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SEF WORKING PAPER 07/2016

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Working Paper 07/2016

ISSN 2230-259X (Print)

ISSN 2230-2603 (Online)

VIABILITY OF AN ECONOMY WITH CONSTRAINED INEQUALITY IN A TWO TAX SYSTEM

MAXIMILIEN DRISCOLL O'KEEFE & JACEK B. KRAWCZYK

ABSTRACT. Motivated by Karacaoglu's treasury paper concerning the sustainability and equity of capital, as well as by Piketty's research suggesting that income inequality will increase if no action is taken to remedy it, we search for a way to reduce inequality while maintaining economic efficiency. Using a developed economic model with evolutions for debt, consumption, capital and relative factor share, which proxies inequality, we look at how tax rates on capital income and labour income can constrain all the quantities above within set bounds. We solve this problem through the mathematics of viability theory and use a program called VIKAASA to solve and display our results in terms of viability kernels. The results tell us that taxation, especially capital taxation, is a powerful tool for reducing inequality. While this taxation usually diminishes consumption and capital, we show that for some economic conditions, these decreases can be negligible. The kernels also tell us how a policy maker should react in a variety of economic situations, including high debt and high capital stocks.

2016 Working Paper

JEL Classification: D31, D33, E25, N17, N37

Authors' keywords: factor income, inequality, viability theory, capital income tax, labour income tax

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1. Introduction

1.1. **New Zealand Living Standards.** In the recent article Karacaoglu (2015), motivated by a desire to improve the living standard of New Zealanders, put forward that it was the responsibility of public policy to 'enhance (the) intergenerational well-being' for all. Classically, it is difficult to get conclusive findings on a nations well being. GDP speaks only to a country's output, not the effect that output has on its constituents. More recently the OECD Better Life Index has been used to measure wellbeing, but that too comes with its faults. [1]

Karacaoglu instead references Arrow, Dasgupta, Goulder, Mumford, and Oleson (2012), who speak of 'collective wellbeing'. Collective wellbeing measures the ability of citizens to consume a variety of assets in order to create wellbeing, calling it 'comprehensive consumption'. As in all models, consumption must be fuelled by a source, which they appropriately call 'comprehensive wealth'.

Comprehensive wealth is made up of not only physical capital, but also of human capital, natural capital and social capital. An individual's wellbeing, and by extension the country's wellbeing, is measured in their ability to consume this capital now and in the future.

Karacaoglu therefore sees it as the role of policy to take 'stewardship of comprehensive wealth', so that it can provide the greatest utility for both current and future generations. More clearly, this entails creating policy that is viable in 5 different facets, which he identifies as growth, equity, social cohesion, resilience and sustainability. Figure 1 below shows this, and how each is defined.

[1]For example, combatting child mortality affects life expectancy more than increased elderly care.

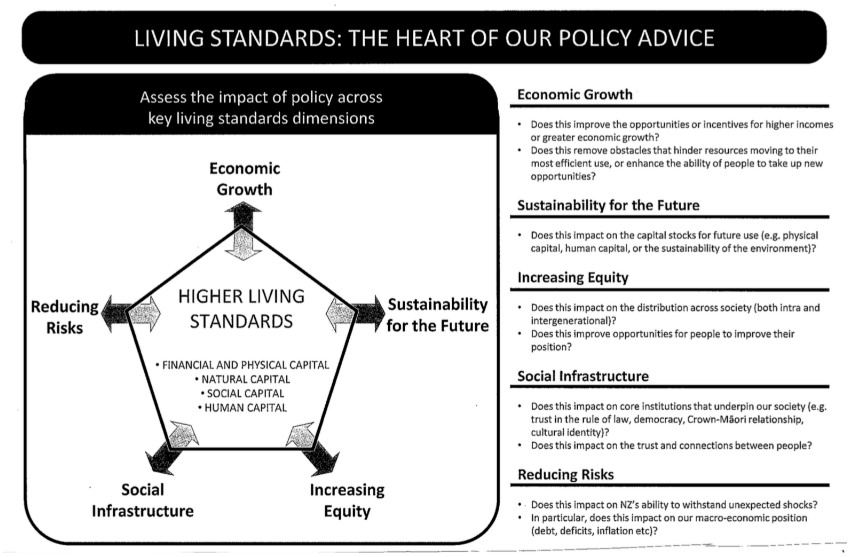

FIGURE 1. New Zealand Treasury's Living Standards Approach

Implicitly, Karacaoglu understands improving living standards as containing the five economic policy outcomes in Figure 1 in a set of socially acceptable intervals. To us, this problem lends itself to viability theory, which will be explained in Section 3.

We take this work as motivation to design a framework for taxation that will give guidance on addressing one of the most pressing issues for this project, namely the equity aspect of policy.[2]

1.2. **Piketty Context.** In Thomas Piketty's 'Capital in the 21st Century' (Piketty (2014)), he speaks of the significant changes in the capital to income ratio since the 18th century as well as the top decile and centile taking an increasing share of income. He finds these two states to be correlated, increasing and suggests equity in living standards can't be achieved while this is the case.

[2]The other issues, apart from *economic growth*, which is easily definable, could also be dealt with by our method, after their bounds have been defined.

Piketty believes that the increase of the capital-income ratio

$$k/y$$

is a historical norm that would have come to prominence earlier had it not been for the vast destruction and devaluation of capital during the World Wars. This belief is backed by the data displayed in Figure 2, which sees the capital-income ratio peak in 1870, where European capital levels were between 650% and 700% of income. Despite the trough actualised by the World Wars a steady increase since 1950 is clear, with levels today standing at 400% in Germany, 510% in the UK and 580% in France.

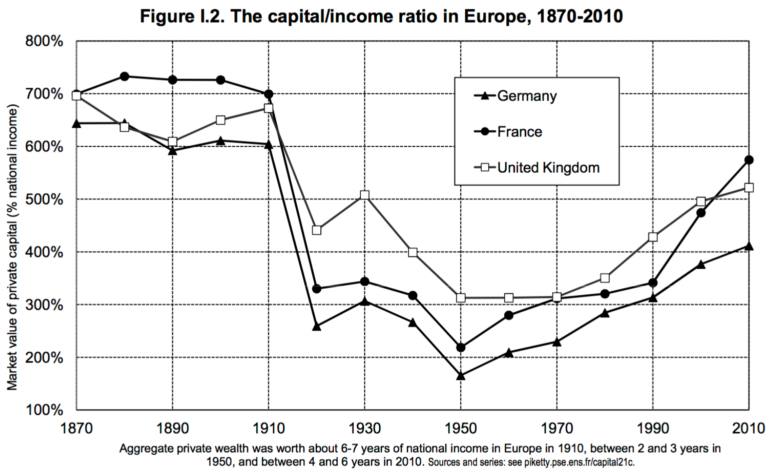

Figure 2. Figure 1.2 from Piketty (2014) showing the evolution of the capital-income ratio in Europe

Piketty explains the continued capital-income ratio's growth with his first Law of Capitalism. He states on page 166 of Piketty (2014) that the capital-income ratio is defined by

$$\frac{k}{y} = r \cdot \frac{s}{(n+g+d)} \tag{1}$$

where k - capital, y - income, s - savings, n - population growth, g - output growth, r - interest rate and d - depreciation.

We see from (1) that if interest rates are large, and not compensated by the other variables in the fraction, then the capital-income ration will grow. In fact, Piketty suggests that with the observed population growth rate decreasing and interest rates not decreasing enough, the capital-income ratio will continue to grow and capital's share of income along with it.

But the capital income ratio is not what concerns Piketty, or indeed us. Rather it is the effect that the ratio has on income inequality. As can be seen in Figure 3, the evolution of the capital-income ratio has been mirrored by the evolution income inequality, at leat in Anglo-Saxon economies.

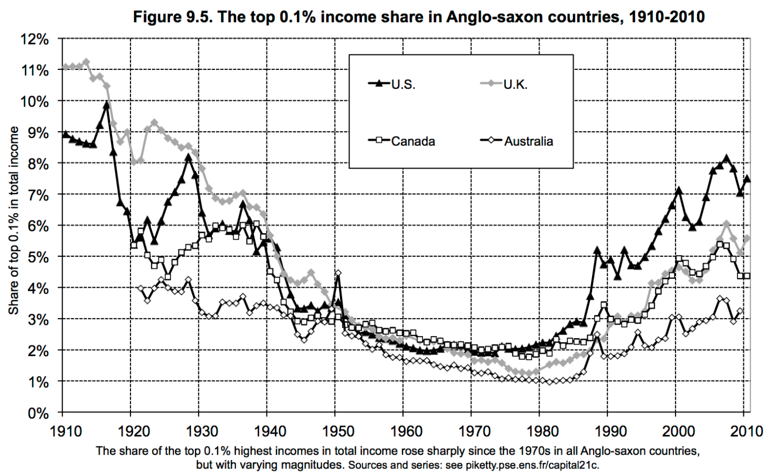

FIGURE 3. Figure 9.5 from Piketty (2014) showing the evolution of the top 0.1%'s share of total income

Neither Piketty nor us pretend that capital accumulation on its own explains the rise in inequality. Various non-normative factors including inequality in access to skill and higher education, changes in labour earnings and the explosion of top managerial position compensation have played a significant part. But these facts fall away when looking at the income of the top centile, and especially the top 0.1%, see Krawczyk and Townsend (2015a)

At these top levels, the increasing income inequality can be linked to the capital holdings these earners have. It has been observed that the income of those who control the majority of capital becomes more and more separated from the masses. Piketty states this is the trend that has fuelled the highest levels of income inequality.

As we accept Piketty's attribution of the growth of inequality to be linked to the historical tendency of $r > g$, (that the interest rate outpaces growth in infinite horizon macroeconomic models), we see his call for redistribution to be synonymous with reducing the capital-income ratio.

2. Background and Motivation

The problem with redistribution is how one finds a trade-off between inequality and economic efficiency. The trade-off itself has been well developed in previous literature all the way back to Okun's 'Equality and Efficiency' Okun (1975), but an explicit framework for enabling analysis hasn't been fully developed.

Motivated by NZ Treasury's desire to improve the *Living Standards of New Zealanders*, see Karacaoglu (2012), we propose the proper framework for this question to be the area of viability theory. As will be explained in Section 3, viability theory determines a set of initial conditions from which the dynamic system can be run while keeping within set constraints.

Although viability theory is not new to economics, few examples exist of its application to macroeconomics (the papers being; Clément-Pitiot and Saint-Pierre (2006), Clément-Pitiot and Doyen (1999), Krawczyk and Kim (2009), Krawczyk and Kim (2014), Bonneuil and Saint-Pierre (2008), Bonneuil and Boucekkine (2008), Krawczyk and Kim (2004), Krawczyk and Sethi (2007)). Notably, this project and its direct predecessors Krawczyk and Townsend (2015a,b,c)), are its first application to income inequality.

The economic model we use was derived in Krawczyk and Judd (2015), while the expression 'factor ratio', a de facto measure of inequality was derived in Krawczyk and Townsend (2015a). The follow up paper Krawczyk and Townsend (2015b) showed the factor ratio to be highly correlated with income shares, especially at comparison levels between the top 1% and 0.1% to the rest. Thus we use the factor ratio as an economic aggregate, which allows us to understand inequality though a representative agent model.

The constraints we set on our model are those that are contrary to an inefficient economy, one realised by low capital formation, restricted consumption and increasing government debt.[3] Thus efficiency is deemed by constraining capital, consumption and debt. In the same way, we limit inequality by putting bounds on the factor ratio.

This paper aims to improve the framework for understanding the trade-off between efficiency and inequality by refining the work of Krawczyk and Townsend (2015c) which showed controlled tax adjustments can be a viable policy instrument for controlling inequality and economic growth. However that paper assumed identical rates of taxation on

[3]Too large capital, too large consumption and not too large savings are also deemed inefficient. For a discussion on the *efficiency* bounds on these variables we refer to Krawczyk and Judd (2015) and Krawczyk and Townsend (2015c).

capital and labour, which we note to be rarely realised in real world economies. [4]

Hence our improvement comes in the form of a more comprehensive tax system, one in which labour income tax and capital income tax are adjusted independently. Through this improvement we can shed more light on how inequality can be reduced by taxation while keeping the economy efficient, in the sense of keeping it within the efficiency bounds.

A model with separated taxes is worth investigating as it should give us more insight and opportunity for analysis on the use of taxation as a policy instrument. It can also further our argument that viability theory should be at the heart of economic and inequality analysis when taxation adjustment is used as a control variable.

3. Viability theory

3.1. **The meaning of viability.** Viability theory is the mathematics which studies constrained dynamic systems. A system's evolution is viable if the for the entire evolution the system is constrained within a constraint set K, given a constrained control set U. A collection of economic positions from which viable evolutions exist is called the *viability kernel*. Viability theory attempts to determine whether a nonempty viability kernel exists and if so what its boundaries are.

In the context of our work, this can be expressed as examining varying economic situations and evolving them through a dynamic model to see if they can be controlled and stabilised within set bounds. Our system's evolution will be governed by the state equations, given in Section 4.1. The constraint set K will be determined by efficient economic bounds

[4]Germany, Denmark and France tax capital and labour differently, while Singapore and Switzerland don't tax capital income at all.

given in Section 4.2. Our control set U will be a reasonable rate of tax adjustment, and will be explained in Section 4.3.

Viability theory provides a more accurate description of some real world decision making than optimisation as it formalises the 'satisficing' policies of Simon (1955), the concept that as long as viability isn't threatened, any policy will be satisfactory. This attribute means viability theory might better represent real world decision making when compared to solutions calculated by optimisation. Krawczyk and Townsend (2015c) cites the example of an inflation-targeting banker. Optimisation would suggest frequent changes to tax rates, whereas viability theory captures the desire to avoid alterations until necessary.

A more in depth analysis of viability theory's use in finance and economics as well as an introduction to the mathematics used in our analysis can be found in Krawczyk and Townsend (2015c) and the publications there cited. We will however reproduce Figure 4 from Krawczyk and Pharo (2013) in order to give visual aid in interpreting the kernels in Section 5.

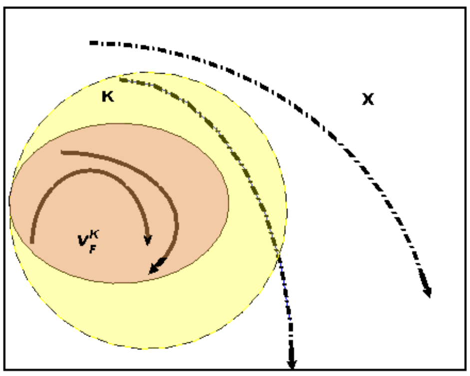

Figure 4. The viable and non viable trajectories for a time-invariant dynamic system.

The yellow sphere represents the state constraints K, which for us are the limits on economic variables. The lines represent possible evolutions based on various starting conditions which stabilise where the arrows end. The inner darker shape is the viability kernel. Evolutions that start in the the kernel are able to stabilise within the constraint set K and are thus viable. Points outside of the kernel stabilise outside of K, so are deemed non-viable.

4. Formulating the viability of constrained relative factor share

We will use the same notation as in Krawczyk and Townsend (2015a) and Krawczyk and Townsend (2015b). In particular, χ is the relative factor share – capital income, less depreciation, divided by labour income. As shown in Krawczyk and Townsend (2015b), χ correlates with the shares of income taken by the highest income 1% and 0.1%. So, we conjecture inequality will diminish in line with the relative factor share.

4.1. **System's Dynamics.** Derived in Krawczyk and Judd (2015) are the formulae that govern the evolutions of consumption, capital and debt. Respectively, they are;

$$\frac{dk}{dt} = A\,k^{\alpha}\left(\frac{\left(A\,k^{\alpha}\,(1-\alpha)\,(1-\tau_{L})\right)^{\frac{1}{\alpha+\eta}}}{(V\,c^{\gamma})}\right)^{1-\alpha} - g - \delta\,k - c \tag{2}$$

$$\frac{dc}{dt} = -\frac{c}{\gamma}\left(\rho + \left(\delta - A\,\alpha\,k^{\alpha-1}\left(\frac{\left(A\,k^{\alpha}\,(1-\alpha)\,(1-\tau_{L})\right)^{\frac{1}{\alpha+\eta}}}{(V\,c^{\gamma})}\right)^{1-\alpha}\right)(1-\tau_{K})\right) \tag{3}$$

$$\frac{dB}{dt} = \tag{4}$$

$$g \quad + \delta\, k\, \tau_{\mathrm{K}} - B \left(\delta - A\, \alpha\, k^{\alpha-1} \left(\frac{(A\, k^{\alpha}\ (1-\alpha)\ (1-\tau_{L}))^{\frac{1}{\alpha+\eta}}}{(V\, c^{\gamma})} \right)^{1-\alpha} \right) (1-\tau_{K})$$

$$- \quad Ak^{\alpha}\ (\tau_{L} + \alpha\ (\tau_{K} - \tau_{L})) \left(\frac{Ak^{\alpha}\ (1-\alpha)\ (1-\tau_{L})}{(V\, c^{\gamma})} \right)^{\frac{1-\alpha}{\alpha+\eta}}$$

These three equations represent the evolution of our economy. As one would expect, capital decreases in labour taxation, as the incentive to work and produce is reduced. Consumption grows in both low income and caoital taxation as people have more money to spend. Debt is reduced by high taxation, but will increase quickly with higher initial debt.

In Krawczyk and Townsend (2015a), the same variables were used to derive an equation for the relative factor share in an economy with a one tax and two tax systems. For the latter case, we have:

$$\chi \equiv \frac{k\bar{r}}{l\bar{w}} = \frac{1-\tau_{K}}{1-\tau_{L}} \left(\frac{\alpha}{1-\alpha} - \delta \left(\left(\frac{V c^{\gamma}}{1-\tau_{L}} \right)^{1-\alpha} k^{\eta(1-\alpha)} (A(1-\alpha))^{-(\eta+1)} \right)^{\frac{1}{\alpha+\eta}} \right). \tag{5}$$

From equation 5, we see that the two tax rates affect χ directly through the equation itself and though the effect on c and k. Diminishing τ_{K} or increasing τ_{L} leads to increasing the initial level of χ, while vice versa will lead to it decreasing. The indirect impact depends on which part of the state space the evolution starts from. For example, for large c and k, the indirect impact of increased labour income taxation may mitigate the inequality growth caused by the direct channel because the term weighted by δ may be large. However, in general, the indirect impact is moderated by the fractional power[5] $\frac{1-\alpha}{\alpha+\eta}$. A more detailed analysis

[5]Unless α is very small, this fraction will be < 1.

of the system's evolutions can be found in Krawczyk and Townsend (2015a).

4.2. **The Viability Kernel.** Let $x(t)$ be the state vector composed of capital k, consumption c, debt B and taxation rates τ_L and τ_K. We ask whether the system dynamics $F(x(t))$, defined through equations (2)-(4), are compatible with the viability constraints K:

$$K \equiv \left\{ (k, c, B, \tau_L, \tau_K) : \left| \begin{array}{c} \underline{k} \leq k(t) \leq \overline{k} \\ \underline{c} \leq c(t) \leq \overline{c} \\ \underline{B} \leq B(t) \leq \overline{B} \\ \tau_L(t) \in [\tau_{L\min}, \tau_{L\max}] \\ \tau_K(t) \in [\tau_{K\min}, \tau_{K\max}] \\ 0 \leq \chi \leq \overline{\chi} \end{array} \right. \right\}. \tag{6}$$

where the constraints on k, c, B, τ_L, τ_L and χ – $\underline{k}, \overline{k}, \underline{c}$, *etc.*– will be explained in the next sub-section.

If the system's dynamics are compatible with K, there will exist a set of economic states from which there exist viable evolutions that respect the entire set of constraints. This is the viability kernel discussed earlier, here given as

(7)

$$\mathcal{V}_F(K) \equiv \left\{ (k(0), c(0), B(0), \tau_L(0), \tau_K(0)) : \begin{array}{|c} \exists\ (k(\cdot), c(\cdot), B(\cdot), \tau_L(\cdot), \tau_K(\cdot))\,, \\ \text{starting from } (k(0), c(0), B(0), \tau_L(0), \tau_K(0)) \\ \text{satisfying dynamics } F(x(t)), \\ u, v \in U \text{ and constraints (6)} \\ \forall\, t\ \in \Theta \end{array} \right\}.$$

where U contains allowable taxation-rate *adjustments* u and v for labour and capital respectively (perhaps ± 20% per year).

A regulator of the economy described by the dynamics $F(x(t))$ and the constraint set K will be seeking strategies $u(\cdot)$ that generate $k(\cdot)$, $c(\cdot)$, $B(\cdot)$, $\tau_L(\cdot)$, $\tau_K(\cdot)$ consistent with the above definition of $\mathcal{V}_F(K)$.

4.3. **The calibration.** Following Krawczyk and Judd (2015), this paper analyses kernels produced for a "reasonably industrialized economy composed of rational agents interested in the near future, drawing a fair satisfaction from consumption and feeling, quite strongly, the burden of labor". We assume $\rho = 0.04$, $\alpha = 0.43$, $\eta = 1$ and $\gamma = 0.5$.

As in Krawczyk and Judd (2015) we are using a stylised steady state $\underline{k} = \underline{\ell} = 1$ with no taxes and no government expenditure and calibrate A and V and obtain $A = 0.2093$, $V = 0.2989$. We then assume that government expenditure g is constant and set at 10% of no-tax steady-state output; $g = 0.1 \cdot A = 0.0209$. As we take total factor productivity to be constant, constant or even small reductions in production can be perceived as a positive result

The constraints come from a combination of positive and normative sources, as well as from the requirement to close K.

(1) **Capital** should be within 10% and 200% of no-tax steady state capital stock, $k \in [0.1,\ 2]$;

(2) **consumption** should not deviate too far from a long-run equilibrium (see Krawczyk and Judd (2015)), $c \in [0.0267, 0.225]$;

(3) **debt** may grow to 150% of the maximum steady-state capital stock and also drop somewhat below zero, $B \in [-1, 3.5]$;

(4) **tax rates** (both on capital and labour) cannot be less than zero, and can at most be equal to 80%, $\tau \in [0. 0.8]$;

(5) **tax-rate adjustment speed** – the amount the regulator increases or decreases the tax rate within a year – will be less than 20 percentage points, $u, v \in [-0.2, 0.2]$.

We will first give results with no upper limit on χ, followed by ≤ 0.4, and then ≤ 0.25. All results will also require that χ be greater than 0, a choice that follows identical reasoning to that of Krawczyk and Townsend (2015c). A negative χ may be a viable point in terms of the mathematics of viability theory, but in real terms it requires negative interest rates as the marginal product of labour, or wages, will be positive in a Cobb-Douglas production function. Negative interest rates would require that $\frac{dy}{dk} < \delta$. That isn't a viable situation, as investment will not occur with negative returns. Thus by requiring $\chi > 0$ we ensure we only get results that are the result of a long run steady state.

From this we have the constraint set K for two taxes from which we will find the viability kernel;

(8)

$$K = [0.1,\, 2] \times [0.0267,\, 0.225] \times [-1,\, 3.5] \times [0,\, 0.8], \times [0,\, 0.8] \times [0,\, \overline{\chi}]\,,$$

where $\overline{\chi}$ is either undefined 0.4 or 0.25.

5. Viability Kernels

5.1. **Viability Kernel Comparison.** This section covers analysis of the viability of economies with different relative factor share constraints.

To derive the viability kernels, we will use VIKAASA, a program developed for graphing viability kernels. An explanation of the program can be found in appendix A.

The derived kernels show whether there are feasible tax adjustment rates for labour and capital tax that deliver both a constrained relative factor share and also maintain our required levels of capital, consumption and debt. We pay particular interest to the effect that the independent tax system has on the shape and size of the viability kernels in relation to the viability kernels derived from Krawczyk and Townsend (2015c) for a one tax (or 'combined') tax system. In that paper it was hypothesised that independent tax rates would increase the size of the viability kernel, so the areas which show increased viability will be paid attention to.

The figures below hold the 3D slices for the kernels produced under our three different inequality constraints. The slices show relationships between capital, consumption and χ, which proxies for inequality, for all values of debt and the two taxation rates. All three kernels require that inequality be greater than 0, while the second and third kernels also require that inequality be less than 0.4 and 0.25 respectively. We have also projected the slices onto the capital-consumption plane, shaded black.

The kernels shrink as tighter constraints on χ are imposed, however the reduction is almost completely in terms of inequality. The projections are reduced little for $\chi < 0.4$, with the only visible reduction appearing in the third kernel with $\chi <0.25$, where points of very low capital lose viability.

5.2. **Viability of low capital, low consumptions states.** In the combined tax model from Krawczyk and Townsend (2015c), it was

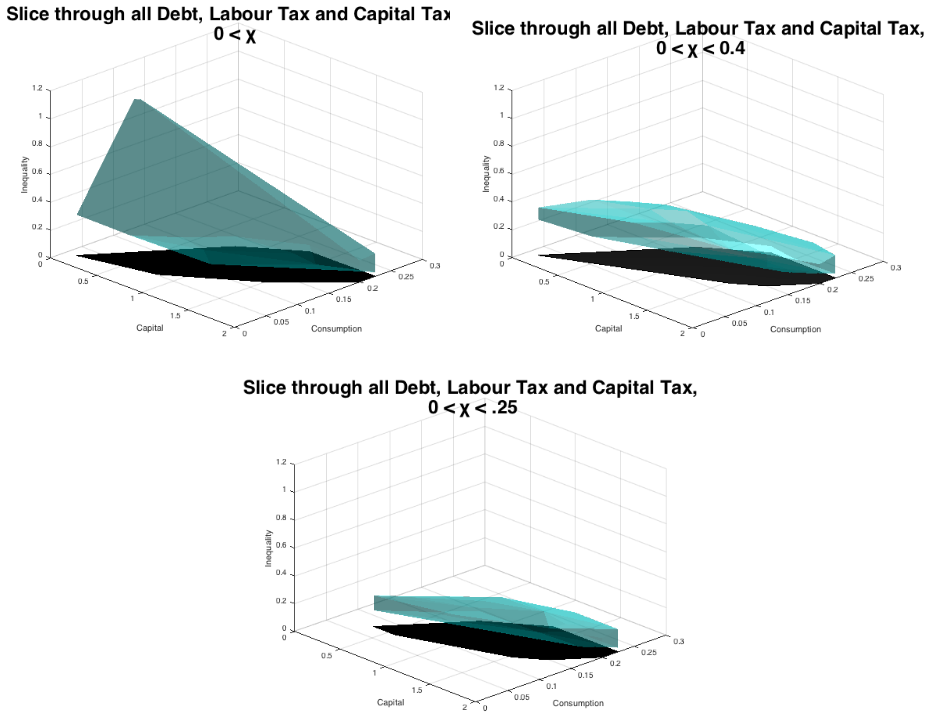

FIGURE 5. Viability kernels for different relative factor share constraints

found that there was a large reduction in the kernel at low capital levels when χ was required to be less than0.25. We therefore see it pertinent to ask why separating tax rates allows low capital economies to be viable in strenuous inequality requirements. We address this by looking at points that are deemed viable in our separate tax model, and look to see why they fail in the same tax model.

The figure below displays the evolution of two viable points in low inequality and low capital. Both evolutions start with capital levels of 0.575 and consumption of 0.12585. The solid line evolution represents a more indebted economy, with debt standing at 200% of reference GDP, and tax rate on labour and capital of 27% and 80% respectively. The dashed evolution shows a more standard economy with debt at 50%

and tax rates of 13% and 67%. As seen by the graphic, both evolutions maintain χ below0.25.

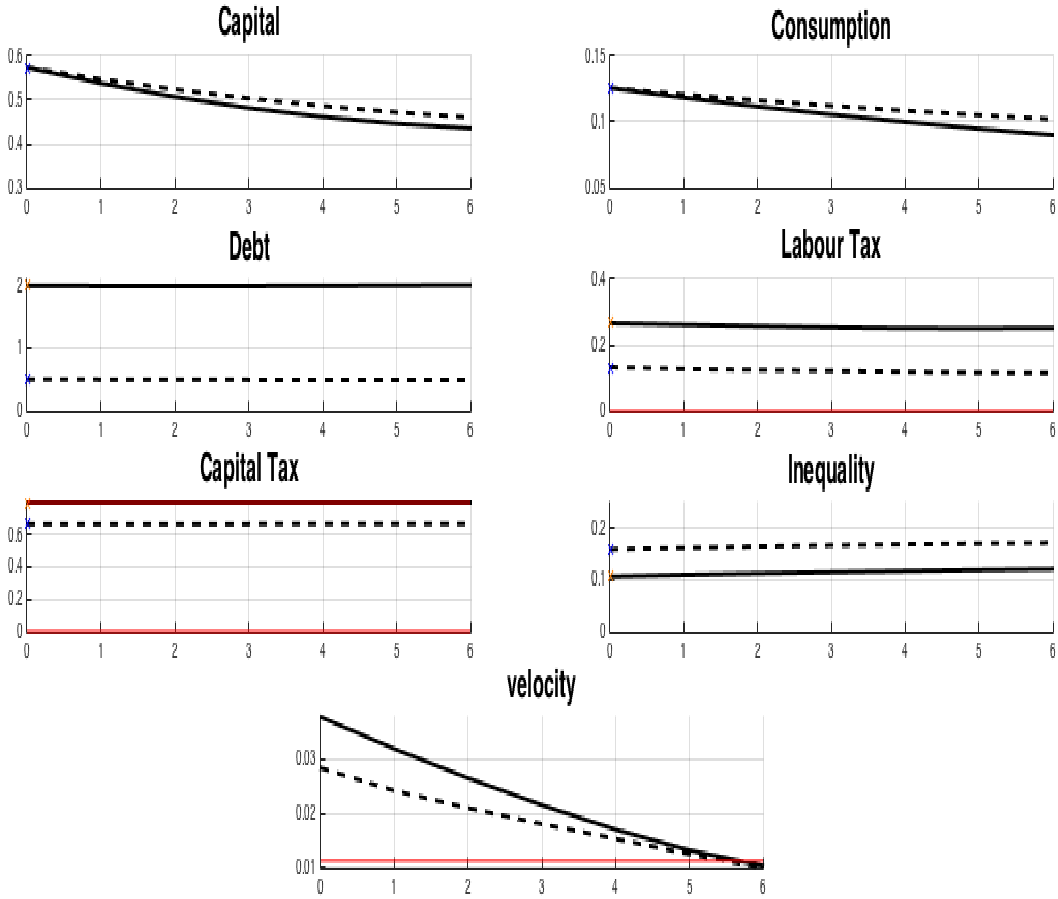

FIGURE 6. Time profiles of viable evolutions in low capital and low inequality

The high debt simulation requires higher taxes in an effort to keep debt down, whereas the lower debt simulation can stabilise with slightly lower starting taxes. The high taxes in both simulations discourage consumption, which helps in reducing the drain on capital. As can be seen in the velocity graph, the system is able to stabilise under these conditions after six time periods.[6] Also of note is that both points have

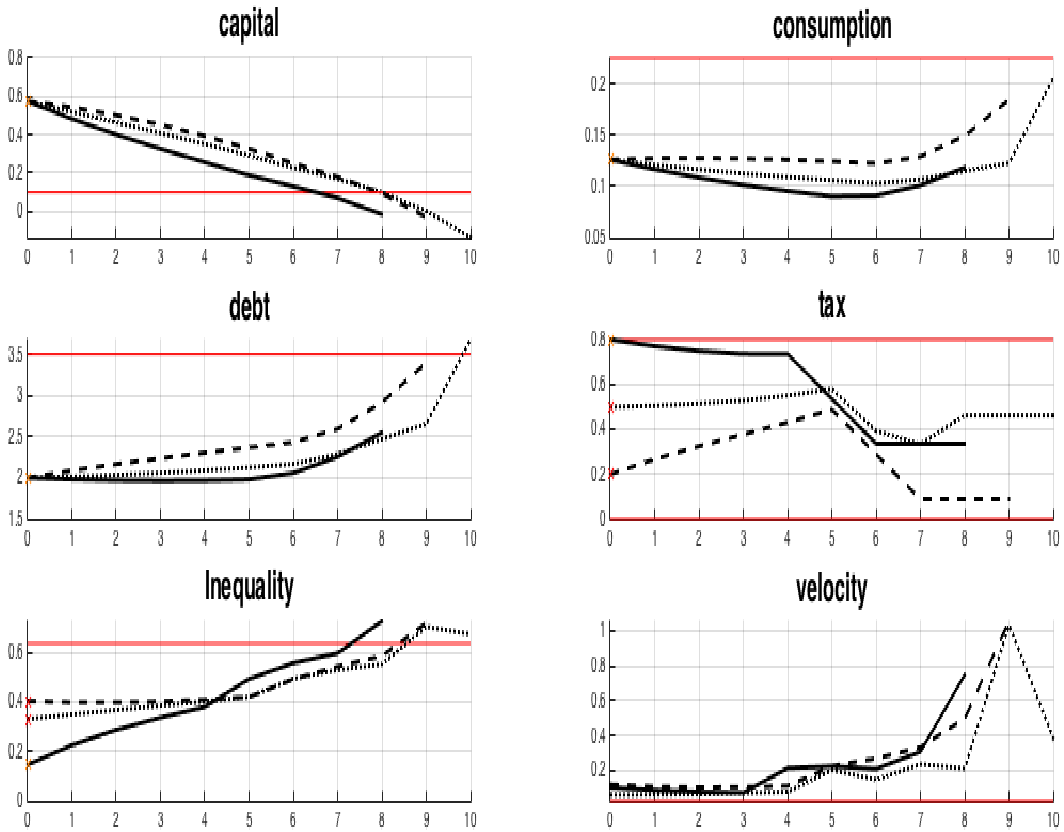

FIGURE 7. Time profiles of non-viable evolutions for a one tax in low capital and low inequality and high debt

[6] VIKAASA stabilises a point by reducing the velocity of the system. From Krawczyk and Pharo (2014a), "The velocity of the system in state x, subjected to some control u is calculated using the Euclidian norm of the system velocities at that point".

capital taxation at a higher rate than labour taxation. This isn't a surprising feature as inequality is significantly and directly affected by the tax ratio: $(1-\tau_L)/(1-\tau_K)$ (see equation (5)).

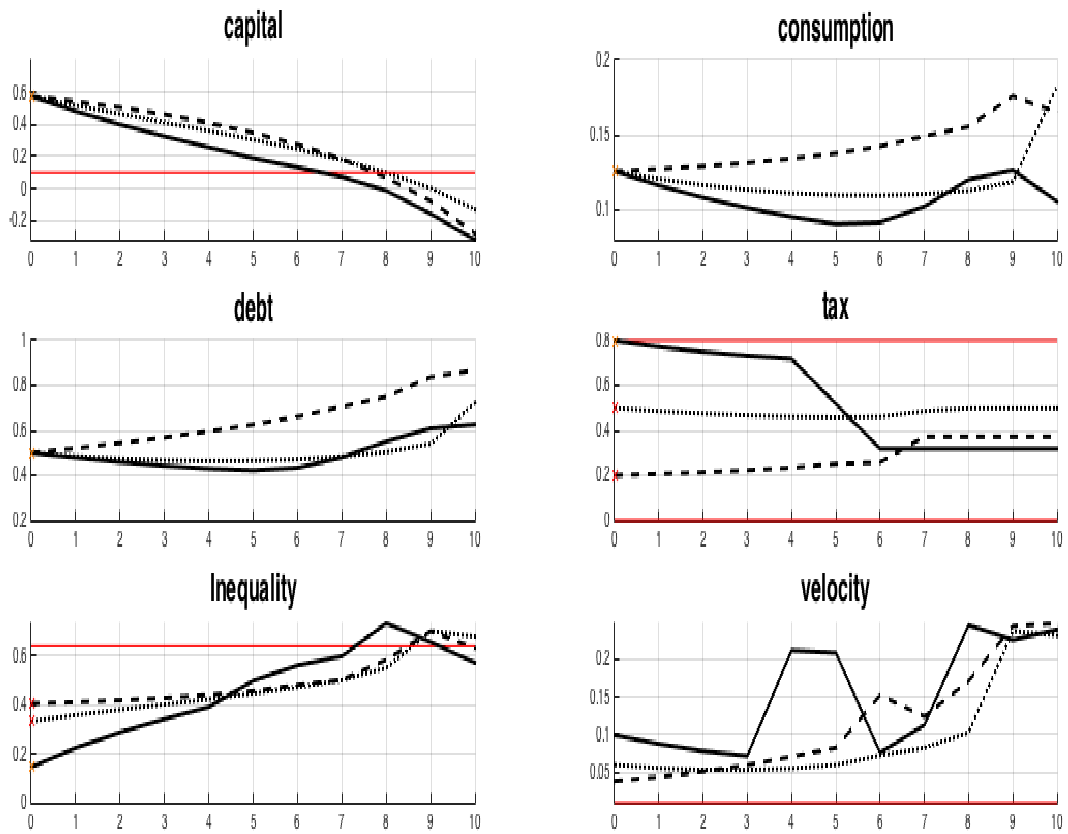

FIGURE 8. Time profiles of non-viable evolutions for a one tax in low capital and low inequality and low debt

We now take these points and simulate them in the 'one' tax model. Figure 7 displays the high debt trajectories given a high (solid line), medium (dotted) and low (dashed) initial starting tax. Figure 8 displays the low debt trajectories with the same set of starting taxes.

5.3. **Why aren't low capital and consumption states viable under the combined tax rates regime?** Figures 7 and 8 show us why low capital economies can't achieve low inequality in a one tax model but succeed as such in a two tax model. In the combined tax model, inequality is decreasing in taxation, capital and consumption.With the latter two being low, our inequality requirement necessitates that taxation rates be high. This can be seen in Figures 7 and 8 where inequality is only at a satisfiable level when we impose the maximum taxation rate of 80%. Capital, however, is decreasing in τ, so a high tax rate leads to a rapid depletion of the capital stock. Clearly these two facts can't be satisfied simultaneously, so the scenario isn't viable.

This explains our point's failure in the one tax model, but does not elucidate us as to why it works in the new two tax model.

We know from the dynamic equations (2), (3) and (4) that consumption is decreasing in taxation, especially capital taxation. With consumption close to its lower bound and a negative consumption rate of change, we would seem to be at risk of breaking the lower consumption bound.

However, the independent tax rates allow capital and consumption to stabilise. A low labour taxation rate increases the incentive to work, and so output and capital increase. This is able to counteract the negative pressure on capital from the high capital taxation rate. The high capital taxation rate also discourages consumption. A lower consumption reduces the drain on capital, which in conjunction with the higher output stabilises capital above its lower bound.

This solution of high capital tax and low labour tax is what enables the viability of the scenario in terms of inequality. Inequality, as measured byχ, can be seen in (5) to be significantly impacted by the fraction $(1-\tau_K)/(1-\tau_L)$. A high capital tax and low labour tax will mean this fraction is small and in turn, inequality will be low.

5.4. **High Debt Economies.** In Krawczyk and Townsend (2015a), it was shown that inequality would be low in high debt economies due to the high taxation rates required to keep below the debt ceiling. Hence we now ask whether this continues to be true in a two tax system, but also whether two tax rates will facilitate low inequality with low capital stocks when combined with high initial debt.

The panels in Figure 9 display kernel slices with the same requirements as Figure 5, but now with the requirement that initial debt start at 350% of GDP.

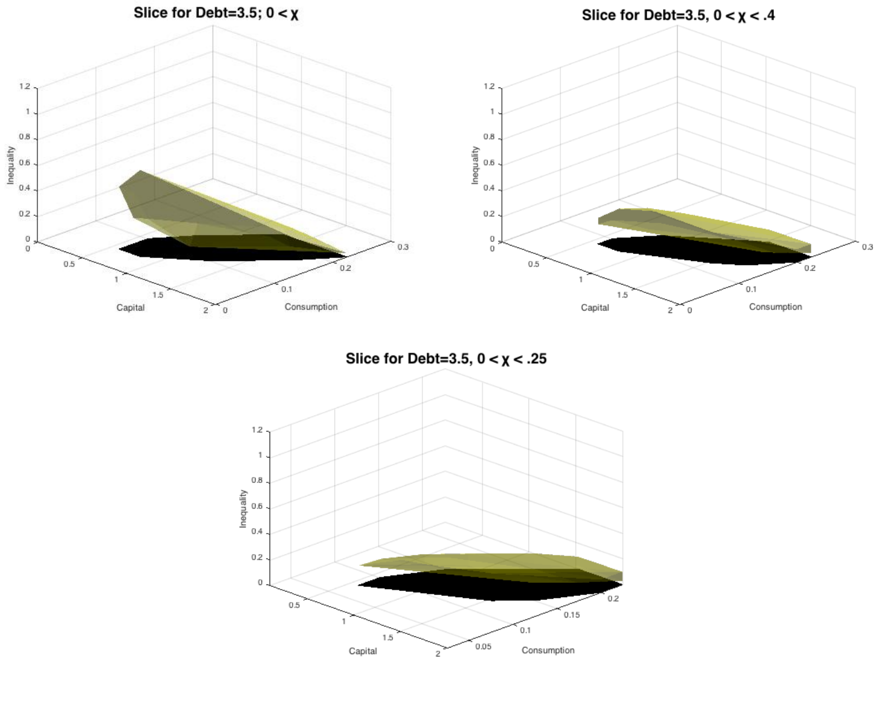

FIGURE 9. Viability kernels with high debt

We observe that the reduction in kernel size under tighter inequality constraints isn't as stark as in the combined tax model in Krawczyk and Townsend (2015a). Where as in the combined tax model only high capital environments were viable in low inequality and high debt, low capital points are deemed viable in the two tax system.

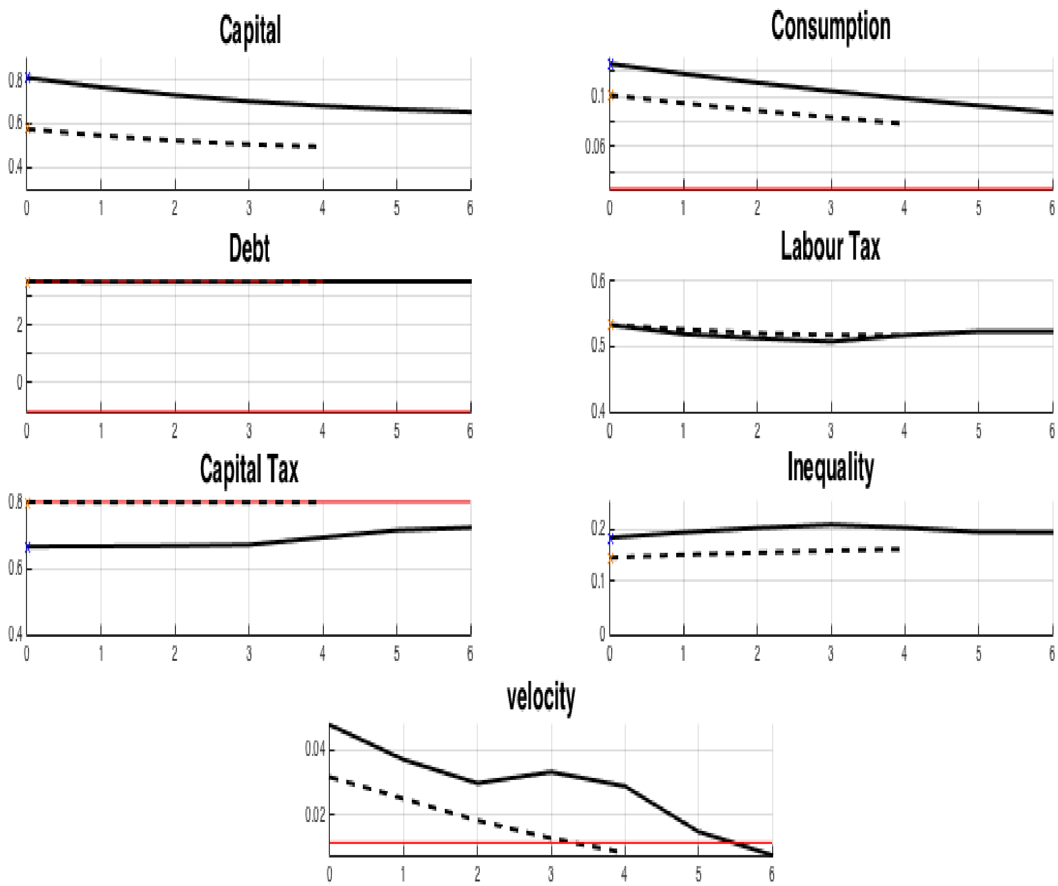

FIGURE 10. Time profiles of viable points in high debt and low capital

In order to investigate this, we again simulate viable points with low initial capital but now from the third kernel in Figure 9. Figure 10 displays this.

The scenario is the same as that in section 5.1. Capital and consumption are decreasing but are able to stabilise at a lower consumption levels as capital is taxed highly and labour moderately. This makes intuitive sense. A high debt level requires a high average tax rate to prevent breach of our upper debt limit. So again, we have that the medium-to-low labour taxation stimulates output and capital while the high capital taxation rate discourages consumption. This reduces the drain on capital, allowing capital to stabilise above its lower bound.

5.5. **Taxation Policy.** In all the scenarios so far, we have seen that the requirement of low inequality lends itself to a higher capital taxation rate than labour tax rate. The reasoning for this was stated earlier, but we now ask whether low inequality targets hinge completely on the balance of taxes targeting capital.

As it turns out this is not the case. Low inequality can be achieved with labour tax higher than capital tax, even to the point where we don't tax capital at all. Two such points and their evolutions are displayed below. We have also graphed production, defined by the standard Cobb Douglas production function

$$y = A\, k^{\alpha}\, l^{1-\alpha}\,. \tag{9}$$

Both of these points achieve low inequality despite the tax ratio being congruent with a high inequality scenario. In this case it is the high capital stock that facilitates low inequality, as from the dynamic equations we can see that inequality is decreasing in capital.

By Okun's law, we expect production to be reduced in falling inequality. In this scenario inequality is low but actually increasing slightly, and so we see a slight increase in production. Given that we have constant total factor productivity, this is a very positive result.

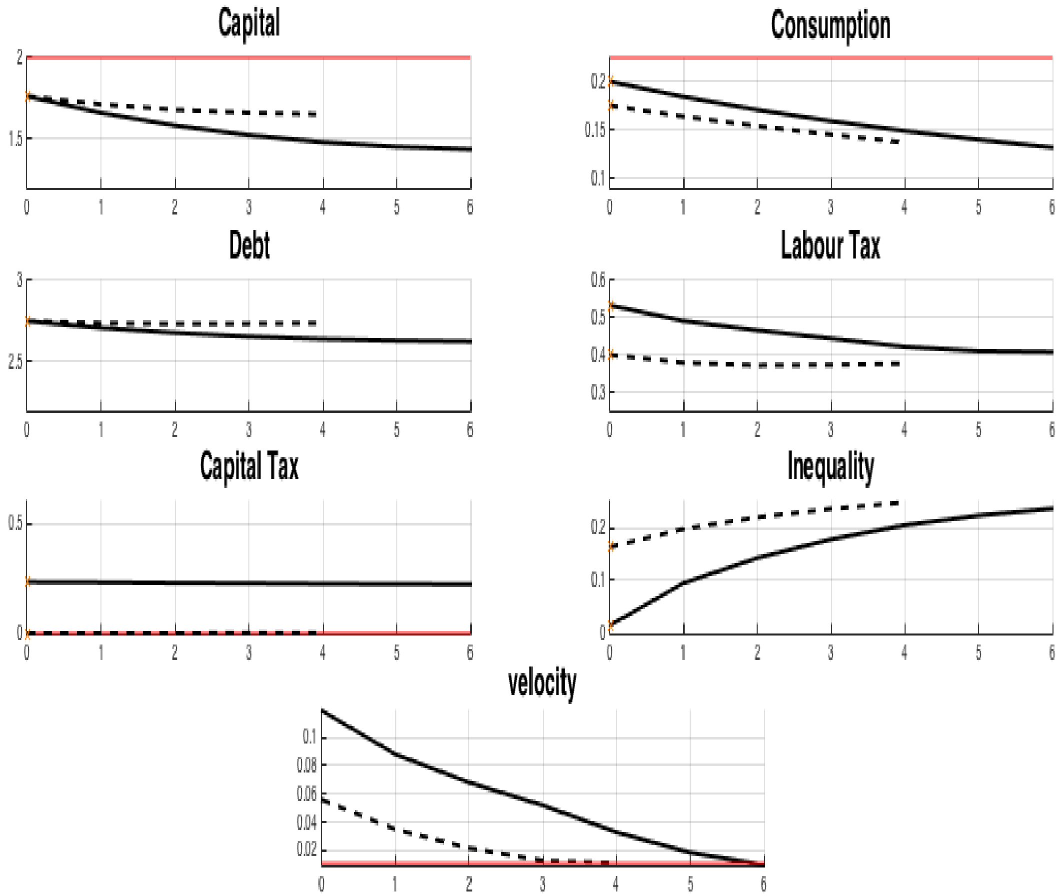

FIGURE 11. Time profiles of economies with higher labour tax than capital tax

This suggests that a country with high capital stocks can achieve low inequality with a labour tax higher than its capital tax. We find this to be represented in the world today, specifically in Scandinavian countries. By Piketty's calculations, some of the most equal countries are Denmark and the Netherlands, where the top 1% take 6.41% and 6.33%

of total income respectively.[7] Both of these countries have higher labour taxation rates (top rates of 60.2% and 52%) than capital tax rates.[8]

5.6. **Evolutions of Low Capital Economies.** Finally, a prevalent policy question is whether a country can grow its capital stock while achieving low inequality. Capital growth would seem to require low tax rates which in conjunction with low capital we have shown to not be conducive to low inequality. In Figure 12, we map the evolution of several viable points with low initial capital [9] and project them onto the capital-consumption axis of the kernel in Figure 5 with requirement $0 < \chi < 0.25$.

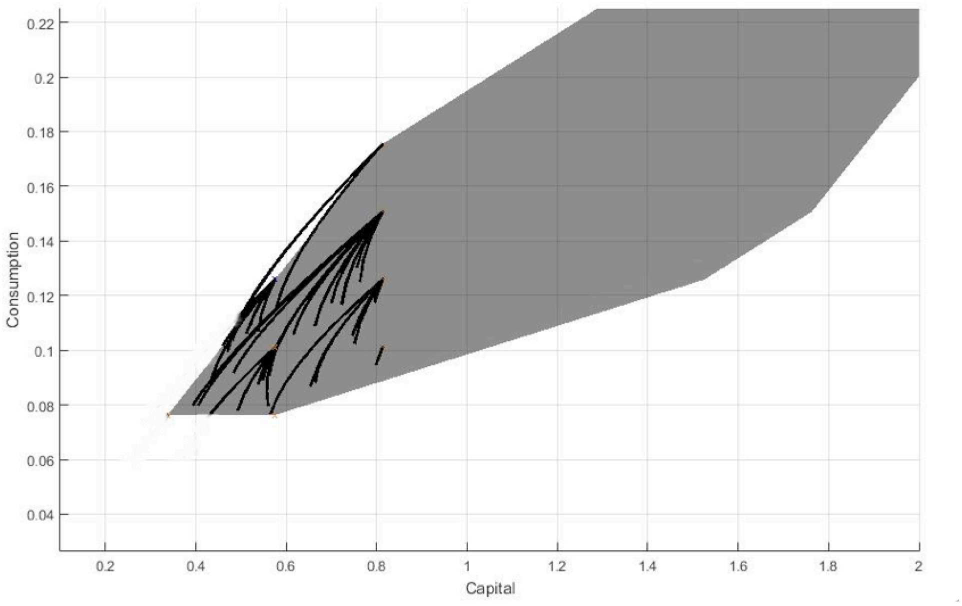

FIGURE 12. Evolutions of low capital points in low inequality

As can be seen in the figure, all viable states have evolutions that lead to a loss of capital stock. This suggests that a country that wants

[7]Compare to the USA where the top 1% took 17.85% of income in 2014 and South Africa where they took 16.68% in 2011

[8]Denmark taxes capital up to 42%, while Netherland's capital tax rate depends on varying factors but is always lower than 52%

[9]Defined as $k \in [0, 0.825]$.

to increase its capital stocks will have to suffer inequality above low target levels. As increasing capital requires lowering taxes, inequality is forced upward by the low capital state and low taxation.

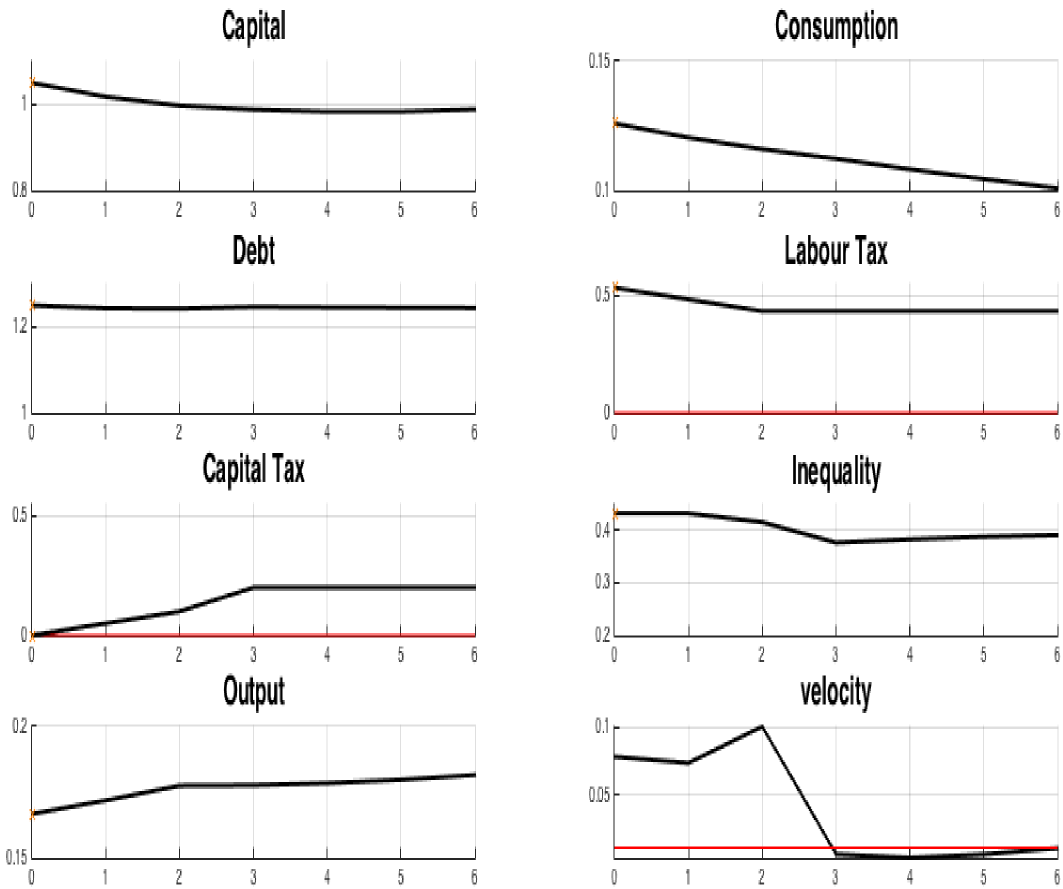

FIGURE 13. Evolutions of an economy with decreasing inequality and high capital stocks

The simulations in Figure 13 show however, that with increasing capital taxation, capital may diminish only slightly but inequality will decrease to below 0.4. From this moment, capital taxation can be kept at the same level (here 20 %) and, with decreasing labour tax, inequality can be maintained medium low. Also, a low labour tax releases more labour which helps maintain and build capital.

These simulations can be representative for a country that has had no capital tax and a medium debt level. By decreasing labour tax and increasing capital tax, the economy achieves low inequality and retains its higher levels of capital stocks.

Also, production actually increases in the reducing inequality, as from equation (9) we can see it increases in capital. This makes sense as by decreasing labour tax we increase the incentive to work which along with the high capital boosts production. It should be noted in this simulation that consumption has decreased. However, this should be only temporal as further labour income tax cuts should stimulate consumption.

5.7. **Kernel Conclusion.** From the kernels of a separated tax system, we can conclude the following.

- A separated tax system facilitates low inequality, even in low capital stocks by having a high capital tax rate and low labour tax rate.
- Secondly, this tax setup works for economies with high initial debt, as the high debt requires the high taxes in the first place.
- Thirdly, low inequality does not require that capital tax be higher than labour tax, but only if the economy starts from a point of high capital stocks.
- Finally, an economy with low capital stocks will not be able to increase its capital stock while meeting low inequality targets but can suffer high inequality to gain higher capital, then reduce inequality later.

Overall this tells us that a global tax on capital will, in most cases, be the most viable path to reducing inequality. This is in keeping with our belief that reducing income inequality can be best achieved by reducing the capital share of income.

6. Inequality Evolutions

6.1. **Stabilising Paths.** While the previous section found points from which our economy can be kept within K, the evolution of the points means that the initial levels of inequality will be different to those after the evolution.

While in some of the kernels we require that inequality be below a certain level, it is unclear whether this will occur naturally. To investigate this we plot the initial levels of inequality as well as the inequality levels after the evolution.

Figure 14 displays the levels of inequality given by the starting set of conditions from which we were able to stabilise the economy. The histogram shows that stabilisation favours low inequality points. This would seem to suggest that starting conditions that give high inequality, *i.e.,* points with high capital tax and low capital and consumption are unstable and likely to violate our bounds. So, decreasing inequality is more difficult to achieve than maintaining medium low inequality.

Figure 15 displays the levels of inequality after the starting points have been stabilised. The histogram shows a movement away from very low inequality toward medium levels of inequality, seen in the high concentration of χ in the range 0.25 to 0.35. High levels of inequality persist after the stabilisation, and in fact increase at χ around 0.7 to 0.8. These results back Pikkety's Laws of Capitalism, that an economy will tend toward an increasing capital income share and high inequality in the medium to long run.

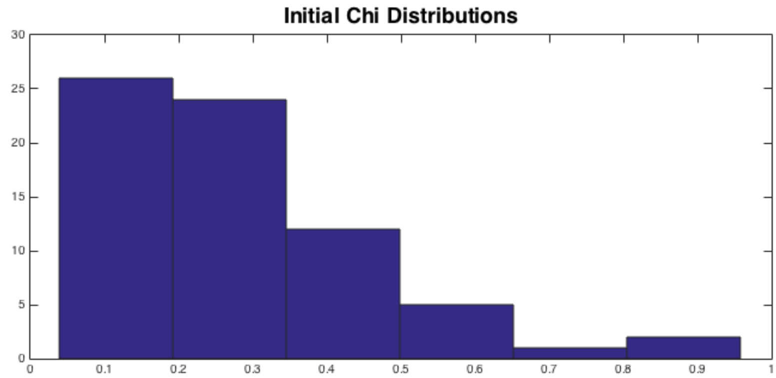

FIGURE 14. Initial χ distributions

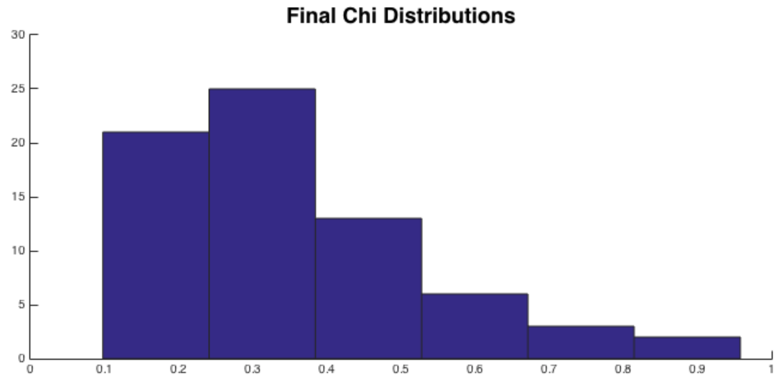

FIGURE 15. Final χ distributions

6.2. **Reducing a Country's Inequality.** In Section 5 we showed that low inequality was viable for various economic states, and that there are multiple taxation policies one can take to achieve that. The previous subsection showed inequality, when not controlled for, flowed from low levels to medium to high levels. We now ask whether trajectories exist that bring inequality from high levels to low levels, all while staying in our set K.

We see from Section 5 that the points with the highest inequality were at levels of low consumption and low capital. The low capital and consumption meant inequality was high, and our lower bounds on these inputs meant that higher taxation, which we showed reduces inequality,

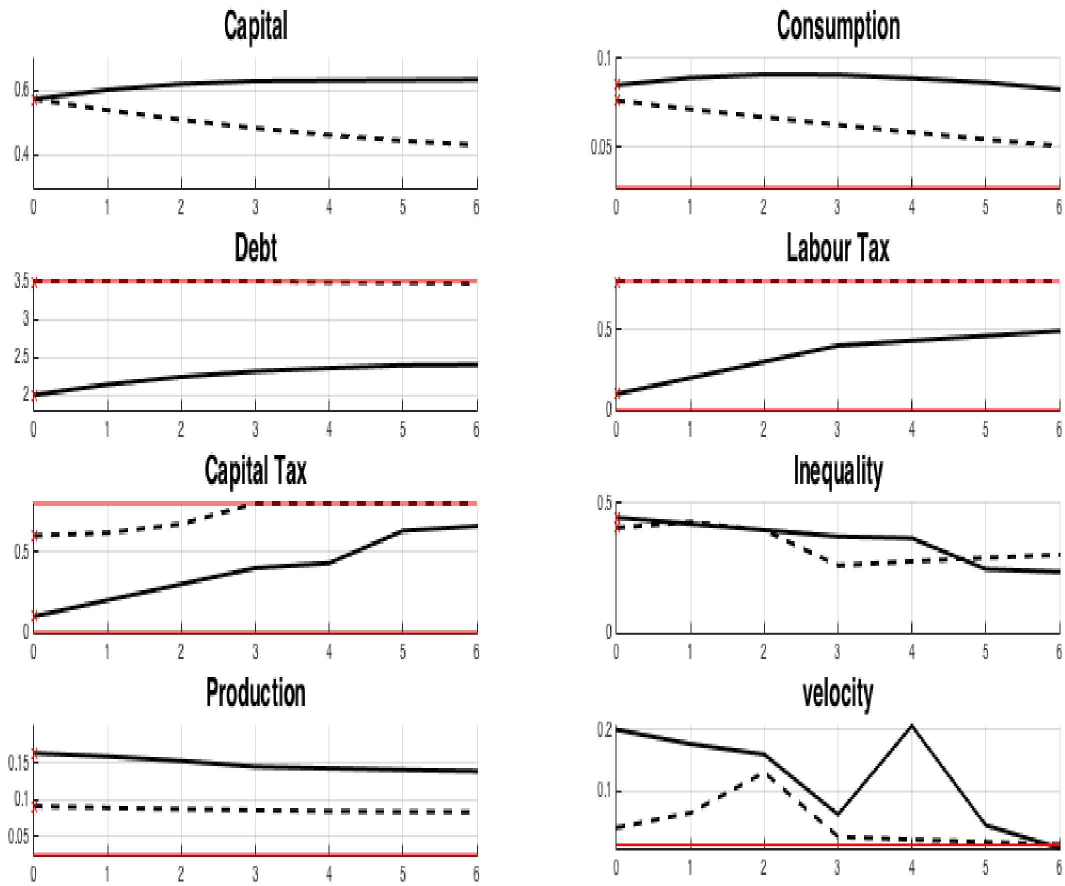

FIGURE 16. Evolutions of an economy from χ >0.4 to below 0.35

would mean violating these bounds. Hence it is from these input levels that we look to reduce inequality from. Figure 16 shows two such points. Both of these points have high initial inequality, with χ at around 0.4 - 0.45.

By slowly imposing increased capital taxation, both economies are able to reduce inequality and stabilise debt, capital and consumption. The slow implementation of the tax policies means that capital is able to stabilise as consumption is slightly discouraged. We also see Okun's law in motion as output falls in time with the decreasing inequality. However the reduction is small[10] when compared to the reduction in inequality, suggesting that Okun's tradeoff is less impactful if treated over a longer time period.

Overall this is in keeping with our previous findings, that a higher capital taxation rate will give lower χ, and that taxation is a viable policy for economic stabilisation and inequality reduction. While this will often decrease output and capital levels, slow implementation can mean that these factors can be revived or even increased.

7. Conclusion

We have analysed a variety of economic states with varying taxation strategies, collectively represented by the kernels $\mathcal{V}_F(K)$. These kernels showed that a two taxation policy yields more viable points than a combined taxation policy. While there is still a a reduction in viable points under increasing constraints on χ, it is of less magnitude than in the combined tax model.

This approach has yielded meaningful results, specifically that capital taxation is a powerful tool for decreasing inequality and therefore for controlling the equity of capital. Our kernel manipulation has shown that this strategy works in a variety of economic states, including in excessive debt. This strategy will follow Okun's tradeoff, that reducing inequality will lead to falling capital and therefore production, but following Piketty's reasoning, this is a small price to pay to prevent greater future inequality. Not withstanding the above, we were able to show which are the economic states from which only a moderate

[10]While Total Productivity Factor is kept constant.

output drop (with Total Factor Productivity as constant) is expected while all other variables of interest are kept in K, deemed containing *efficient* states.

The kernels did show however that once a country has reached high levels of capital, more taxation policies conducive to low inequality are available and the Okun's tradeoff between inequality and production ceases, but reaching that point will likely mean suffering high inequality during the transition.

Importantly, we have also shown that viability theory is a useful tool for not only understanding the validity of economic states and the tradeoffs that occur with different policies, but also as a way of understanding the reasons behind policy makers decisions. Denmark and the Netherlands have low inequality despite capital tax being lower than income tax, and the kernels suggest this is valid for high capital levels. Thus if New Zealand were to attempt to reduce inequality and improve living standards under Karacaoglu's stylised model, viability theory could prove to be a key tool in assisting that.

Clearly viability theory and its applications could prove to be invaluable to policy makers and economists in the future, and deserves attention in a wide array of economic literature.

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Appendix A. A method for finding viability kernels

VIKAASA[11] is a suite of MATLAB® programmes that approximate viability kernels. VIKAASA follows the approach suggested in Gaitsgory and Quincampoix (2009).

VIKAASA can be used either as a set of MATLAB® functions, or via a GUI.[12] The GUI can specify the viability problem, run the kernel approximation algorithms and display the results. A detailed (though somewhat outdated) manual for VIKAASA can be found in Krawczyk and Pharo (2011). The latest version of VIKAASA is available for download at Krawczyk and Pharo (2014b). In Figure 17, we show the main window of VIKAASA.

In this paper, our algorithm solves a truncated optimal stabilisation problem for each element of $K^h \subset K$, a discretisation of K. For each $x^h \in K^h$, VIKAASA assesses whether a dynamic evolution originating at x^h can be controlled to a (nearly) steady state without leaving the

[11]See Krawczyk and Pharo (2011) and Krawczyk and Pharo (2014b); also Krawczyk, Pharo, Serea, and Sinclair (2013).

[12]VIKAASA is also compatible with GNU Octave, though its GUI is not.

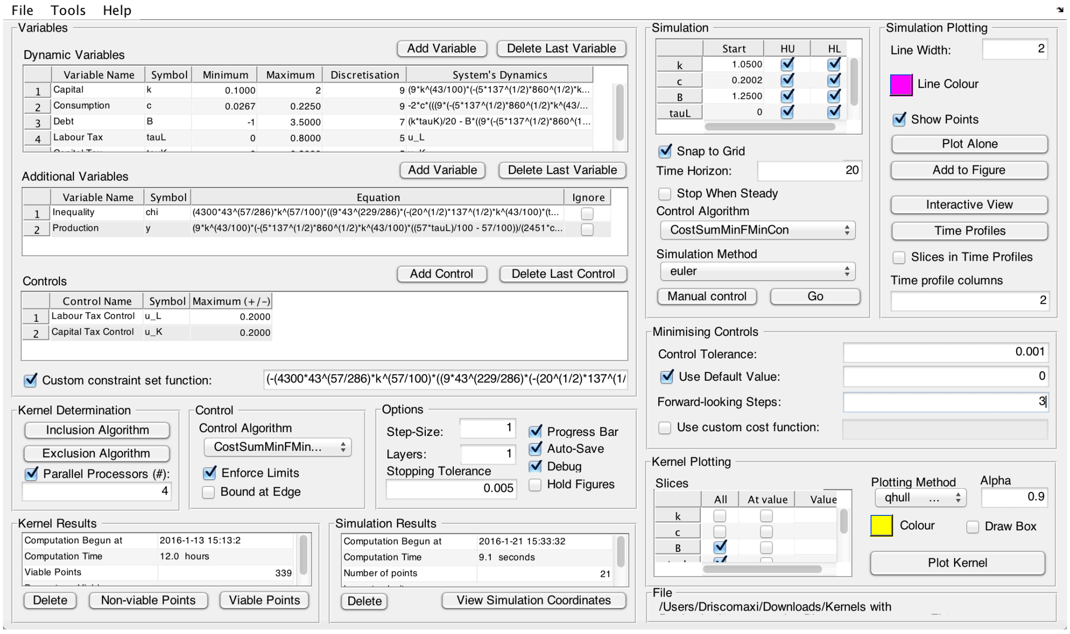

FIGURE 17. VIKAASA main window.

constraint set in finite time. Those points that can be brought close enough to such a state are included in the kernel while those that are not are excluded. This algorithm (called the *inclusion* algorithm, see Krawczyk et al (2013)) will miss viable points that cannot reach a steady state, such as those which form orbits.

APPENDIX B. INCREASING CONSUMPTION UPPER BOUND

In Krawczyk and Judd (2015), the calculation of the bounds on consumption were at 500% and 20% of the steady state. However that paper assumed δ to be 0, where we took it to be 0.05. As such, the steady state for consumption appears to be higher for this paper, signified by our kernel appearing to be cut by the constraint set K.

Hence an improvement to this model would increase the bound on consumption in order to find viable points that VIKAASA dismissed for having consumption slightly above our constraint. We have recalculated the kernels with a top constraint on consumption of 0.33 and displayed them below.

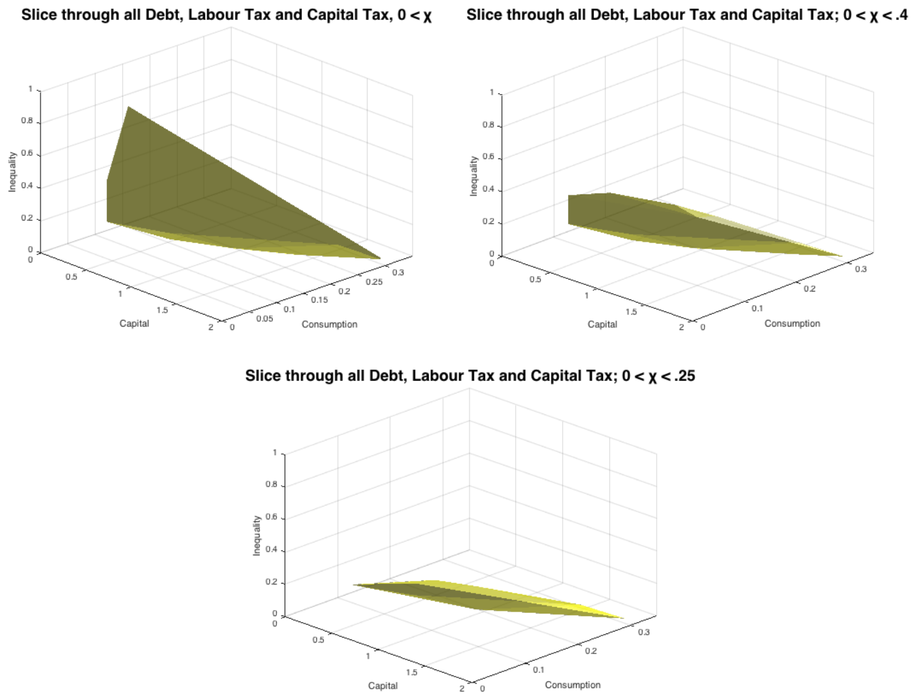

FIGURE 18. Viability kernels for different relative factor share constraints with increased consumption bound

A comparison between Figure 5 and Figure 18 reveals that the kernel shape is the same, but some new high levels of consumption and capital now exist. An extension of this work could involve looking at these higher consumption levels and comparing them to the combined tax model.

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