foster the announcement effect as much as possible. Therefore, the MMLR would always increase the price to P_{\max} conditional on using the effect. When $\alpha > 1$, Proposition 6 implies that even an

TABLE 1

<i>Parameter Values</i>						
λ	κ	S	δ	x	r	π
10	1	4	1	0.035	0.05	0.99

intervention at P_{\min} can generate an announcement effect when the intervention is sufficiently delayed. Hence, for any intervention $T < \tilde{T}$ whenever the announcement effect is used, it is also optimal to set $P = P_{\max}$. In the numerical section, we establish this result more generally.

Proposition 12. *Whenever the announcement effect is used, it is optimal to foster it by setting $P = P_{\max}$ if either*

(i) $\alpha < 1$

or

(ii) $\alpha > 1$ and the intervention takes place at $T < \tilde{T}$, where \tilde{T} satisfies $\alpha(\tilde{T}) = 1$.

Finally, we establish a special case in which it is optimal to use the announcement effect. According to Proposition 9, when a market is sufficiently important, the MMLR should intervene early at a minimum price. However, such a policy cannot be implemented if there is an operational delay. In this case, the MMLR can still achieve continuous trading from $T = 0$ onwards by intervening at a higher price (and possibly quantity) provided the operational delay is not too large. Furthermore, this is optimal despite the higher cost of intervention whenever the market is important enough.

Proposition 13. *Suppose the intervention can take place only for $T \geq T^D$. There exists a $\bar{T}^D > 0$ such that for any $T^D \leq \bar{T}^D$, it is optimal to intervene at T^D , $P > P_{\min}$ and $Q \geq Q_{\min}$ to induce an announcement effect whenever $\theta \leq \underline{\theta}(T^D)$.*

Proof. See Appendix.

4.4. Numerical Results on the Announcement Effect

4.4.1. Calibration. Since one cannot derive closed form solutions for equilibrium trading in terms of policy, we use now a numerical analysis to study the optimal intervention and optimal use of the announcement effect further. We calibrate our economy to capture a typical market for structured finance products such as asset-backed securities (ABS) or collateralized debt obligations (CDO). Table 1 summarizes the values of the exogenous parameters. Details of our calibration are described in an Online Appendix.

We consider a negative quality shock at $t = 0$ such that $\pi(0) = 0.73 < \bar{\pi}$. Hence about a quarter of the assets turn from being good to being lemons. To resurrect trading, this requires an intervention that purchases at least an amount of $Q_{\min} = 0.9878$ which corresponds to roughly 90% of the total number of lemons. Our calibration then implies that $\alpha \approx 0.925$.

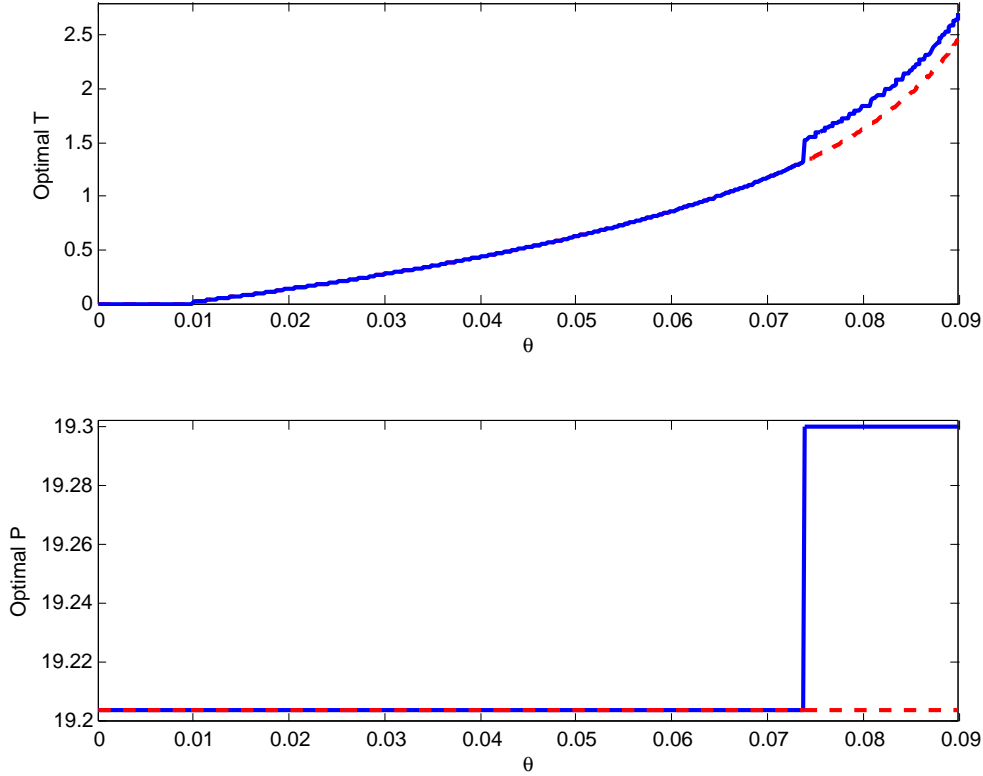


FIGURE 4
Optimal Intervention for the Benchmark Economy

4.4.2. Optimal Trade-off between Timing and Price. The optimal intervention depends on the importance of the market which is captured by the parameter θ that weighs the cost relative to the benefits of the intervention. Figure 4 shows the optimal timing and pricing of the intervention as a function of θ for our benchmark economy. The solid line indicates the optimal intervention. For comparison, we also plot the optimal intervention with the restriction that $P = P_{\min}$ as a dashed line. Note that our calibration implies that $\alpha < 1$.

The optimal price is either set to P_{\min} or P_{\max} . When θ is small, an immediate intervention is optimal so that there is no reason to increase the price above P_{\min} . As the value of θ increases, it is optimal to keep the price at P_{\min} , but to delay the intervention ($T > 0$) more and more. Given our parameter values, there cannot be an announcement effect from delaying the intervention. For a sufficiently high θ however, it becomes optimal to set the price to P_{\max} in order to achieve the maximum announcement effect. As suggested by Proposition 9, when P_{\max} is chosen, there is a discrete jump in the optimal T , relative to the optimal time for the minimal intervention. Hence, increasing the announcement effect allows the MMLR to delay the intervention through which he can partially recuperate the additional costs of paying a high price for lemons. This will be important for understanding our comparative statics results to which we turn next.

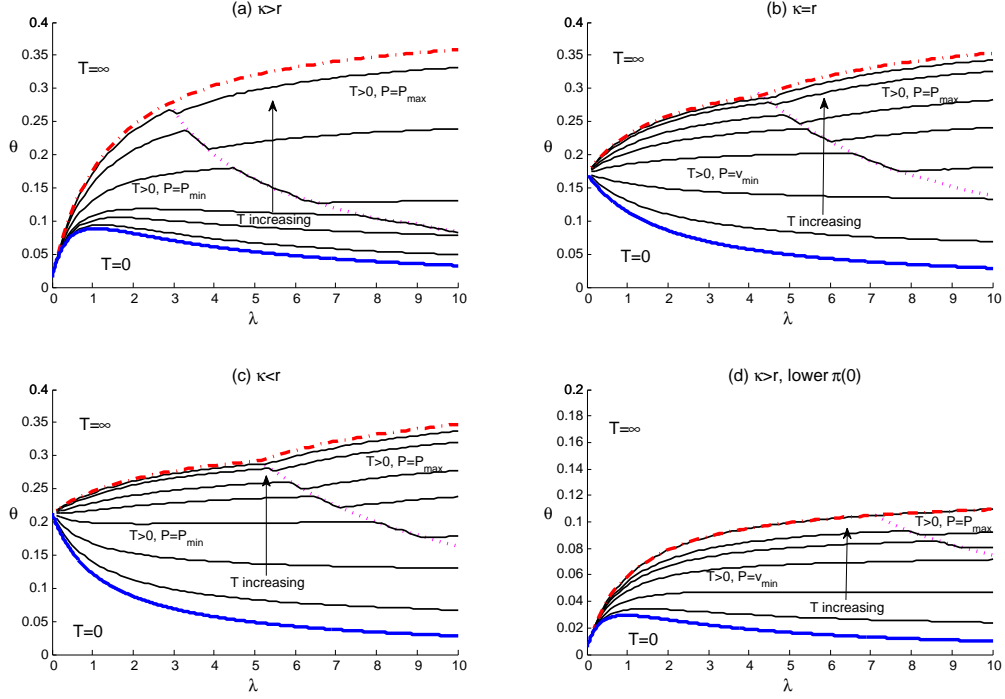


FIGURE 5
Optimal Intervention for different λ and θ

4.4.3. Comparative Statics. We now look at how the optimal time and price of the intervention changes with key parameters. Each subfigure of Figure 5 shows isoquants for the optimal timing of the intervention; i.e., combinations of trading frictions (λ) and market importance (θ) that lead to the same optimal time of intervention. There are two important insights that confirm and extend our previous results.

First, the optimal time of intervention is increasing in θ . Furthermore, the bounds for an immediate intervention and no intervention at all vary non-monotonically with trading frictions and liquidity needs of traders as given by the parameter κ . Also, the figure on the bottom right confirms that the optimal time of intervention increases with a larger shock to quality.

Second, the isoquants bend downward precisely when the optimal intervention relies on the announcement effect by increasing the price to P_{\max} . This demonstrates the main trade-off for the optimal policy between the price and the timing of the intervention. The MMLR can take advantage of the announcement effect through a higher price and delay the intervention. Here, the higher price is only being used once the intervention is delayed sufficiently. The reason for this is once again the quality effect. As the intervention is delayed, selling pressure builds up in the market. Hence, the average quality increases over time causing the announcement effect to be stronger. Consequently, intervening at P_{\max} increases the time interval of trading before the intervention $\tau_1 - T$, and thus allows the MMLR to delay the intervention further.

Third, search frictions and the importance of the market are both important for when it is optimal to increase the price to P_{\max} . This is shown by the lense shaped

region that is defined by the dashed line in the graphs. The parameter θ shifts the weight between the costs of the intervention and the costs of misallocation. When the weight on the costs of the intervention is large, delaying the intervention while increasing the price becomes more attractive. Here, small search frictions (large λ) are important once again for two reasons. First, they cause the price increase $P_{\max} - P_{\min}$ to be small. Second, since the stock of assets held by owners in steady state is large whenever λ is large, good assets flow faster into the market without trading when trading frictions are small. Hence, small search frictions cause the announcement effect to be stronger due to a stronger quality effect.

5. DISCUSSION

Our paper has shown that trading is fragile in asset markets where trading frictions matter and adverse selection is present. Small shocks to the quality of the asset being traded can bring trading to a halt. We have looked at interventions that buy bad assets to raise the average quality sufficiently so that trading in the market is again an equilibrium. The most interesting feature of such interventions is that the mere announcement of them can induce trading in the market before the actual intervention takes place. While our paper has provided guidance for policy makers how to intervene and use this effect, we have abstracted from some important and interesting aspects.

An interesting detail of the announcement effect is that there is a time-consistency problem. Suppose that investors believe the announcement that the intervention will take place at some time $T > 0$ at a price $P > P_{\min}$ and quantity $Q > Q_{\min}$, and, as a consequence, trading starts already before the intervention. The MMLR has then an incentive at T to surprise the market by lowering the price and the quantity to save costs. Hence, only minimum interventions can be time-consistent so that for $\alpha < 1$ such policies can never involve an announcement effect.²⁹ More generally, there cannot be full trade in the market before the intervention, since otherwise the MMLR would have an incentive to delay the intervention further by a small amount of time and save costs without bringing trading to a halt. Consequently, for any policy that involves an announcement effect to be time-consistent, the cost savings from delaying the intervention must be smaller than the losses from the additional misallocation of assets due to less trading in the market.

Our analysis has assumed that the shock to the quality of the asset is permanent. With a random recovery time for $\pi(0)$ to jump back to its original level our results might change. If the initial shock is small and recovery very likely, the market might just function continuously on its own. Furthermore, the possibility of a recovery can influence the optimal timing of the intervention. On the one hand, there can be additional incentives to delay the intervention, since delay can save costs in expected terms, even if this requires an increase in the size of the intervention in case the recovery does not happen quickly. On the other hand, once the asset quality recovers, there is the option to sell back some of the assets into the market. This can induce the MMLR to intervene earlier as now the expected total cost of the intervention have decreased.

We have also not looked at another, related problem. The quality shock is exogenous in our model. Suppose, however, that investors can create new assets. Anticipating that a MMLR will resurrect the market, investors will have an incentive to create lemons. In other words, a moral hazard problem arises from intervening in the event of a market freeze. This shifts the emphasis from intervention to improving the infrastructure in asset

29. See the Online Appendix for a characterization of optimal minimum interventions.

markets. In this context, our paper points here to improving the transparency of assets traded and to increasing market liquidity as possible improvements.

Finally, when studying trading dynamics, we abstract from learning in a dynamic market with adverse selection. As Daley and Green (2012) show, it can be optimal to delay trade when there is the potential for news to arrive in the market. We do not allow for a situation where investors or the government would need to learn over time the quality of an asset or the severity of a shock to average quality in the market. Similarly, we abstract from information percolation in decentralized markets. Duffie, Malamud and Manso (2009) for example investigate the incentives to search for information which could be applied to a situation of asset trading when there is asymmetric information. However, we do not capture how information is relayed through trading in the market place, but instead assume that all information acquired through a trade is always lost once the asset has been resold. Notwithstanding, such considerations are important for thinking about how a government as a large player could learn the average quality of assets through small, possibly repeated interventions in the market. Delaying asset purchases is an important factor here, as it could induce information revelation at the cost of a longer period for the market freeze. We leave a detailed analysis of these last two issues for future work.

APPENDIX A.

Appendix A.1. *Proof of Proposition 3*

If there is no trade for any t , the law of motion for good assets that are for sale is given by

$$\dot{\mu}_s(t) = -\dot{\mu}_o(t) = \kappa\mu_o(t).$$

Since the fraction of good assets drops to $S\pi(0)$ at time $t = 0$, the initial condition is given by $\mu_s(0) = \frac{\kappa}{\kappa+\lambda\gamma}S\pi(0)$. This implies that the fraction of good assets on the market for sale at time t , $\tilde{\pi}(t)$, is increasing monotonically to $\pi(0)$.

Since $v_\ell(t) = 0$ for all t , we are left to verify that

$$\tilde{\pi}(t)v_o - v_s \leq 0.$$

for all t . We have that $\tilde{\pi}(t) < \pi(0)$ for all t and $\tilde{\pi}(t) \rightarrow \pi(0)$ as $t \rightarrow \infty$. Hence, there exists an equilibrium with no trade as long as

$$\pi(0)v_o - v_s \leq 0$$

or, equivalently, $\pi(0) \leq \underline{\pi}$.

To show uniqueness, consider the buyer's surplus if there is trade ($\gamma(t) = 1$) for all t . Since $\sup_t \tilde{\pi}(t) \leq \pi(0)$, it suffices to show that

$$\pi(0)v_o + (1 - \pi(0))v_\ell - v_s \leq 0$$

where $v_\ell = \frac{\lambda}{\lambda+r}v_s$. Hence, if

$$\pi(0) \leq \frac{v_o - v_\ell}{v_o - v_\ell} = \frac{r\underline{\pi}}{r + (1 - \underline{\pi})\lambda} = \tilde{\pi},$$

it is a strictly dominant strategy not to buy an asset at any time t , which completes the proof.

Appendix A.2. *Proof of Proposition 5*

The proposition is established by a series of lemmata. We first establish that after a feasible intervention, trade is always an equilibrium independent of equilibrium trading prior to the intervention.

Lemma A1. *Consider any feasible intervention at time T . Trade with $\gamma(t) = 1$ for $t \in [T, \infty)$ is an equilibrium independent of $\gamma(t) \in [0, T)$.*

Proof. We need to show that $\alpha(t) \geq 1$ for all $t \in [T, \infty)$. To do so, we show that this function is monotonically decreasing over the interval to a level that is larger than 1.

Note first that $\alpha(t)$ is decreasing if and only if $\tilde{\pi}(t)$ is decreasing. For an interval $[\tau, \tau']$ with $\tau \geq T$ where $\gamma(t) = 1$, we then have

$$\tilde{\pi}(t) = \frac{S\pi(0)\frac{\kappa}{\kappa+\lambda}(1 - e^{-(\kappa+\lambda)(t-\tau)}) + \mu_s(\tau)e^{-(\kappa+\lambda)(t-\tau)}}{S\pi(0)\frac{\kappa}{\kappa+\lambda}(1 - e^{-(\kappa+\lambda)(t-\tau)}) + \mu_s(\tau)e^{-(\kappa+\lambda)(t-\tau)} + (1 - \pi(0))S - Q}$$

where $\mu_s(\tau)$ is the measure of sellers at the beginning of the interval. Differentiating, we obtain up to a multiplying constant

$$\frac{\partial \tilde{\pi}(t)}{\partial t} = -(\kappa + \lambda)e^{-(\kappa+\lambda)(t-\tau)} [(1 - \pi(0))S - Q] \left(\mu_s(\tau) - S\pi(0)\frac{\kappa}{\kappa+\lambda} \right).$$

If there has been continuous trade from $t = 0$ until τ – i.e., $\gamma(t) = 1$ for all $t \in [0, \tau]$ – we have that $\mu_s(\tau) = \mu_s(0) = S\pi(0)\frac{\kappa}{\kappa+\lambda}$. Hence, $\tilde{\pi}(t)$ is constant.

If there has not been full trade at some time before τ – i.e. $\gamma(t) < 1$ for some $[t_1, t_2] \subset [0, \tau]$ – it must be the case that $\mu_s(\tau) > \mu_s(0) = S\pi(0)\frac{\kappa}{\kappa+\lambda}$. Hence, $\tilde{\pi}(t)$ is decreasing.

By continuity, we have that with continuous trade $\tilde{\pi}(t)$ converges to a long-run steady-state value given by

$$\frac{\frac{\kappa}{\kappa+\lambda}S\pi(0)}{\frac{\kappa}{\kappa+\lambda}S\pi(0) + S(1 - \pi(0)) - Q} \geq \frac{\frac{\kappa}{\kappa+\lambda}S\pi(0)}{\frac{\kappa}{\kappa+\lambda}S\pi(0) + S(1 - \pi(0)) - Q_{\min}} = \tilde{\pi}.$$

Hence, conditional on $Q \geq Q_{\min}$, we have that $\alpha(t) \geq 1$ for all $t \geq T$ which completes the proof.

We now prove the second part of the proposition, which states that trading before the intervention can be characterized by two breaking points $\tau_1(T) \geq 0$ and $\tau_2(T) \in [\tau_1, T)$. To do so, we take full trade after the intervention as given and show first that once the surplus function $\Gamma(t)$ becomes positive it has to stay positive. This implies that, after there has been some trade in the economy ($\gamma(t) > 0$), we cannot have no trade ($\gamma(t) = 0$) anymore, since the surplus function $\Gamma(t)$ has to stay non-negative. We then show that with full trade ($\gamma(t) = 1$) in some interval, the surplus function has to be strictly convex.

Lemma A2. *If $\Gamma(t_0) \geq 0$ for some $t_0 < T$, then $\Gamma(t_1) \geq 0$ for all $t_1 \in (t_0, T)$.*

Proof. Suppose not. Then, there exists a $t_1 \in (t_0, T)$ such that $\Gamma(t_1) < 0$. As Γ is continuous on $[0, T)$, this implies that there must be an interval $(\tau_0, \tau_1) \subset (t_0, t_1)$ where there is no trade, i.e. $\gamma(t) = 0$. But then, over this interval, the average quality $\tilde{\pi}(t)$ increases and we have that $\dot{v}_\ell(t) = v_\ell(\tau_1)e^{-r(\tau_1-t)}r > 0$. Hence, $\Gamma(t)$ must be strictly increasing over this interval starting out at $\Gamma(\tau_0) = 0$. A contradiction.

Lemma A3. *If $\gamma(t) = 1$ for some interval $[t_0, t_1]$ with $t_1 < T$, then $\Gamma(t)$ is strictly convex over this interval.*

Proof. We have

$$\begin{aligned} \Gamma(t) &= \tilde{\pi}(t)(v_o - v_\ell(t)) + (v_\ell(t) - v_s) \\ \dot{\Gamma}(t) &= \dot{\tilde{\pi}}(t)(v_o - v_\ell(t)) + (1 - \tilde{\pi}(t))\dot{v}_\ell(t) \\ \ddot{\Gamma}(t) &= \ddot{\tilde{\pi}}(t)(v_o - v_\ell(t)) - 2\dot{\tilde{\pi}}(t)\dot{v}_\ell(t) + (1 - \tilde{\pi}(t))\ddot{v}_\ell(t). \end{aligned}$$

We will show that $\Gamma(t)$ is strictly convex if it is positive and strictly increasing. Assuming $\gamma(t) = 1$ for the asset quality and omitting time indexes, we have

$$\begin{aligned} \dot{\tilde{\pi}} &= (1 - \tilde{\pi})\frac{\dot{\mu}_s}{\mu_s + \mu_\ell} \\ \ddot{\tilde{\pi}} &= -\dot{\tilde{\pi}}\frac{\dot{\mu}_s}{\mu_s + \mu_\ell} + (1 - \tilde{\pi})\frac{(\mu_s + \mu_\ell)\ddot{\mu}_s - (\dot{\mu}_s)^2}{(\mu_s + \mu_\ell)^2}. \end{aligned}$$

Since $\gamma(t) = 1$ for $t \in [t_0, t_1]$, $\dot{\tilde{\pi}}(t) < 0$.

By the previous lemma, we either have continuous trade so that $\Gamma(t) \geq 0$ with $\gamma(t) = 1$ for all $t \in [0, T)$ or there is less than full trade ($\gamma(t) < 1$) for $t < t_0$. Without loss of generality, we can look only at the second case. Since $\Gamma(t)$ is continuous on $[0, T)$, it must be the case that $\Gamma(t_0) = 0$. Furthermore, $\gamma(t) = 1$ for $t \in [t_0, t_1]$ implies that $\dot{\tilde{\pi}}(t) < 0$. Hence, at t_0 , we need that the right-hand derivative $\dot{\Gamma}(t_0^+) \geq 0$. This can only be the case if $\dot{v}_\ell(t_0^+) > 0$. Also, we have that

$$v_\ell(t) = \frac{\lambda}{\lambda + r} v_s \left(1 - e^{-(r+\lambda)(t_1-t)} \right) + v_\ell(t_1) e^{-(r+\lambda)(t_1-t)}.$$

Hence, $\dot{v}_\ell(t) > 0$ for $t \in [t_0, t_1]$ if and only if $\dot{v}_\ell(t_0^+) > 0$. This implies that $v_\ell(t)$ is a strictly increasing and strictly convex function over this interval. Hence, the last two terms are positive in the expression for $\ddot{\Gamma}(t)$. Note that $v_o - v_\ell(t) > 0$. If $\ddot{\tilde{\pi}}(t)$ is positive we are done.

Suppose to the contrary that $\ddot{\tilde{\pi}}(t) < 0$. As long as $\dot{\Gamma}(t) \geq 0$ for $t \in (t_0, t_1)$, it must be the case that

$$0 < v_o - v_\ell(t) \leq -\frac{1 - \tilde{\pi}(t)}{\dot{\tilde{\pi}}(t)} \dot{v}_\ell(t).$$

Using this in the expression for $\ddot{\Gamma}(t)$, it suffices to show that

$$-\ddot{\tilde{\pi}}(t) \frac{1 - \tilde{\pi}(t)}{\dot{\tilde{\pi}}(t)} \dot{v}_\ell(t) - 2\dot{\tilde{\pi}}(t) \dot{v}_\ell(t) + (1 - \tilde{\pi}(t)) \ddot{v}_\ell(t) > 0$$

or that

$$-\frac{\ddot{\tilde{\pi}}(t)}{\dot{\tilde{\pi}}(t)} - 2\frac{\dot{\tilde{\pi}}(t)}{1 - \tilde{\pi}(t)} + \frac{\ddot{v}_\ell(t)}{\dot{v}_\ell(t)} > 0.$$

Note that $\dot{v}_\ell(t) = (r + \lambda)v_\ell(t)$. Hence, rewriting and using the fact that $\dot{\mu}_s = -(\kappa + \lambda)\dot{\mu}_s$, we obtain

$$-\left[-\frac{\dot{\mu}_s}{\mu_s + \mu_\ell} + \frac{-(\mu_s + \mu_\ell)(\kappa + \lambda) - \dot{\mu}_s}{\mu_s + \mu_\ell} \right] - 2\frac{\dot{\mu}_s}{\mu_s + \mu_\ell} + (r + \lambda) > 0$$

or

$$(\kappa + \lambda) + (r + \lambda) > 0$$

which completes the proof.

The proposition follows now directly as a corollary from this lemma. Without trade, it must be the case that the surplus function $\Gamma(t)$ is increasing over time, as both $\dot{\tilde{\pi}} > 0$ and $\dot{v}_\ell > 0$. Once the surplus function becomes strictly positive, it cannot stay constant anymore, as it is strictly convex. Hence, $\dot{\Gamma}(t) > 0$ whenever $\gamma(t) = 1$ is an equilibrium.

Appendix A.3. Proof of Proposition 6

Consider any feasible intervention (T, P, Q) . By Proposition 5, there is no trade before the intervention ($\tau_1(T) = T$) if and only if

$$\begin{aligned} \Gamma(T^-) &= \tilde{\pi}(T^-)v_o + (1 - \tilde{\pi}(T^-))(v_\ell(T) + V_I) - v_s \\ &= (1 - \tilde{\pi}(T^-)) \left[\left(\frac{\tilde{\pi}(T^-)}{1 - \tilde{\pi}(T^-)} \right) (v_o - v_s) + V_I - \frac{r}{r + \lambda} v_s \right] \\ &= (1 - \tilde{\pi}(T^-)) \left[\alpha(T^-) \left(\frac{\tilde{\pi}}{1 - \tilde{\pi}} \right) (v_o - v_s) + V_I - \frac{r}{r + \lambda} v_s \right] \\ &= (1 - \tilde{\pi}(T^-)) \left[\alpha(T^-) \frac{r}{r + \lambda} v_s + V_I - \frac{r}{r + \lambda} v_s \right] < 0. \end{aligned}$$

Hence, an announcement effect occurs if and only if

$$V_I \geq (1 - \alpha(T^-)) \frac{r}{r + \lambda} v_s.$$

Suppose $\alpha > 1$. Without trade, we have that $\alpha(t)$ increases monotonically to α . This implies that for all (P, Q) there exist some T large enough so that the condition is satisfied. Conversely, suppose

$\alpha < 1$. For $P = P_{\min}$, we have $V_I = 0$ independent of Q . Since with no trade α describes an upper bound for $\alpha(t)$, the right-hand side of the condition is always positive so that there cannot be an announcement effect.

Appendix A.4. Proof of Proposition 7

To prove the result we rely on two lemmata. The first one characterizes the minimum cost for achieving any option value V_I . Importantly, this cost function exhibits a kink at the value for V_I where $Q = Q_{\min}$ and $P = P_{\max}$.

Lemma A4. Define $\hat{V}_I = \frac{Q_{\min}}{S(1-\pi(0))} \frac{r}{r+\lambda} v_s$. The minimum cost to achieve an option value V_I is given by

$$C(V_I) = \begin{cases} V_I S(1-\pi(0)) + Q_{\min} v_\ell & \text{if } V_I \in [0, \hat{V}_I] \\ \frac{r+\lambda}{r} V_I S(1-\pi(0)) & \text{if } V_I \in (\hat{V}_I, \frac{r}{r+\lambda} v_s]. \end{cases}$$

Proof. For any given price $P \in (v_\ell, v_s)$ increasing the quantity of the intervention above Q_{\min} involves a deadweight cost, as increasing the quantity at such prices implies a transfer of utility to current lemons that otherwise is provided by future buyers of lemons after the intervention resurrects the trading. Hence, it is cheaper to increase the price to achieve a particular $V_I < \hat{V}_I$. Note that by definition, we have

$$V_I = \frac{Q}{S(1-\pi(0))} (P - v_\ell).$$

The costs of any feasible policy at T are simply given by PQ . The result then follows from first holding Q_{\min} constant and increasing P to v_s , then also increasing Q to $S(1-\pi(0))$ for $P = v_s$ to achieve all option values associated with feasible policies.

The second lemma looks at how the net present value of the minimum cost $C(V_I)e^{-rT}$ of an intervention changes by varying this policy in a way that allows us to keep the surplus $\Gamma(T)$ constant conditional on trading behaviour not changing. This net present value is minimized precisely at the kink of the minimum cost function established above.

Lemma A5. Let $\gamma \in (0, 1]$ and change the policy (T, V_I) according to $dT/dV_I = \frac{1}{r+\lambda\gamma} \frac{1}{V_I}$ for all (T, V_I) with $T > 0$ and $V_I > 0$. For such changes, the net present value of costs is minimized at \hat{V}_I .

Proof. Consider the cost isoquants for any policy (T, V_I) . These isoquants are given by

$$\frac{\partial C(V_I)e^{-rT(V_I)}}{\partial V_I} = e^{-rT(V_I)} \left[\frac{dC}{dV_I} - rC(V_I) \frac{dT}{dV_I} \right] = 0.$$

Let $V_I > \hat{V}_I$. Then, we have

$$\frac{1}{r} \frac{dC/dV_I}{C(V_I)} = \frac{1}{r} \frac{1}{V_I} > \frac{1}{r+\lambda\gamma} \frac{1}{V_I} = \frac{dT}{dV_I}.$$

for all V_I . Hence, the net present value of costs is increasing in the policy change dT/dV_I .

Now, let $V_I < \hat{V}_I$. We have

$$\frac{1}{r} \frac{dC/dV_I}{C(V_I)} - \frac{dT}{dV_I} < 0$$

if and only if

$$V_I S(1-\pi(0)) < \frac{r}{r+\lambda\gamma} (V_I S(1-\pi(0)) + Q_{\min} v_\ell)$$

or

$$1 < \frac{r}{r+\lambda\gamma} \left(1 + \frac{Q_{\min} v_\ell}{V_I S(1-\pi(0))} \right) = \frac{1}{r+\lambda\gamma} \left(r + \lambda \frac{Q_{\min} \frac{r}{r+\lambda} v_s}{V_I S(1-\pi(0))} \right).$$

Since $V_I S(1 - \pi(0)) < Q_{\min} \frac{r}{r+\lambda} v_s$, we have that the right-hand side of the last expression is bounded below by $(r + \lambda)/(r + \lambda\gamma) > 1$. Hence, the net present value of costs is decreasing in the policy change dT/dV_I .

We now prove the result that any optimal intervention has $Q = Q_{\min}$. Consider first any policy (T, V_I) for which $V_I > \hat{V}_I$ and $\tau_2(T) < T$. We construct a cheaper policy (T', V'_I) that leaves the incentives to trade unchanged at any t . Define the new time of intervention by $T' = T - \Delta \in (\tau_2(T), T)$ and define the new size of the intervention corresponding to a marginal policy change by

$$V'_I = V_I e^{-(r+\lambda)(T-T')}.$$

Since $\Gamma(t) > 0$ for all $[T', \infty)$, this leaves $v_\ell(T'^-)$ unaffected. This implies that the old equilibrium strategy $\gamma(t)$ is still an equilibrium for $[0, T)$, as both $v_\ell(t)$ and $\tilde{\pi}(t)$ remain unchanged leading to the same surplus function $\Gamma(t)$ as before for all $t \in [0, T']$. For small enough $\Delta > 0$, we have $V'_I \geq \hat{V}_I$. By Lemma A5 the net present value of costs has decreased which implies that the original policy (T, V_I) cannot be optimal.

Consider then a policy (T, V_I) such that $V_I > \hat{V}_I$ and $\tau_1(T) < \tau_2(T) = T$. We have that at any $t \in (\tau_1(T), T)$,

$$\begin{aligned} v_\ell(t) &= v_\ell(T) e^{\int_t^T -(r+\lambda\gamma(s))ds} - \int_t^T e^{\int_t^s -(r+\lambda\gamma(s))ds} (-\gamma(s)) \lambda ds \\ &= v_\ell(T) e^{-(r+\lambda\bar{\gamma})(T-t)} + \frac{\lambda\bar{\gamma}}{\lambda\bar{\gamma} + r} v_s \left(1 - e^{-(r+\lambda\bar{\gamma})(T-t)}\right) \\ &= \left(\frac{\lambda}{\lambda + r} v_s + V_I\right) e^{-(r+\lambda\bar{\gamma})(T-t)} + \frac{\lambda\bar{\gamma}}{\lambda\bar{\gamma} + r} v_s \left(1 - e^{-(r+\lambda\bar{\gamma})(T-t)}\right), \end{aligned}$$

for some $\bar{\gamma} \in (0, 1)$. This allows us to define a new policy with $T' = T - \Delta \in (\tau_1(T), T)$ and the option value given by

$$\begin{aligned} V'_I &= V_I e^{-(r+\lambda\bar{\gamma})(T-T')} + \left(1 - e^{-(r+\lambda\bar{\gamma})(T-T')}\right) \left(\frac{\lambda\bar{\gamma}}{\lambda\bar{\gamma} + r} - \frac{\lambda}{\lambda + r}\right) v_s \\ &< V_I e^{-(r+\lambda\bar{\gamma})(T-T')}. \end{aligned}$$

For Δ sufficiently small, we again have $V'_I > \hat{V}_I$. The new policy saves more costs than are saved by the policy change of Lemma A5 given $\bar{\gamma} \in (0, 1)$. As in the argument above, since $v_\ell(T'^-)$ stays constant for the new policy (T', V'_I) , the old equilibrium strategies for $[0, T')$ and $\gamma(t) = 1$ for $[T', \infty)$ form an equilibrium. Since costs decrease and trading in the market improves, the policy (T, V_I) cannot be optimal. This completes the proof, since for any policy with $V_I < \hat{V}_I$ it is optimal to set $Q = Q_{\min}$.

Appendix A.5. Proof of Proposition 8

By Proposition 6, an announcement effect can only occur when the intervention is sufficiently delayed and the option value is sufficiently large. Using $Q = Q_{\min}$, this implies that we need a minimum price equal to

$$\underline{P}(T) - P_{\min} = \left(\frac{1 - \alpha(T)}{1 - \alpha(0)}\right) \left(\frac{r}{r + \lambda}\right) v_s.$$

for the effect to arise. Hence, any price in the interval $(P_{\min}, \underline{P}(T)]$ cannot be optimal, since it increases the cost of the intervention without providing additional benefits in the form of inducing trading before the intervention.

We show next that any policy with $P \in (\underline{P}(T), P_{\max})$ is dominated by P_{\max} . We proceed via two lemmata, where the first one establishes the result for the case $\tau_1 \leq \tau_2 < T$ and the second deals with the case $\tau_1 < \tau_2 = T$.

Lemma A6. *Any policy with $P \in (\underline{P}(T), P_{\max})$ and $\tau_2(T) < T$ is not optimal.*

Proof. Consider any policy (T, P) with $\tau_2(T) < T$. Define a new policy by $T' = T + \Delta$ and $V_I' = V_I e^{-(r+\lambda)(T-T')}$. By Lemma A5 this defines a policy change that saves costs and leaves $v_\ell(T)$ unchanged. Furthermore, for Δ sufficiently small we have that $\Gamma(t) > 0$ for all $t \in [T, T')$, since at the original policy $\Gamma(T^-) > 0$. Hence, the old equilibrium trading strategy γ is still an equilibrium, but the new policy is cheaper. Hence, (T, V_I) cannot be optimal.

Lemma A7. *Any policy with $P \in (\underline{P}(T), P_{\max})$ and $\tau_1(T) < \tau_2(T) = T$ is not optimal.*

Proof. Consider any policy (T, P) with $\tau_1(T) < \tau_2(T) = T$, so that $\Gamma(T^-) = 0$. Since we have an equilibrium with partial trading before T , we have that $\Gamma(t) = 0$ and $\dot{\Gamma}(t) = 0$. Hence,

$$\begin{aligned} \frac{\mu_s(t)}{\mu_\ell(t)}(v_o - v_s) + (v_\ell(t) - v_s) &= 0 \\ \frac{\dot{\mu}_s(t)}{\mu_\ell(t)}(v_o - v_s) + \dot{v}_\ell(t) &= 0 \end{aligned}$$

for all $t \in [\tau_1(T), T)$. Moreover, we have over this interval that

$$\begin{aligned} \dot{v}_\ell(t) &= (r + \lambda\gamma(t))v_\ell(t) - \lambda\gamma(t)v_s \\ \dot{\mu}_s(t) &= \kappa S\pi(0) - (\kappa + \lambda\gamma(t))\mu_s(t). \end{aligned}$$

Let $t \rightarrow T^-$. This yields a boundary condition for $v_\ell(t)$ which is given by

$$\lim_{t \rightarrow T^-} v_\ell(t) = \frac{\lambda}{\lambda + r}v_s + V_I.$$

Since $\mu_s(t)$ and $v_\ell(t)$ are left-continuous, these five conditions determine $\gamma(T^-)$ which depends on V_I , but is independent of T . We then have that

$$\dot{\mu}_s(T^-) = \left(\frac{r}{\lambda + r} \lambda(\gamma(T^-) - 1) - (r + \lambda\gamma(T^-)) \frac{V_I}{v_s} \right) S(1 - \pi(0)) \frac{\pi}{1 - \pi},$$

which also determines $\mu_s(t)$.

Consider again delaying the intervention, but increasing the price according to the cost saving policy change of Lemma A5 that is given by

$$\frac{dV_I}{dT} = (r + \lambda)V_I.$$

Note that this change leaves $v_\ell(T^-)$ unchanged and yields

$$\dot{v}_\ell(T) = (r + \lambda)V_I.$$

Hence, as long as $\gamma(t) = 1$ for $t \in [T, \infty)$, trading for $t \in [0, T)$ will also remain the same.

Set $\gamma(t) = 1$ for $t \in [T, \infty)$. With this change, we have an upward jump in $\gamma(t)$ at T which causes a kink in the differential equation that determines $\mu_s(t)$, but leaves $\mu_s(T)$ the same. For this to be an equilibrium, we need to show that $\Gamma(t) \geq 0$ when $\gamma(t) = 1$ in the interval $[T, \infty)$. Since the increase in v_ℓ is the smallest at T and the decrease in μ_s is the largest at T for $t \in [T, \infty)$, it is sufficient to show that

$$\dot{v}_\ell(T) \geq -\dot{\mu}_s(T) \frac{\mu_\ell(T)}{v_o - v_s}.$$

Since $\gamma(T) = 1$, we have

$$\dot{v}_\ell(T) \geq (r + \lambda) \frac{V_I}{v_s} S(1 - \pi(0)) \frac{\pi}{1 - \pi} \frac{v_o - v_s}{\mu_\ell} = (r + \lambda)V_I$$

which completes the proof.

Appendix A.6. Proof of Proposition 9

We first show that there exists some $\bar{\theta}$ such that for all $\theta \geq \bar{\theta}$ it is optimal to set $T = \infty$. To do so, we proceed in three steps. First, we derive a bound on the maximum announcement effect that is

independent of T . Then we use the bound on the announcement effect to derive bounds on the gains from intervening. Finally, we can use these bounds to show that for large enough θ delaying the intervention further is optimal independent of T .

Consider any intervention at T and $P = v_s$. We derive a lower bound $\tau_{\min} \leq \tau_1(T)$ such that there is no trade before τ_{\min} . Note that at any time $t < T$ before the intervention a buyer would have the maximum incentive to trade when (i) $\tilde{\pi} = \pi(0)$, (ii) there is full trade after t , and (iii) trading before T has no effect on the discounting of the option value V_I . Hence, a lower bound on $\tau_1(T)$ is given by τ_{\min} such that

$$\Gamma(\tau_{\min}) = \pi(0)v_o + (1 - \pi(0)) \left(\frac{\lambda}{\lambda + r} v_s + V_I e^{-r(T - \tau_{\min})} \right) - v_s = 0$$

where we have used the maximum option value

$$V_I = Q_{\min} \frac{v_s - v_\ell}{S(1 - \pi(0))} = \frac{1 - \frac{\pi(0)}{\tilde{\pi}}}{1 - \pi(0)} \frac{r}{\lambda + r} v_s.$$

Note that the bound on the maximum announcement effect $\Delta_{\max} = T - \tau_{\min} > 0$ is thus independent of T and given by

$$e^{-r\Delta_{\max}} = \frac{1 - \frac{\pi(0)}{\tilde{\pi}}}{1 - \pi(0)}.$$

We now use Δ_{\max} to derive a lower and an upper bound on the benefits of any intervention. For these bounds, we assume that there is no trade before the lower bound $\tau_{\min} = T - \Delta_{\max}$ so that all assets are misallocated according to the inflow of assets from no trade which we denote $\hat{\mu}_s$. For the lower bound, we assume that all assets are misallocated ($\mu_s(t) = 1$) after τ_{\min} . For the upper bound, we assume that no assets are misallocated ($\mu_s(t) = 0$) after τ_{\min} . Using $Q = Q_{\min}$ for any optimal policy, the lower bound of the welfare with any intervention (T, P) is then given by

$$\underline{W}(T, P) = \int_0^{T - \Delta_{\max}} (S\pi\delta - \hat{\mu}_s(t)x) e^{-rt} dt + \int_{T - \Delta_{\max}}^{\infty} S\pi(\delta - x) e^{-rt} dt - \theta PQ_{\min} e^{-rT}$$

and the upper bound by

$$\bar{W}(T, P) = \int_0^{T - \Delta_{\max}} (S\pi\delta - \hat{\mu}_s(t)x) e^{-rt} dt + \int_{T - \Delta_{\max}}^{\infty} S\pi\delta e^{-rt} dt - \theta PQ_{\min} e^{-rT}.$$

Consider now delaying the intervention from T to some $T + \epsilon$, for some arbitrary $\epsilon > 0$. Using the bounds, the welfare gain is then at least

$$\begin{aligned} \underline{W}(T + \epsilon, P) - \bar{W}(T, P) &= \\ &= - \int_{T - \Delta_{\max}}^{T + \epsilon - \Delta_{\max}} \hat{\mu}_s(t)x e^{-rt} dt - S\pi x \int_{T + \epsilon - \Delta_{\max}}^{\infty} e^{-rt} dt + \theta PQ_{\min} (e^{-rT} - e^{-r(T + \epsilon)}) \\ &> -S\pi \left(\frac{x}{r} \right) e^{-r(T - \Delta_{\max})} + \theta PQ_{\min} (e^{-rT} - e^{-r(T + \epsilon)}) \\ &= e^{-rT} \left[-S\pi \left(\frac{x}{r} \right) e^{r\Delta_{\max}} + \theta PQ_{\min} (1 - e^{-r\epsilon}) \right], \end{aligned}$$

since $\hat{\mu}_s(t) < S\pi$. The expression in brackets does not depend on T , since Δ_{\max} is a constant independent of T and PQ . For $\epsilon > 0$, it is positive for θ finite, but sufficiently large. Since the argument does not depend on the cost PQ , there exists some $\bar{\theta} < \infty$ such that for $\theta > \bar{\theta}$ the optimal intervention is $T = \infty$.

We now show that there exists $\underline{\theta} > 0$ such that for $\theta < \underline{\theta}$ it is optimal to intervene immediately. Consider any policy that is delayed sufficiently to be cheaper than an intermediate intervention with P_{\min} . By Proposition 7, any optimal policy sets $Q = Q_{\min}$. A policy with $P > P_{\min}$ is cheaper if and only if it is sufficiently delayed or, equivalently, if and only if

$$e^{-rT} \geq \frac{P}{P_{\min}}.$$

Hence, any cheaper policy with $T > 0$ and P needs to satisfy

$$T \geq T_1 = \frac{1}{r} \ln \left(\frac{P}{v_\ell} \right).$$

Note that $T_1 \rightarrow 0$, as $P \rightarrow v_\ell$.

Any policy with $P > P_{\min}$ and $T > 0$ can have an announcement effect. By Proposition 6, we have that given P the first time an announcement effect arises is given by the solution $T_2(P)$ to

$$P - v_\ell = \frac{(1 - \alpha(T_2))}{1 - \alpha(0)} \frac{r}{r + \lambda} v_s.$$

Note that for any α , we have $T_2(P_{\max}) = 0$. For any $P < P_{\max}$, $T_2(P)$ is positive and monotonically increasing as $P \rightarrow v_\ell$.

We now compare the welfare of policies with delay that are cheaper, but have an announcement effect to an immediate intervention at $T = 0$. Since $T_1(P_{\max}) > 0$, there exists some \hat{P} such that $T_1(\hat{P}) = T_2(\hat{P}) = \underline{T} > 0$. It must then be the case that $\max\{T_1(P), T_2(P)\} \geq \underline{T}$ for all $P > v_\ell$. This implies that the welfare loss from delaying the intervention with any policy (T, P) is given by

$$\tilde{W}(T, P) - \tilde{W}(0, v_\ell) + \theta Q_{\min} (v_\ell - e^{-rT} P) < \tilde{W}(\underline{T}, P) - \tilde{W}(0, v_\ell) + \theta Q_{\min} v_\ell,$$

where \tilde{W} expresses the welfare from allocating asset across traders.

Since $\tilde{W}(\underline{T}, P) < \tilde{W}(0, v_\ell)$, there exists $\underline{\theta} > 0$ such that for any $\theta < \underline{\theta}$ this expression is negative. Hence, for θ sufficiently small it is never optimal to delay the intervention, but increase its price. The result now follows from the fact that is also not optimal to delay the intervention, but keeping $P = P_{\min}$ when θ is sufficiently close to 0 (see the Online Appendix).

For the final part of the proposition, we can restrict ourselves to $P \in \{P_{\min}, P_{\max}\}$. Denote the optimal policy given θ by $(T^*(\theta), P^*(\theta))$. By definition, we then have for the welfare function as a function of policy and the parameter θ that

$$\begin{aligned} W(T^*(\theta_1), P^*(\theta_1); \theta_1) &\geq W(T^*(\theta_2), P^*(\theta_2); \theta_1) \\ W(T^*(\theta_2), P^*(\theta_2); \theta_2) &\geq W(T^*(\theta_1), P^*(\theta_1); \theta_2). \end{aligned}$$

This implies that

$$\begin{aligned} W(T^*(\theta_1), P^*(\theta_1); \theta_1) - W(T^*(\theta_2), P^*(\theta_2); \theta_1) &\geq \\ W(T^*(\theta_1), P^*(\theta_1); \theta_2) - W(T^*(\theta_2), P^*(\theta_2); \theta_2). \end{aligned}$$

This inequality reduces to

$$(\theta_2 - \theta_1) \left(P^*(\theta_1) e^{-rT^*(\theta_1)} - P^*(\theta_2) e^{-rT^*(\theta_2)} \right) \geq 0.$$

Hence, if $\theta_2 > \theta_1$, we have that $P^*(\theta_2) \geq P^*(\theta_1)$ implies $T^*(\theta_2) \geq T^*(\theta_1)$.

Appendix A.7. Proof of Proposition 10

We first derive a sufficient condition for the optimal time T to be sufficiently large so that an announcement effect occurs. Consider interventions with $P = P_{\min}$ and $T > 0$ sufficiently small. Then, there is no announcement effect for this policy. From the Online Appendix, the optimal time of intervention for this class of policies is characterized by

$$\alpha(T^*) = \theta(1 - \alpha(0)) \left(\frac{r + \kappa + \lambda}{r + \kappa} \right).$$

Note that there is no trade at $T < T^*$ only if $\alpha(T^*) \leq 1$. Consequently, $\alpha(T^*) \leq 1$ is a necessary condition for a minimal intervention without an announcement effect being optimal. Hence, a sufficient condition for an announcement effect to be optimal is that the optimal T^* is sufficiently large so that $\alpha(T^*) > 1$. There will be an announcement effect associated with a minimal intervention whenever

$$\theta > \bar{\theta} = \frac{1}{1 - \alpha(0)} \left(\frac{r + \kappa}{r + \kappa + \lambda} \right).$$

since it is optimal to delay the intervention sufficiently.

It is then straightforward to verify that this condition is satisfied when θ is sufficiently close to $\bar{\theta}$ as defined in the Online Appendix or when λ is sufficiently large.

Appendix A.8. *Proof of Proposition 11*

The first part follows directly from Proposition 9 that an immediate intervention is optimal for θ sufficiently close to 0. To prove the second and the third part, we will show that for sufficiently small $\pi(0)$ or λ , any intervention with P_{\max} is dominated by a minimum intervention with $P = v_\ell$.

Consider an intervention at T and $P = v_s$. From the proof of Proposition 9, the upper bound on the announcement effect is given by $\tau_{\min} = T - \Delta_{\max}$ where Δ_{\max} is given by the solution to the equation

$$e^{-r\Delta_{\max}} = \frac{1 - \frac{\pi(0)}{\bar{\pi}}}{1 - \frac{\pi(0)}{\pi}}.$$

We now compare a policy with P_{\max} at T to an intervention with P_{\min} at $\hat{T} = \min\{0, T - \Delta_{\max}\}$. By construction, the welfare from trading with the minimum intervention is at least as high as that with the original intervention. The cost of the minimum intervention is lower if

$$\frac{P_{\min}}{P_{\max}} = \frac{v_\ell}{v_s} < e^{-r \min\{\Delta_{\max}, T\}}.$$

Hence, a sufficient condition for the minimum intervention to be better is given by $v_\ell/v_s < e^{-(r+\lambda)\Delta_{\max}}$ or, equivalently,

$$\frac{\lambda}{\lambda + r} < \frac{1 - \frac{\pi(0)}{\bar{\pi}}}{1 - \frac{\pi(0)}{\pi}}.$$

For $\pi(0) \rightarrow 0$ this condition is satisfied. For $\lambda \rightarrow 0$, the left-hand side converges to 0, while the right-hand side converges to 1. This completes the proof.

 Appendix A.9. *Proof of Proposition 13*

Suppose that one can only intervene at $T \geq T^D$. We first find conditions such that (i) the optimal timing for a minimal intervention is then at T^D , (ii) an intervention at T^D with P_{\max} and Q_{\max} can induce continuous trade, and (iii) the intervention at T^D at P_{\max} and Q_{\max} is strictly better than a minimal intervention at T^D . These conditions establish the result, since by Proposition 9 and 11 we can choose a strictly positive θ small enough so that a minimum intervention is optimal without the additional restriction on T and takes place sufficiently early so that there are no announcement effects independent of α .

Consider then first an intervention with P_{\min} and Q_{\min} . From Equation 37, it is optimal to intervene at T^D when

$$T^D \geq T^* = -\frac{1}{\kappa} \ln \left(\frac{\kappa \bar{\theta} - \theta}{\lambda \underline{\theta}} \right)$$

or whenever

$$\theta \leq \theta_1(T^D) = \bar{\theta} - \underline{\theta} \frac{\lambda}{\kappa} e^{-\kappa T^D}.$$

Next, consider a maximal intervention at P_{\max} and Q_{\max} which implies an option value of $V_I = \frac{r}{r+\lambda} v_s$. This intervention at T^D can lead to an equilibrium with continuous trade if

$$\bar{\pi}(0)v_o + (1 - \bar{\pi}(0)) \left(v_\ell + \frac{r}{r+\lambda} v_s e^{-(r+\lambda)T^D} \right) - v_s \geq 0.$$

which is the case for sufficiently small T^D since $v_\ell = \frac{\lambda}{\lambda+r} v_s$ with continuous trade. More generally, this condition defines a cutoff value $\bar{T}^D > 0$ such that for $T^D \leq \bar{T}^D$ the maximum intervention achieves continuous trade.

Finally, we compare the welfare of an intervention with (P_{\min}, Q_{\min}) and (P_{\max}, Q_{\max}) at T^D . Note that the trading gain of a maximum intervention is always larger than the minimum intervention at T^D since the former achieves continuous trade while the latter does not. Hence, a maximum intervention at T^D is better whenever

$$\theta \leq \theta_2(T^D) \equiv \frac{\tilde{W}(T^D, P_{\max}, Q_{\max}) - \tilde{W}(T^D, P_{\min}, Q_{\min})}{(P_{\max}Q_{\max} - P_{\min}Q_{\min})e^{-rT^D}},$$

where \tilde{W} expresses the welfare from allocating assets across traders.

These three conditions thus imply that it is optimal to intervene at some price $P > P_{\min}$ and $Q > Q_{\min}$ to induce an announcement effect if $T^D \leq \bar{T}^D$ and $\theta \in (0, \underline{\theta}(T^D))$ where $\underline{\theta}(T^D) = \min\{\theta_1(T^D), \theta_2(T^D)\}$.

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Supplementary Material

Supplementary material is available at *Review of Economic Studies* online.

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Trading Dynamics with Adverse Selection and Search:
Market Freeze, Intervention and Recovery

APPENDICES

FOR ONLINE PUBLICATION ONLY

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Appendix A An Alternative Set-up Similar to DGP

Set-up We briefly study an alternative model set-up similar to Duffie et al. (2005) (DGP).¹ In this set-up, an investor with low valuation, after selling his asset will change his valuation only after a preference shock arrives (for a graphical illustration of this set-up see the Figure A.1 below). Hence, each trader can be in an additional state where he does not own an asset and has low valuation. We denote the value of this state as v_n and the measure of investors in this state by μ_n . The value functions for this set-up are then given by

$$rv_s(t) = (\delta - x) + \gamma(t)\lambda\mu_b(t) \max\{p(t) + v_n(t) - v_s(t), 0\} + \dot{v}_s(t). \quad (\text{A.1})$$

$$rv_o(t) = \delta + \kappa(v_s(t) - v_o(t)) + \dot{v}_o(t) \quad (\text{A.2})$$

$$rv_\ell(t) = \gamma(t)\lambda\mu_b(t) \max\{p(t) + v_n(t) - v_\ell(t), 0\} + \dot{v}_\ell(t) \quad (\text{A.3})$$

$$rv_n(t) = \kappa(v_b(t) - v_n(t)) + \dot{v}_n(t) \quad (\text{A.4})$$

$$rv_b(t) = \gamma(t)\lambda(\mu_s(t) + \mu_\ell(t)) \cdot \max\{\max_{p(t)} \tilde{\pi}(p)v_o + (1 - \tilde{\pi}(p))v_\ell(t) - p(t) - v_b(t), 0\} + \dot{v}_b(t) \quad (\text{A.5})$$

while the measure of different types of traders evolves according to the flow equations

$$\dot{\mu}_b(t) = -\gamma(t)\lambda\mu_b(t) (\mu_s(t) + \mu_\ell(t)) + \kappa\mu_n(t) \quad (\text{A.6})$$

$$\dot{\mu}_o(t) = -\kappa\mu_o(t) + \gamma(t)\lambda\mu_b(t)\mu_s(t) \quad (\text{A.7})$$

$$\dot{\mu}_s(t) = \kappa\mu_o(t) - \gamma(t)\lambda\mu_b(t)\mu_s(t) \quad (\text{A.8})$$

$$\dot{\mu}_\ell(t) = -\gamma(t)\lambda\mu_b(t)\mu_\ell(t) + \gamma(t)\lambda\mu_b(t)\mu_\ell(t) = 0 \quad (\text{A.9})$$

$$\dot{\mu}_n(t) = -\kappa\mu_n(t) + \gamma(t)\lambda\mu_b(t) (\mu_s(t) + \mu_\ell(t)). \quad (\text{A.10})$$

For the buyer, the probability of buying a good asset is still given by

$$\tilde{\pi}(t) = \begin{cases} \frac{\mu_s(t)}{\mu_s(t) + \mu_\ell(t)} & \text{if } p(t) \geq v_s(t) - v_b(t) \\ 0 & \text{if } p(t) < v_s(t) - v_b(t). \end{cases} \quad (\text{A.11})$$

so that the buyer's expected surplus from buying the asset is

$$\Gamma(t) = \tilde{\pi}(p)v_o(t) + (1 - \tilde{\pi}(p))v_\ell(t) - p(t) - v_b(t), \quad (\text{A.12})$$

¹Duffie, D., Garleanu, N. and Pedersen, L. (2005). "Over-the-Counter Markets", *Econometrica*, 73, pp. 1815-1847.

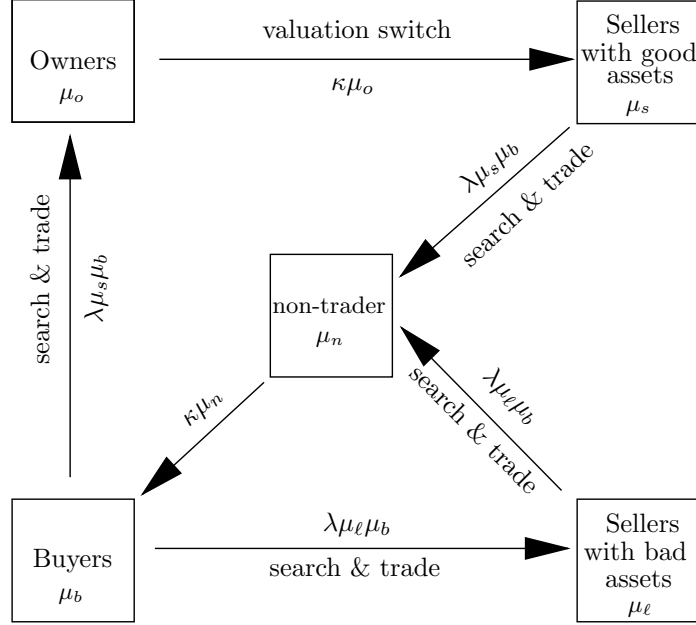


Figure A.1: Flow Diagram

where $p(t) = v_s(t) - v_n(t)$. In steady state, the measure of traders of different types are given by

$$\lambda\mu_b(\mu_s + \mu_\ell) = \kappa\mu_n \quad (\text{A.13})$$

$$\kappa\mu_o = \lambda\mu_b\mu_s \quad (\text{A.14})$$

$$\kappa\mu_o = \lambda\mu_b\mu_s \quad (\text{A.15})$$

$$\kappa\mu_n = \lambda\mu_b(\mu_s + \mu_\ell). \quad (\text{A.16})$$

Steady State Equilibria As in the original setup, there exists a steady state without trade when π is sufficiently low and a steady state with trade when π is sufficiently high. Specifically, if $\pi \leq \bar{\pi}$, we have that $\gamma = 0$ is a steady state equilibrium in pure strategies. Furthermore, there exists an equilibrium with trade ($\gamma = 1$) with μ_s satisfying

$$\mu_s = \frac{\sqrt{\kappa^2 + 4\lambda(\kappa + \lambda\mu_\ell)} - \kappa - 2\lambda\mu_\ell}{2\lambda}. \quad (\text{A.17})$$

and with the measures of other types given by

$$\mu_o = S\pi - \mu_s, \quad (\text{A.18})$$

$$\mu_\ell = S(1 - \pi), \quad (\text{A.19})$$

$$\mu_b = \frac{\kappa}{(\lambda\mu_s + \lambda\mu_\ell + \kappa)}, \quad (\text{A.20})$$

$$\mu_n = 1 - \mu_b. \quad (\text{A.21})$$

The corresponding value functions are described by

$$v_s = \frac{\delta - x}{r} \quad (\text{A.22})$$

$$v_o = \frac{1}{r + \kappa}(\delta + \kappa v_s) \quad (\text{A.23})$$

$$v_\ell = \frac{\lambda\mu_b}{\lambda\mu_b + r}v_s \quad (\text{A.24})$$

$$v_n = \frac{\kappa}{r + \kappa}v_b \quad (\text{A.25})$$

$$v_b = \lambda(\mu_s + \mu_\ell)\Gamma \quad (\text{A.26})$$

Hence, an steady state equilibrium with $\gamma = 1$ exists if

$$\begin{aligned} \Gamma &= \tilde{\pi}v_o + (1 - \tilde{\pi})v_\ell - p - v_b \\ &= [\tilde{\pi}v_o + (1 - \tilde{\pi})v_\ell - v_s] \frac{r + \kappa}{r + \kappa + r\lambda(\mu_s + \mu_\ell)} > 0 \end{aligned} \quad (\text{A.27})$$

where $\tilde{\pi} = \mu_s/(\mu_s + \mu_\ell)$. Consequently, for sufficiently large π , we have $\gamma = 1$.

Appendix B Equilibrium Price Dynamics

This Appendix characterizes how market prices change in response to the intervention. Due to the take-it-or-leave-it offer, market prices in an equilibrium with trading are given by

$$p(t) = v_s - v_b(t). \quad (\text{B.1})$$

Hence, they are inversely related to the value function of the buyer $v_b(t)$, which is a continuous function. The dynamics of prices thus depends on the trading behaviour over time which influences $v_b(t)$. The next result summarizes these dynamics.

Proposition B.1. *Given an intervention at T , market prices $p(t)$*

(i) jump to a strictly positive value at τ_1 ,

(ii) decline at rate r with partial trade in the interval $[\tau_1, \tau_2]$,

(iii) decline at a rate lower than r or increase with full trade in the interval $(\tau_2, T]$

(iv) increase monotonically to the steady state price after the intervention with a positive, discrete jump in their growth rate at T .

Proof. Since there is no trade in the interval $[0, \tau_1)$, we set $p(t) = 0$. For $[\tau_1, \tau_2)$, there is partial trade. The expected surplus $\Gamma(t)$ is constant at zero in this interval which implies for the value function of the buyer that

$$v_b(t) = v_b(\tau_2)e^{-r(\tau_2-t)}. \quad (\text{B.2})$$

Since prices are given by $p(t) = v_s - v_b(t)$, they decrease at a rate r .

For the interval $[\tau_2, T)$, we have that the differential equation of buyers is given by

$$\dot{v}_b(t) - rv_b(t) = -\lambda(\mu_s(t) + \mu_\ell(T^-))\Gamma(t). \quad (\text{B.3})$$

Rewriting, we have

$$\Gamma(t) = \frac{1}{\lambda(\mu_s(t) + \mu_\ell(T^-))} (rv_b(t) - \dot{v}_b(t)) > 0. \quad (\text{B.4})$$

Hence,

$$r > \frac{\dot{v}_b(t)}{v_b(t)} \quad (\text{B.5})$$

which means that the price can decline at most at rate r which is less than with partial trade.

Finally, we turn to the the interval $[T, \infty)$. After the intervention we have for the differential equation of buyers

$$\dot{v}_b(t) - rv_b(t) = -\lambda \left(\mu_s(t)(v_o - v_s) - \mu_\ell(T) \frac{r}{r + \lambda} v_s \right). \quad (\text{B.6})$$

Note that the right-hand side of this expression is strictly negative and continuously increasing as $\dot{\mu}_s(t) < 0$. Differentiating the differential equation for $v_b(t)$, we obtain

$$\frac{\partial \dot{v}_b(t)}{\partial t} = r\dot{v}_b(t) - \lambda \dot{\mu}_s(t)(v_o - v_s). \quad (\text{B.7})$$

If $\dot{v}_b(t) > 0$, $v_b(t)$ is strictly convex and continuity of $v_b(t)$ in (T, ∞) would imply that it diverges. This is a contradiction since $v_b(t)$ is bounded from above.

Hence, $v_b(t)$ is strictly decreasing implying that prices are increasing or $\dot{p}(t) > 0$. Finally, denoting the steady state level of the buyers value function by $v_b(Q)$, we have

$$\lim_{t \rightarrow \infty} v_b(t) = \frac{\lambda}{r} \left\{ S\pi(0) \frac{\kappa}{\kappa + \lambda} + (S(1 - \pi(0)) - Q) \frac{r}{r + \lambda} v_\ell \right\} \equiv v_b(Q), \quad (\text{B.8})$$

which implies that $\lim_{t \rightarrow \infty} p(t) = p = v_s - v_b(Q)$.

For the last statement in the proposition, note that $v_b(t)$ is continuous, but has a discrete jump in its derivative as $\Gamma(T)$ jumps discretely at the time of intervention. The left- and right-hand derivatives both exist at T and are given by

$$\dot{v}_b(T^-) = rv_b(T) - \lambda \mu_s(T)(v_o - v_s) - \lambda \mu_\ell(T^-) \left(-\frac{r}{r + \lambda} v_s + V_I \right) \quad (\text{B.9})$$

$$\dot{v}_b(T) = rv_b(T) - \lambda \mu_s(T)(v_o - v_s) - \lambda \mu_\ell(T) \left(-\frac{r}{r + \lambda} v_s \right). \quad (\text{B.10})$$

We thus obtain for their difference

$$\begin{aligned} \dot{v}_b(T) - \dot{v}_b(T^-) &= \lambda \left[\mu_\ell(T^-) \left(V_I - \frac{r}{r + \lambda} v_s \right) + \mu_\ell(T) \frac{r}{r + \lambda} v_s \right] \\ &= \lambda \left[S(1 - \pi(0)) \left(V_I - \frac{r}{r + \lambda} v_s \right) + \left(S(1 - \pi(0)) - Q \right) \frac{r}{r + \lambda} v_s \right] \\ &= \lambda \left[S(1 - \pi(0)) V_I - Q \frac{r}{r + \lambda} v_s \right]. \end{aligned} \quad (\text{B.11})$$

Using the definition of V_I ,

$$\begin{aligned} \dot{v}_b(T) - \dot{v}_b(T^-) &= \lambda \left[Q(P - v_\ell(T)) - Q \frac{r}{r + \lambda} v_s \right] \\ &= \lambda Q (P - v_s) \\ &\leq 0, \end{aligned} \quad (\text{B.12})$$

where the last inequality follows from $P \leq v_s$. Hence, at T , the derivative jumps down discretely and thus, the derivative of the price function increases discretely. \square

In general, the equilibrium dynamics of market prices are non monotone. First, as trade restarts at time τ_1 , there is a discrete jump of price from zero to a strictly positive number. Second, when there is partial trade in the interval $[\tau_1, \tau_2)$, buyers are indifferent between trading now or waiting subject to the discount rate r . Therefore, $v_b(t)$ is increasing at a rate r and, consequently, the price is decreasing at the rate r . The idea is that, as the market recovers, the opportunity cost of a seller giving up a good asset and turning into a buyer goes down. As a result, the price offered to the seller also drops. Third, as the market fully recovers in the interval $[\tau_2, T)$, the quality deteriorates faster and generates a negative effect on the buyers' incentives to trade. This negative effect will offset the above-mentioned increasing trend of $v_b(t)$. As a result, the price declines at a rate lower than r or even increases. After the intervention at time T , there is continuous trade and thus only the quality in the market evolves over time. In particular, $v_b(t)$ drops monotonically as the quality deteriorates over time. As a result, the price increases monotonically to its steady state level.

Appendix C Optimal Minimal Interventions

C.1 Time of Intervention

For this Appendix, we assume throughout that $\alpha < 1$. This implies that, for a minimal intervention at any T , there cannot be an announcement effect, since $\alpha(t) < \alpha < 1$ for any $t < T$. Due to the absence of an announcement effect, we can express the trading gains from a minimal intervention at T explicitly. These gains are given by two parts – a steady state part and a part that captures the transitions associated with an intervention at T

$$S\pi(0) \left[\left(\frac{\delta - x}{r} \right) + e^{-rT} \left(\frac{x}{r} \right) \left(\frac{\lambda}{\lambda + \kappa} \right) + \left(\frac{x}{r} \right) \left(\frac{\lambda}{\lambda + \kappa} \right) \left(\frac{r}{r + \kappa} (1 - e^{-(r+\kappa)T}) - \frac{r}{r + \kappa + \lambda} (e^{-rT} - e^{-(r+\kappa)T}) \right) \right]. \quad (\text{C.1})$$

The first term is a constant and expresses the welfare cost from an asset that is permanently misallocated, while the second term is the gain from going back immediately to a steady state with trading at period T , the time of the intervention, discounted to period 0. Hence, the first two terms overstate the loss from a market breakdown and the gain from an intervention by neglecting welfare effects from transitions. The third and fourth terms express the welfare effects due to transitions. The third one is the additional gain from the slow movement (according to κ) of traders from high to low valuations until the intervention takes place. The last term expresses the welfare loss from moving slowly to the steady state after the intervention at T .

The net present value of the costs of a minimal intervention are decreasing in T . This gives rise to the primary trade-off for the policy maker where delaying the intervention saves costs, but at the expense of less trading in the market. Using the expressions for P_{\min} and Q_{\min} , we have

$$\frac{\partial W}{\partial T} = S\pi(0) \left(\frac{x}{r} \right) \left(\frac{\lambda}{\lambda + \kappa} \right) r e^{-rT} \cdot \left[\underbrace{\theta \left(\frac{1 - \alpha(0)}{\alpha} \right) \left(\frac{\lambda + \kappa}{r + \kappa} \right)}_{\text{marginal benefit}} - \underbrace{\frac{1}{r + \kappa + \lambda} (\kappa + \lambda(1 - e^{-\kappa T}))}_{\text{marginal cost}} \right]. \quad (\text{C.2})$$

The optimal time of intervention T^* at an interior solution equates the marginal benefit with the marginal costs from delaying the intervention. The marginal benefit arises from costs savings which correspond to the first term in the square bracket. The marginal costs are given by the second term and stem from less trading in the market. Both these terms

are expressed in current values as a fraction of the total instantaneous gain from allocating good assets to traders with high valuation in steady state which are given by $S\pi(0)x\frac{\lambda}{\lambda+\kappa}$. An interior solution for the welfare maximizing policy is thus given by

$$\alpha(T^*) = \theta(1 - \alpha(0)) \left(\frac{r + \kappa + \lambda}{r + \kappa} \right). \quad (\text{C.3})$$

Furthermore, there is an upper bound on θ for an immediate intervention given by

$$\underline{\theta} = \frac{\alpha(0)}{1 - \alpha(0)} \left(\frac{r + \kappa}{r + \lambda + \kappa} \right), \quad (\text{C.4})$$

and a lower bound for no intervention at all

$$\bar{\theta} = \left(\frac{\lambda + \kappa}{\kappa} \right) \underline{\theta} = \frac{\alpha}{1 - \alpha(0)} \left(\frac{r + \kappa}{r + \lambda + \kappa} \right). \quad (\text{C.5})$$

For $\theta \in [\underline{\theta}, \bar{\theta}]$, the optimal time of intervention is given by

$$T^* = -\frac{1}{\kappa} \ln \left(\frac{\kappa \bar{\theta} - \theta}{\lambda \underline{\theta}} \right). \quad (\text{C.6})$$

This confirms the result in the general case, that larger shocks to quality (smaller $\alpha(0)$) and less important markets (larger θ) imply later interventions.

C.2 Role of Search Frictions

What is the role of search frictions for the optimal timing of the intervention? Figure C.1 below summarizes qualitatively how search frictions influence the marginal benefits and costs of delaying the intervention. Marginal benefits are a constant function, while marginal costs are an increasing and concave function of T . The optimal time T^* is determined by their intersection.

The marginal benefits change with the severity of the adverse selection problem. We have that

$$\frac{\partial(1 - \alpha(0))}{\partial\lambda} < 0 \text{ if and only if } \kappa > r. \quad (\text{C.7})$$

Hence, whenever $\kappa > r$, the adverse selection problem becomes worse (higher $1 - \alpha(0)$) as search frictions increase (lower λ). Thus a minimum intervention is more costly which increases the marginal benefits to delay. For $r > \kappa$, search frictions alleviate the adverse selection problem so that the marginal benefit of delaying the intervention decreases.

The marginal costs of delaying the intervention depend on the time of the intervention. They decrease for late interventions, but increase for early interventions. Delaying the intervention

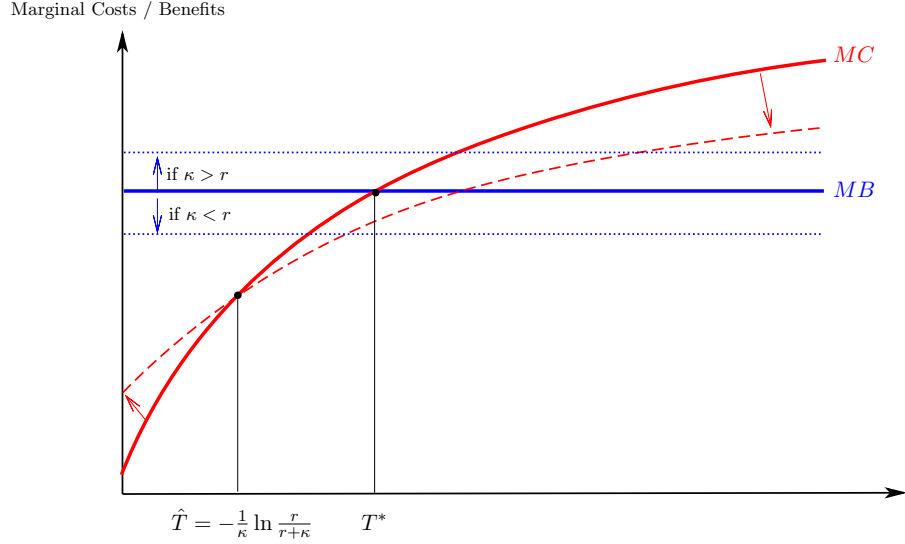


Figure C.1: Marginal Costs and Benefits as Search Frictions Increase (Lower λ)

implies less trade before the intervention and more selling pressure after it. When search frictions increase, selling pressure dissipates more slowly which increases the marginal costs of delaying the intervention. However, at the same time fewer potential trades are lost by delay. As the intervention is postponed further, the second effect becomes stronger as more potential trades could be carried out. Hence, for T sufficiently large, we have that the marginal costs decline when search frictions increase. Finally, differentiating the optimal time of intervention with respect to λ we obtain that $\frac{dT^*}{d\lambda} > 0$ if and only if

$$\frac{\partial \underline{\theta}}{\partial \lambda} \frac{\lambda}{\underline{\theta}} < \underline{\theta} / \theta - 1. \quad (\text{C.8})$$

Hence, market importance drives the effects of search frictions on the timing of the intervention. For important enough markets, larger search frictions imply that the MMLR should intervene earlier. Further analytical insights cannot be obtained since the lower boundary $\underline{\theta}$ can be non-monotonic in λ .

Appendix D Pooling vs. Separating Contracts

We show in this Appendix that offering a pooling contract is a dominant strategy for buyers. We assume that there is positive trade surplus for good assets, but non-positive trade surplus for lemons.¹ Denote the *net* value of a good asset to a seller as v_s^g and to a buyer as $v_b^g > v_s^g$. Denote the net value of a lemon to a seller as v_s^ℓ and to a buyer as $v_b^\ell \leq v_s^\ell$. Also, we assume that $v_s^g > v_s^\ell$.²

Consider any contract (p, q) , where p is the price paid by the buyer and q is the probability that the seller transfers the asset. We want to show that buyers always prefer a pooling contract or, in other words, do not have an incentive to separate sellers by using contracts with lotteries (i.e., $q < 1$).

Pooling contract with (p, q) Sellers with a good asset will sell at price p if and only if

$$p + (1 - q)v_s^g \geq v_s^g. \quad (\text{D.1})$$

Hence, given q , the buyer will offer the price $p = qv_s^g$. The best contract for the buyer is then given by the solution to

$$\max_{q \in [0,1]} \tilde{\pi} q v_b^g + (1 - \tilde{\pi}) q v_b^\ell - p = q (\pi v_b^g + (1 - \pi) v_b^\ell - v_s^g). \quad (\text{D.2})$$

Since the return is linear in q , we have that no lottery will be used and the solution is either $(p, q) = (0, 0)$ or $(p, q) = (v_s^g, 1)$.

Separating contract (p^g, q^g) and (p^ℓ, q^ℓ) To separate and trade with the two types, the two contracts have to satisfy incentive constraints and make all sellers willing to sell. These constraints are given by

$$p^g - q^g v_s^g \geq 0 \quad (\text{D.3})$$

$$p^g - q^g v_s^g \geq p^\ell - q^\ell v_s^g \quad (\text{D.4})$$

$$p^\ell - q^\ell v_s^\ell \geq 0 \quad (\text{D.5})$$

$$p^\ell - q^\ell v_s^\ell \geq p^g - q^g v_s^\ell. \quad (\text{D.6})$$

¹We could dispense of this assumption. There could then be a separating equilibrium where only lemons are traded, but not the good asset. We would interpret such a situation still as a market freeze.

²In our set-up, these net values are defined by $v_s^g = v_o - v_b$, $v_b^g = v_o - v_b$, $v_s^\ell = v_\ell - v_b$ and $v_b^\ell = v_\ell - v_b$.

Since there is no trade surplus from trading lemons, any equilibrium with trade must have some trade of the good asset or $q^g > 0$. This implies that (D.5) is never binding or that lemons will always be sold (see $v_s^g > v_s^\ell$). Also, both (D.3) and (D.6) must be binding, otherwise it is profitable for the buyer to lower both p^g and p^ℓ by some $\varepsilon > 0$ or p^ℓ by some $\varepsilon > 0$. Then, (D.4) and (D.6) imply

$$(q^g - q^\ell)v_s^\ell = p^g - p^\ell \geq (q^g - q^\ell)v_s^g. \quad (\text{D.7})$$

Therefore, (D.4) is satisfied if and only if $q^\ell \geq q^g$. We also obtain for the prices

$$p^g = q^g v_s^g \quad (\text{D.8})$$

$$p^\ell = (q^\ell - q^g)v_s^\ell + q^g v_s^g \quad (\text{D.9})$$

so that $p^\ell \leq p^g$.

The buyer then solves

$$\begin{aligned} & \max_{q^\ell, q^g} \pi[q^g v_b^g - p^g] + (1 - \pi)[q^\ell v_b^\ell - p^\ell] \\ & = \max_{q^\ell, q^g} \pi[q^g v_b^g - q^g v_s^g] + (1 - \pi)[q^\ell v_b^\ell - (q^\ell - q^g)v_s^\ell - q^g v_s^g] \\ & = \max_{q^\ell, q^g} q^g (\pi[v_b^g - v_s^g] + (1 - \pi)(v_b^\ell - v_s^\ell)) + q^\ell (1 - \pi)[v_b^\ell - v_s^\ell] \end{aligned} \quad (\text{D.10})$$

subject to $q^\ell \geq q^g$. If $v_b^\ell < v_s^\ell$, it is thus optimal to set $q^\ell = q^g$ implying that $p^g = p^\ell$. We then have a pooling contract. To the contrary, if $v_b^\ell = v_s^\ell$, the problem becomes

$$\max_{q^g} q^g (\pi v_b^g + (1 - \pi)v_b^\ell - v_s^g) \quad (\text{D.11})$$

which gives exactly the same payoff as with a pooling contract. In particular, if $\pi v_b^g + (1 - \pi)v_b^\ell - v_s^g > 0$, then $q^g = q^\ell = 1$ and $p^g = p^\ell = v_s^g$. If $\pi v_b^g + (1 - \pi)v_b^\ell - v_s^g < 0$, then there exist only trades with lemons which generates zero trade surplus.

Trade by good or bad sellers only at (p^g, q^g) or (p^ℓ, q^ℓ) To exclude lemons, we need $p^\ell - q^\ell v_s^\ell < 0$, which by (D.6) implies that $p^g - q^g v_s^\ell < 0$ contradicting (D.3). By assumption, trade of bad assets only cannot generate any positive trade surplus for buyers.

Appendix E Anticipated Quality Shock

E.1 Set-up

Suppose the quality shock follows a Poisson process with rate ρ ; i.e, the timing for the quality shock is exponentially distributed with arrival rate ρ . Hence, at a random time we have that the fraction of good assets switches from π to $\pi(\tau)$. We assume that (i) $\pi(\tau) < \tilde{\pi}$ and (ii) that up to the shock the economy is stationary. In what follows, we first show that the trade dynamics are identical to the analysis in the paper. The only difference is that the thresholds for trade and no trade also depend on the parameter ρ . We then briefly discuss that the optimal intervention does not change, since one can view the economy after the shock as one that starts out after an unanticipated shock.

E.2 Trade Dynamics

We denote the value functions after the shock has occurred at random time τ by $\bar{v}_s(t)$, $\bar{v}_\ell(t)$ and $\bar{v}_o(t)$. After the shock has occurred, the economy is identical to the one analyzed in the main text. Hence, without an intervention there is a unique no trade equilibrium where $\bar{v}_s(t) = v_s$, $\bar{v}_o(t) = v_o$ and $\bar{v}_\ell(t) = 0$ for all $t \geq \tau$.

When there is an intervention (P, Q, T) where $T \geq \tau$, only the value function of lemons will change and is given by

$$\bar{v}_\ell(\tau) = \lambda v_s \int_{\tau}^T \gamma(s) e^{\int_{\tau}^s -(r+\lambda\gamma(\nu))d\nu} ds + \left(\frac{\lambda}{\lambda+r} v_s + V_I \right) e^{\int_{\tau}^T -(r+\lambda\gamma(s))ds} \quad (\text{E.1})$$

where V_I is again the option value implied by (P, Q) .

The value functions before the shock are now given by

$$v_s(t) = \frac{\delta - x + \rho \frac{\pi(\tau)}{\pi} v_s + \rho \frac{\pi - \pi(\tau)}{\pi} \bar{v}_\ell(\tau) + \dot{v}_s(t)}{r + \rho} \quad (\text{E.2})$$

$$v_o(t) = \frac{\delta + \kappa(v_s(t) - v_o(t)) + \rho \frac{\pi(\tau)}{\pi} v_o + \rho \frac{\pi - \pi(\tau)}{\pi} \bar{v}_\ell(\tau) + \dot{v}_o(t)}{r + \rho} \quad (\text{E.3})$$

$$v_\ell(t) = \frac{\lambda v_s(t) + \rho \bar{v}_\ell(\tau) + \dot{v}_\ell(t)}{r + \lambda + \rho} \quad (\text{E.4})$$

reflecting that investors anticipate that there is a fall in quality from π to $\pi(\tau)$ with rate ρ .

Suppose first that there is no intervention at all so that $\bar{v}_\ell(\tau) = 0$. For a stationary economy,

we have that at $t < \tau$ full trade is an equilibrium before the shock if and only if

$$\Gamma = \tilde{\pi}v_o + (1 - \tilde{\pi})v_\ell - v_s > 0 \quad (\text{E.5})$$

where we have again that

$$\tilde{\pi} = \frac{\kappa\pi}{\kappa + (1 - \pi)\gamma\lambda} \quad (\text{E.6})$$

and the value functions solve

$$v_s^* = \frac{\delta - x + \rho \frac{\pi(\tau)}{\pi} v_s}{r + \rho} \quad (\text{E.7})$$

$$v_o^* = \frac{\delta + \kappa(v_s^* - v_o^*) + \rho \frac{\pi(\tau)}{\pi} v_o}{r + \rho} \quad (\text{E.8})$$

$$v_\ell^* = \frac{\lambda v_s^*}{r + \lambda + \rho}. \quad (\text{E.9})$$

This implies that we have continuous trade for all $t < \tau$, provided π is sufficiently large and ρ is sufficiently small.

E.3 Optimal Interventions

Suppose the MMLR can commit to purchase Q lemons at price P at time $\tau + T$. Denote the optimal intervention in the economy where the quality shock is unanticipated by (P^*, Q^*, T^*) . We show that this intervention is still optimal in an economy where the quality shock is anticipated.

Suppose not. Then there exists a better intervention (P', Q', T') . When equation (E.5) holds, there must always be trade before the shock even without any intervention. Therefore the two interventions generate exactly the same welfare before the shock arrives. But the alternative policy must generate higher welfare after the shock. This contradicts that (P^*, Q^*, T^*) is optimal when shocks are unanticipated, since after the shock has been realized the economy is identical to the one analyzed in the main paper.

Remark: There is, however, one difference when looking at the policy design. When equation (E.5) is violated, it matters for the optimal policy that the shock is anticipated. The reason is that investors anticipate the intervention so that there can be trade for lower values of π . For example, with an extreme shock such as $\pi(\tau) = 0$, it is optimal not to intervene when the shock is unanticipated because there are no gains from the market functioning again. As a consequence, the mere anticipation of this extreme shock can freeze the market

even before its arrival for sufficiently low π if there is no intervention. With an immediate intervention at τ , however, that causes a sufficiently high $\bar{v}_\ell(\tau)$, the market can function for $t < \tau$. This policy could be optimal depending on the parameter θ that expresses how important the market is relative to the costs for an intervention. Interestingly, this set-up moves the analysis closer to one where commitment to an intervention can cause moral hazard.

Appendix F Free Disposal of Lemons

This Appendix derives a sufficient condition on the number of securities S that rules out incentives for traders to dispose of assets in steady state in order to become buyers again. Note first that the value of sellers is larger than the value of lemons independent of the buyers' trading strategy

$$v_s \geq v_\ell. \quad (\text{F.1})$$

Hence, it is sufficient to show that $v_\ell \geq v_b$ which also implies that $p > 0$ for any $\gamma > 0$. For $\gamma > 0$, we have

$$\begin{aligned} \frac{v_\ell(\gamma) - v_b(\gamma)}{v_s} &= \frac{\lambda\gamma}{\lambda\gamma + r} - \frac{1}{r}\lambda\gamma(\mu_s + \mu_\ell) \left[\tilde{\pi} \left(\frac{1}{\underline{\pi}} - 1 \right) + (1 - \tilde{\pi}) \frac{r}{\lambda\gamma + r} \right] \\ &= \frac{\lambda\gamma}{\lambda\gamma + r} - \frac{1}{r} \left[\mu_s \lambda\gamma \left(\frac{1}{\underline{\pi}} - 1 \right) + \mu_\ell \lambda\gamma \frac{r}{\lambda\gamma + r} \right] \\ &= \left(\frac{\lambda\gamma}{\lambda\gamma + r} \right) \left((1 - \mu_\ell) - \mu_s \left(\frac{\lambda\gamma + r}{r} \right) \left(\frac{(\xi - 1)r}{r + \kappa} \right) \right). \end{aligned} \quad (\text{F.2})$$

Hence, the sign of this expression depends on

$$\left(1 - (1 - \pi)S \right) - S\pi \left(\frac{\kappa(\lambda\gamma + r)}{(\kappa + \lambda\gamma)r} \right) \left(\frac{(\xi - 1)r}{r + \kappa} \right). \quad (\text{F.3})$$

This expression is increasing in γ whenever $\kappa > r$ and decreasing otherwise.

Assume first that $r > \kappa$. Then, with $\gamma = 0$, the expression above is positive as long as

$$(1 + S) \geq S\pi \left(1 + \frac{(\xi - 1)r}{r + \kappa} \right) \quad (\text{F.4})$$

or

$$(1 + S) \geq S \frac{\pi}{\underline{\pi}}. \quad (\text{F.5})$$

Similarly, for $\kappa > r$, set $\gamma = 1$ and the expression is positive as long as

$$(1 + S) \geq S \frac{\pi}{\bar{\pi}}. \quad (\text{F.6})$$

This implies that a sufficient condition for traders not to dispose of any assets is given by

$$\frac{\max\{\underline{\pi}, \bar{\pi}\}}{1 - \max\{\underline{\pi}, \bar{\pi}\}} \geq S \quad (\text{F.7})$$

whenever $\pi \geq \min\{\underline{\pi}, \bar{\pi}\}$.

When $\pi < \min\{\underline{\pi}, \bar{\pi}\}$, free disposal does not matter, as there is no trade in steady state and it cannot be an optimal strategy for any trader with a lemon to dispose of his asset with positive probability so that there is trade again, since he would then have a strictly higher utility from retaining the lemon.

Appendix G Avoiding a Self-fulfilling Freeze through Guarantees

Depending on parameter values, trading on the market can cease simply because of a coordination failure among buyers across time. We show here for such a case that guaranteeing a floor on the value of the asset can resurrect trading. The reason is that such a policy makes purchasing an asset in a meeting a strictly dominant strategy for buyers for any steady state equilibrium. More interestingly, such a guarantee would be costless in equilibrium. Having bought a lemon, a buyer is always worse off taking the guarantee than waiting to trade his lemon to another trader in the functioning market.

To show this, let $\kappa > r$ and $\pi \in (\bar{\pi}, \underline{\pi})$. Then there exists a no trade and a partial trade steady state equilibrium. Define the guarantee offered by the MMLR by the price

$$P_G = v_\ell - \epsilon - v_b(\gamma). \quad (\text{G.1})$$

In steady state, the surplus from trading for a buyer is given by

$$\tilde{\pi}(\gamma)v_o + (1 - \tilde{\pi}(\gamma)) \max\{P_G + v_b(\gamma), v_\ell(\gamma)\} - p - v_b(\gamma). \quad (\text{G.2})$$

where $v_\ell(\gamma) = \frac{\lambda\gamma}{\lambda\gamma+r}v_s < \frac{\lambda}{\lambda+r}v_s$. The max operator expresses the fact that a buyer with a lemon has the option to receive a utility transfer equal to $v_\ell - \epsilon$ or can wait for a trade in the market anticipating to receive an offer with probability γ when he meets a buyer.

Suppose first that $\gamma \in [0, 1)$. Recall that the market price is given by $p = v_s - v_b(\gamma)$ if $\gamma \in (0, 1]$. Then, since $\tilde{\pi}$ decreases with γ , we have that

$$\begin{aligned} & \tilde{\pi}(\gamma)v_o + (1 - \tilde{\pi}(\gamma)) \max\{P_G + v_b(\gamma), v_\ell(\gamma)\} - p - v_b(\gamma) \\ & > \tilde{\pi}(\gamma = 1)v_o + (1 - \tilde{\pi}(\gamma = 1)) (v_\ell - \epsilon) - v_s > 0 \end{aligned} \quad (\text{G.3})$$

for $\epsilon > 0$ sufficiently small, since $\pi > \tilde{\pi}$. Since the trading surplus is strictly positive independent of the future buyers' trading strategy, it is a strictly dominant strategy to set $\gamma = 1$. Hence, there cannot be a steady state equilibrium with $\gamma < 1$. For $\gamma = 0$, we have $v_b(\gamma) = v_\ell(\gamma) = 0$. The result follows then directly, since offering $p = v_s$ yields again

$$\pi v_o + (1 - \pi) \max\{P_G + v_b(\gamma), v_\ell(\gamma)\} - v_s > \tilde{\pi}(\gamma = 1)v_o + (1 - \tilde{\pi}(\gamma = 1)) (v_\ell - \epsilon) - v_s > 0. \quad (\text{G.4})$$

with $\pi > \bar{\pi}$ and ϵ sufficiently small.

Finally, with $\gamma = 1$, buyers that obtain a lemon will not take advantage of the guarantee as they can obtain a higher value by waiting to trade in the market for any positive ϵ . Hence,

$\gamma = 1$ is the only steady state equilibrium and the guarantee is never used in equilibrium. Notice, however, that the floor that the guarantee provides depends explicitly on how much trading there is on the market as reflected by $v_b(\gamma)$.

Appendix H Avoiding Deadweight Costs when Intervening

The MMLR incurs a deadweight cost when increasing the quantity of lemons bought. For any additional quantity $\Delta_Q = Q - Q_{\min}$ of lemons bought at price P , only the portion $(P - P_{\min})\Delta_Q$ provides an additional transfer to lemons, since the market functions again after the intervention and a lemon has an expected market value of P_{\min} . The MMLR could, however, avoid the deadweight cost by selling the additional quantity Δ_Q back to buyers immediately after the intervention has taken place at a price just below P_{\min} . The market will still function continuously after the sale, since the average quality of the asset in the market remains above the threshold to sustain trading in equilibrium. Also, buyers would be willing to purchase lemons at this price, as they make non-negative profits from this transaction in expected terms. The reason is that later on they can sell the lemon again on the market. Thus, the deadweight cost is simply shifted from the MMLR to future buyers.

When selling additional lemons back to the market at price v_ℓ , the cost of the intervention is then approximately given by¹

$$QP - (Q - Q_{\min}) \left(\frac{\lambda}{\lambda + r} \right) v_s = Q_{\min} P_{\min} + Q \left(P - \left(\frac{\lambda}{\lambda + r} \right) v_s \right). \quad (\text{H.1})$$

The costs are thus given by a constant for the minimum intervention plus the net cost for providing an option value $V_I > 0$. The MMLR can now recover the deadweight cost that the purchase of every additional lemon involves. Of course, the MMLR can only sell lemons in the market again, if future buyers cannot observe which assets were once owned and sold again by the MMLR.

Importantly, the costs of providing an option value $V_I \geq \hat{V}_I$ have now decreased making it more attractive to rely on the announcement effect and thus delaying the intervention. Indeed, the cost function is now smooth at \hat{V}_I when assuming immediate exit from any additional purchases of lemons. As a consequence, it does not matter for the costs anymore whether the MMLR increases Q or P . Price and quantity are now perfect substitutes in terms of the costs of providing any particular option value V_I .

¹The sale would occur at $T + \epsilon$ at price $P_{\min} - \epsilon$. To ease the exposition, we neglect the infinitesimally small terms.

Appendix I Gradual Interventions

We consider here that the MMLR can intervene gradually in the market. To gain some insights into how this possibility will affect trade and the optimal policy we look first at a minimal intervention that achieves trade after its start, but not before. Such interventions are cheaper than one-time interventions, but do not change the trading patterns before the intervention as long as they achieve full trade. Then, we provide a discussion of what happens in the more general case where trade can start before the intervention.

I.1 Set-up

Let $\lambda_0(t)$ be the Poisson rate at which lemons can contact the large player. The total number of assets bought during the interval $[t, t + \Delta]$ is given by

$$\int_t^{t+\Delta} \lambda_0(t) \mu_\ell(t) dt. \quad (\text{I.1})$$

We denote the stock of assets bought at time t by $Q(t)$ so that an intervention can be characterized by $\{Q(t)\}_{t=T}^\infty$.

We restrict ourselves to minimal interventions. Given an initial shock, we require that as $t \rightarrow \infty$, the quality has just improved enough to have trade in steady state

$$\lim_{t \rightarrow \infty} Q(t) = Q_{\min}(\pi_0) \equiv \frac{S(\bar{\pi} - \pi_0)}{\bar{\pi}}. \quad (\text{I.2})$$

Note that this restriction implies that $\lambda_0(t) \rightarrow 0$ for $t \rightarrow \infty$. Furthermore, we restrict attention to the case where there is no announcement effect or $\alpha(T^-) < 1$ and

$$P(t) = P_{\min}. \quad (\text{I.3})$$

I.2 Minimum Gradual Interventions

Consider any intervention that starts at T ; i.e., $Q(t) = 0$ for $t \in [0, T)$. Since there is no announcement effect, we have that before the intervention the law of motions are characterized by

$$\mu_\ell(t) = S(1 - \pi(0)) \quad (\text{I.4})$$

$$\dot{\mu}_s(t) = \kappa \mu_0(t) \quad (\text{I.5})$$

for all $t \in [0, T)$. Since $\alpha(T^-) < 1$, it must be the case that $\tilde{\pi}(T^-) < \tilde{\pi}$. In order to have continuous trade starting at T , there must be a discrete jump in buying assets at T , or $Q(T) > 0$, so that the surplus function becomes positive. We have thus that $\Gamma(T) = 0$ with continuous trade for $t \geq T$ if and only if

$$Q(T) = S(1 - \pi(0)) - \mu_s(T) \frac{1 - \tilde{\pi}}{\tilde{\pi}}. \quad (\text{I.6})$$

For $t \geq T$, with continuous trade and interventions according to $\lambda_0(t)$, the law of motions are given by

$$\dot{\mu}_\ell(t) = -\lambda_0(t)\mu_\ell(t) \quad (\text{I.7})$$

$$\dot{\mu}_s(t) = \kappa\mu_0(t) - \lambda\mu_s(t). \quad (\text{I.8})$$

In order to ensure continuous trade, we can restrict attention to policies that achieve $\alpha(t) = 1$ or $\tilde{\pi}(t) = \tilde{\pi}$ for all $t \in [T, \infty)$. This condition is equivalent to requiring that

$$\frac{\dot{\mu}_\ell(t)}{\mu_\ell(t)} = \frac{\dot{\mu}_s(t)}{\mu_s(t)}. \quad (\text{I.9})$$

Hence, after T a minimal gradual intervention is characterized by

$$-\lambda_0(t) = \kappa \left(\frac{S\pi(0) - \mu_s(t)}{\mu_s(t)} \right) - \lambda. \quad (\text{I.10})$$

I.3 Remarks

When there is no announcement effect, a minimal gradual intervention is indeed the cheapest policy that achieves full trading after T . Any other policy needs to either have a jump at some time $t > T$ or have a faster rate of purchases earlier on. For all such policies, we can shift the costs of the intervention into the future without affecting trading. For the optimal timing of such an intervention the trade-off is similar as in the main text. When delaying the intervention the change in the minimum gradual intervention comes entirely from the law of motion of μ_s . With delay the initial purchases required are smaller, but the rate at which one needs to buy assets later on is larger, since at T , the average quality is now higher, but falls faster at any time $T + \Delta$ after the intervention has taken place.

The same reasoning applies when there is an announcement effect, but the intervention is still minimal. Relative to a one-time intervention at T , one can reduce the intervention so that $\tilde{\pi}(T) = \tilde{\pi}$ and spread out purchases into the future to again keep the quality of assets

that are for sale at this threshold. The intuition is again that what happens after T is of no consequence to the trading incentives before T as long as there is trade which only depends on the average quality in the market.

Finally, note that an announcement effect can only be fostered through increases in prices. Simply shifting purchases towards T cannot increase the effect. The reason is that the quality in the market does not improve before T , the time of the intervention. Hence, one needs to increase the price of the intervention in order to achieve trading before T . This implies that the strongest announcement effect happens precisely when there is a single intervention at T . Delaying purchases even at a higher price reduces the option value due to discounting. Hence, if one would like to use the announcement effect, a point intervention at T achieves the maximum effect. The optimal policy design, however, is much more complex as one needs to consider how many lemons to buy at what price over which time horizon.

Appendix J Numerical Analysis – Calibration and Additional Results

J.1 Benchmark Calibration

We calibrate our economy to capture a typical market for structured finance products such as asset-backed securities (ABS) or collateralized debt obligations (CDO). We then interpret private information concerning the quality π of the assets as reflecting the opaqueness associated with the tranches of these assets. In our benchmark, assets are of very high quality with an average of 99% being good assets ($\pi = 0.99$). This is consistent with Aaa rated corporate debt which historically has a default rate of 1.09% and 2.38% for 10 and 20 year maturities respectively and a recovery rate of about 50% (see [Moody’s Investor Service, 2000](#)). Similar impairment probabilities were associated with Aaa rated tranches of structured debt products before 2007 (see [Moody’s Investor Service, 2010](#)). In accordance with [Duffie, Garleanu and Pedersen \(2007\)](#) we set the annual interest rate r to 5% and the fraction of investors holding an asset to $S/(1 + S) = 0.8$.

The two key parameters in our analysis, the arrival rate of a liquidity shock κ and the degree of search frictions λ are chosen to match annual turnover rates for debt products. We set κ to 1 so that an asset holder remains an owner for an average of one year. With the proportion of lemons being 1%, we set λ to 10 to obtain a turnover rate of

$$\frac{\lambda(\mu_s + \mu_\ell)}{S} = \lambda \left(\frac{\kappa\pi}{\lambda + \kappa} + 1 - \pi \right) = 1, \tag{J.1}$$

so that assets change hands once a year on average. This is consistent with the typical turnover rate reported in the literature.¹

Table J.1: **Benchmark Parameter Values**

λ	κ	S	δ	x	r	π
10	1	4	1	0.035	0.05	0.99

Table J.1 summarizes the values of the exogenous parameters while Table J.2 describes the resulting steady-state equilibrium. As in [Duffie, Garleanu and Pedersen \(2007\)](#), we have

¹[Bao, Pan and Wang \(2008\)](#) give turnover rates between one and two years for corporate bonds, while [Goldstein, Hotchkiss and Sirri \(2007\)](#) report a lower annual rate in the range of 0.8-1.2 (see also [Edwards, Harris and Piwowar, 2007](#)). Data for structured products are not readily available.

normalized $\delta = 1$. Finally, the valuation shock is chosen to match the spread of highly rated structured finance products over the risk-free rate r . The yield is $1/17.6682 = 5.6599\%$ or 66 pbs above the risk-free rate.² As shown in Table J.2, due to search frictions, a fraction $\mu_s/(S\pi)$ of good assets – or 9% of the total – is misallocated to investors with a liquidity shock. Also, since $\kappa > r$, the resale effect dominates the quality effect, implying that $\bar{\pi} < \underline{\pi}$. The steady state equilibrium then falls in the range of multiple equilibria so that it matters whether a trader can resell the asset when receiving a liquidity shock.

Table J.2: **Benchmark Steady-state Equilibrium**

μ_b	μ_o	μ_s	μ_ℓ	p	$\tilde{\pi}$	$\bar{\pi}$	$\underline{\pi}$
1	3.600	0.360	0.04	17.6682	0.9000	0.9694	0.9983

J.2 Low Search Frictions and Small Quality Shock

In our benchmark calibration, the asset turnover rate is 1. We now consider a different situation where the search frictions are lower and hence the turnover rate is higher. The purpose is to examine in details an economy in which the assumption of $\alpha < 1$ is violated. Keeping other parameters fixed, increasing λ from 10 to 100 implies a turnover rate of 1.98. Table J.3 reports the steady-state masses, asset price, and critical quality levels.

Table J.3: **Steady-state Equilibrium**

μ_b	μ_o	μ_s	μ_ℓ	p	$\tilde{\pi}$	$\bar{\pi}$	$\underline{\pi}$
1	3.9208	0.0392	0.04	17.4578	0.4950	0.9669	0.9983

Consider a negative quality shock at $t = 0$ such that 10% of the assets turn from good to lemons, i.e., $\pi(0) = 0.89 < \bar{\pi}$. Since the shock is smaller than that in the benchmark case, $\alpha > 1$, and hence even a minimal intervention can generate an announcement effect. Figure J.1 depicts the equilibrium trading response $\gamma(t)$ and the associated market price $p(t)$ for three different values of λ , when there is a minimum intervention at time $T = 0.25$.

The price jumps up when the market starts to recover at τ_1 , and drops slightly over time at the rate of r as trading activity increases. After the intervention, full trade is restored

²This is consistent with the total spread in [Krishnamurthy and Vissing-Jorgenson \(2010\)](#) for corporate debt of the highest quality. [Gorton and Metrick \(2010\)](#) report a range of 50 to 100 bps on highly rated ABS before 2007.

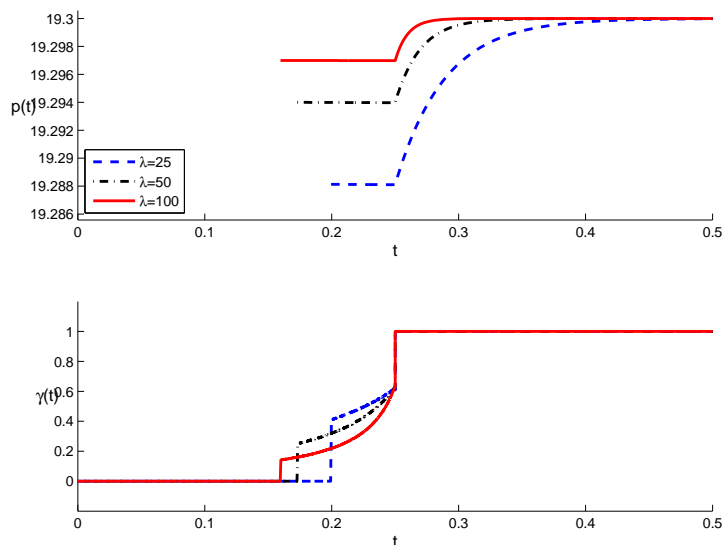


Figure J.1: Equilibrium Prices and Trading Dynamics ($T = 0.25$, $V_I = 0$) – Impact of Search Friction

with the price increasing monotonically towards the steady-state level. With smaller search frictions (higher λ), the market recovers earlier. But with partial trading the fraction of buyers making offers, $\gamma(t)$, is also decreasing in λ . The reason is that with less search frictions there cannot be too much trading, as otherwise the quality of assets for sale would drop too fast in order to maintain a mixed strategy equilibrium. Furthermore, a higher λ tends to increase the asset price and speed up its convergence.

Figure J.2 examines the effects of increasing the price of the intervention from $P = P_{\min}$ to $P = P_{\max}$ while holding Q fixed at Q_{\min} . Such an increase in V_I strengthens the strategic complementarity. Hence, it induces the market to recover earlier. It also raises trading activity before the intervention which in turn increases the market price. This is due to a faster drop in the average quality of assets that are for sale when there is more trading in the market.

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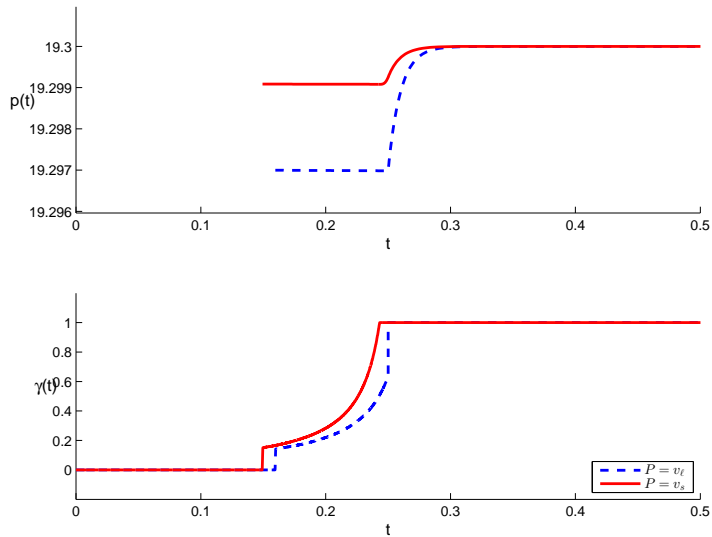


Figure J.2: Equilibrium Prices and Trading Dynamics ($T = 0.25$) – Impact of Option Value V_I

EDWARDS, A., HARRIS, L. AND PIWOWAR, M. (2007). “Corporate Bond Market Transaction Costs and Transparency”, *Journal of Finance*, 62, pp. 1421-1451. [23](#)

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Appendix K The Liquidity Channel of Policy

We have assumed throughout that when traders sell lemons to the MMLR they exit the market forever. This assumption kept the market tightness constant in the long-run. Suppose now instead that traders who sell lemons to the MMLR can stay in the market and become buyers again after they sold to the MMLR. An intervention can then have more powerful effects, since it increases market liquidity permanently by raising the number of buyers in the market from 1 to $1 + Q$. This reinforces the strategic complementarity, since we have now for the value of a lemon after full recovery that

$$v_\ell(t) = \frac{\lambda(1+Q)}{\lambda(1+Q)+r}v_s > \frac{\lambda}{\lambda+r}, \quad (\text{K.1})$$

for all $t \geq \tau_2(T)$. We call this additional effect of an intervention *liquidity channel*.

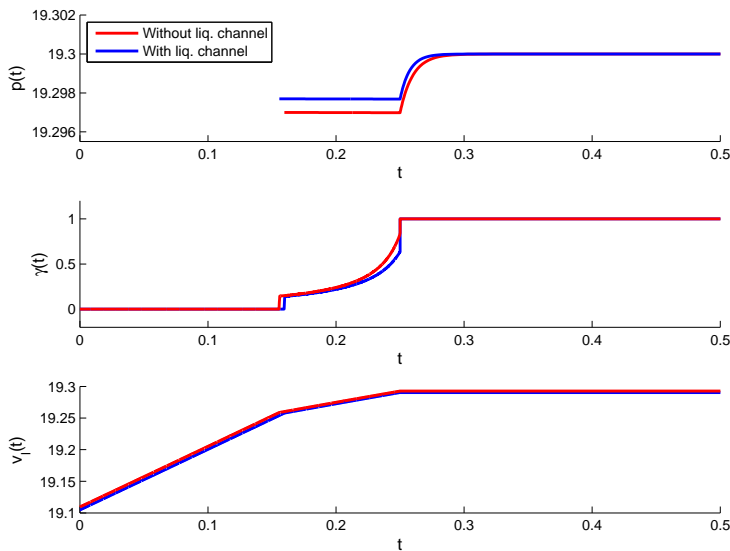


Figure K.1: Equilibrium Price and Trading Dynamics – Liquidity Channel

Using the parameter values in Appendix J, Figure K.1 compares the equilibrium trading dynamics of an intervention with and without this liquidity channel of policy. As shown, the liquidity channel quantitatively plays only a small role. The value function for lemons is almost identical to our benchmark economy, and thus the market price and trading dynamics are only slightly altered by this additional effect.¹

¹It is not clear how relevant the liquidity channel would be empirically, as there could be entry and exit of traders in any specific market. Moreover, the shock to quality could be temporary with the MMLR being able to lay off some of the assets purchased after recovery leaving market tightness unchanged in the long-run.



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