



Taxation and the User Cost of Capital: An Introduction

John Creedy and Norman Gemmell

WORKING PAPER 04/2015
March 2015

The Working Papers in Public Finance series is published by the Victoria Business School to disseminate initial research on public finance topics, from economists, accountants, finance, law and tax specialists, to a wider audience. Any opinions and views expressed in these papers are those of the author(s). They should not be attributed to Victoria University of Wellington or the sponsors of the Chair in Public Finance.

Further enquiries to:
The Administrator
Chair in Public Finance
Victoria University of Wellington
PO Box 600
Wellington 6041
New Zealand

Phone: +64-4-463-9656
Email: cpf-info@vuw.ac.nz

Papers in the series can be downloaded from the following website:
<http://www.victoria.ac.nz/cpf/working-papers>

Taxation and the User Cost of Capital: An Introduction*

John Creedy[†] and Norman Gemmell[‡]

Abstract

The aim of this paper is to provide an introduction to the concept of user cost and its determinants. Particular attention is given to the influence of taxation. The concept of user cost relates to the rental, the rate of return to capital, that arises in a profit maximising situation in which further investment in capital produces no additional profit. This paper sets out in some detail the range of assumptions involved in obtaining alternative expressions for the user cost. The user cost refers to a before-tax capital rental, the rate of return that ensures that the (after-tax) cost of capital is equal to the post-tax returns over its life. Hence, associated with the user cost measure is an effective marginal tax rate. This can differ substantially from the statutory marginal rate applicable to the investor. A related effective average tax rate is also defined.

*We should like to thank Martin Keene, Helen Miller and Florian Misch for their comments on an earlier draft.

[†]New Zealand Treasury and Victoria University of Wellington.

[‡]Victoria University of Wellington.

Executive Summary

The concept of *user cost* relates to the rate of return to capital, referred to as the rental, that arises in a profit maximising situation. This is one in which further investment in capital produces no additional profit. Despite this apparently simple statement, the concept gives rise to a complex range of cases which need to be distinguished.

The importance of taxation and the link with optimising behaviour by firms means that the user cost concept has a central role in investment and location decisions. Differences in tax regimes among countries can influence the decision regarding where to locate production and the head office of multinational firms. The relevant features of tax regimes relate not only to the treatment of companies but to the individuals who are the ultimate owners. Any change in a tax rate or tax structure which implies an increase in the user cost of capital implies that firms need to obtain a higher pre-tax rate of return or rental for an investment to be worthwhile.

The aim of this paper is to provide an introduction to the concept of user cost and its determinants, paying particular attention to the influence of taxation. This paper sets out in some detail, using a consistent terminology, the range of assumptions involved in obtaining alternative expressions for the user cost.

The user cost refers to a before-tax capital rental, the rate of return that ensures that the (after-tax) cost of capital is equal to the post-tax returns. Hence, associated with the user cost measure is an effective marginal tax rate. This can differ substantially from the statutory marginal rate applicable to the investor. Particular attention is given in this paper to the properties of the effective marginal tax rate in different circumstances, drawing attention to the difference between tax-inclusive and exclusive rates. It is shown that the relationship between the statutory tax rate and the effective tax rate can vary substantially, depending on the rate of interest.

A related effective average tax rate is also defined for the context in which the firm obtains economic rents (that is, earnings above those needed for it to remain in its present position). This may be important in the context of multinational investment where the firm is operating below its profit maximising output.

The link between the user cost, effective tax rates and investment is also briefly discussed.

1 Introduction

The aim of this paper is to provide an introduction to the concept of *user cost* and its determinants, paying particular attention to the influence of taxation. In the standard neoclassical model of production, the term ‘cost’ generally refers to an opportunity cost rather than simply a direct pecuniary cost of a good or service. In the present context, user cost relates to the *rental*, the rate of return to capital, that arises in a profit maximising situation in which further investment in capital produces no additional profit. This paper hopes to make this rather loose statement more precise and clear by setting out in some detail the range of assumptions involved.¹

To provide a little more context at this preliminary stage, consider a firm’s decision to increase its investment in a capital asset. This decision depends on a complex range of factors, including the cost of financing the investment and their tax treatment. Suppose, for simplicity, that a single type of capital good is used in production. Assuming that capital can be varied continuously, a basic implication of profit maximisation is that the firm increases its investment until its total cost is equal to the present value of the after-tax and depreciation returns from the flow of capital services, discounted at a suitable rate over the life of the project. With an assumption of decreasing marginal returns, capital is increased until the condition is satisfied. It is not necessary to suppose that the firm actually owns the capital good, the firm may be considered to rent the corresponding capital services and for this reason the return is referred to as a rental, for comparison with a wage rate.

Once this profit maximising position is achieved, a further increase in the use of capital services produces zero profit. The marginal revenue product at that point determines the capital rental. It is the before-tax rate of return, the capital rental at the profit maximising level of investment, that defines the user cost. This concept refers to the rate of return that ensures that the (after-tax) cost of capital is equal to the after-tax return. Hence, associated with the user cost measure is an effective marginal tax rate. As shown below, the effective marginal tax rate is typically not equal to the statutory marginal rate applicable to the investor: they are equal only under special conditions.

¹This paper is not a literature review, so only selected references are made. The seminal paper is Hall and Jorgensen (1967). See also Auerbach (1983, 2002), King and Fullerton (1984), Bengtsson (1997, 1998) and Fabling *et al.* (2013).

Additional important distinctions, other than between before- and after-tax values, must also be made. The existence of inflation means that in practice a distinction must be made between real and nominal values. The existence of depreciation means that a distinction must be made between gross and net values. The simple statement above must therefore be modified. The *gross user cost* is defined as the before-tax and before-depreciation real capital rental, obtained from a marginal investment which must be earned if the after-tax real rate of return (the rental adjusted for taxation, depreciation and capital gains or losses) is equated to the cost of capital. The latter cost is given by the after-tax real rate of interest. The *net user cost* is the gross user cost adjusted for depreciation. As shown below, care is needed to disentangle the various components in view of the complexity of tax structures.

The importance of taxation and the link with optimising behaviour by firms means that the user cost concept has a central role in investment and location decisions. Differences in tax regimes among countries can influence the decision regarding where to locate production and the head office of multinational firms. The relevant features of tax regimes relate not only to the treatment of companies but to the individuals who are the ultimate owners.² Any change in a tax rate or tax structure which implies an increase in the user cost of capital implies that firms need to obtain a higher pre-tax rate of return or rental for an investment to be worthwhile. An understanding of precisely how taxation can affect the user cost in different circumstances is needed in order to appreciate the incentives facing firms. Although the concept is central in the neoclassical analysis of firms, and has important policy implications, its treatment is typically given very little attention in economics texts, despite the extensive and often technical literature in which it features. Therefore, the present introductory review seems warranted.

Discussions of user cost are often not easy for the newcomer to follow. There appears to be no settled terminology, let alone notation, and even the concept itself is often described merely ‘in passing’ and is given various definitions when put into words rather than equations, which may appear confusing to the uninitiated. The assumptions behind its use are often not clear. Some authors even use the term ‘cost

²The situation is considerably complicated if the firm is not taxed as a separate entity, and is owned by a number of individuals who may themselves face different marginal tax rates, as well as different borrowing rates of interest. With such a diversity, there may not necessarily be unanimity regarding the desired level of investment.

of capital' when referring to user cost, while others use the term 'cost of capital' (as above) to refer to an appropriate rate of interest. Other authors refer to 'gross user cost' when what they have in mind is a before-tax, but after depreciation, concept.

The present paper therefore attempts to provide a synthesis using a consistent terminology. It adopts the standard in the national income accounting literature and uses the terms 'gross' and 'net' to refer only to before- and after-depreciation quantities respectively, and reserves the terms before- and after-tax (rather than pre- and post-tax, or gross and net of tax) to refer respectively to quantities before and after any tax is imposed. Some authors adopt without comment a widely-used approximation in discrete-time contexts in which a real rate of return is expressed simply as a nominal return minus an inflation rate, whereas other authors use the more precise Fisher expression. The latter relationship is used here. Discussions are complicated by the considerable complexity of corporate (and to a lesser extent personal) tax regimes, involving the different treatment of forms of financing for corporations and individuals (in their capacity as investors). This leads to a wide and often confusing range of cases, following a taxonomy that is seldom fully articulated. The approach adopted here, as befitting an introduction, is to concentrate on the relatively simpler cases rather than providing an exhaustive catalogue.

Section 2 introduces the concept of user cost by considering the simplest case of profit maximisation and investment in the absence of taxation. Section 3 extends the derivation to include a simple treatment of taxation. Section 4 examines the after-tax real interest rate. Section 5 considers the effective marginal tax rate. The question of whether the user cost concept (necessarily associated with a marginal change) can be extended to define an effective average tax rate is examined in Section 6. Section 7 turns briefly to the relationship between investment, the desired capital stock and the user cost. Conclusions are in Section 8

2 User Cost: The Simplest Case

The concept of user cost, expressed somewhat imprecisely in section 1, can perhaps most easily be understood by considering the simplest case where there is no taxation, no inflation, and the capital good is not subject to depreciation. Furthermore, there are no capital gains or losses to be made at the end of the project. In a discrete-time

framework, consider an increase in capital in a period, where a single type of capital is used in production and other inputs are held constant.³ This takes the form of an investment in capital equipment in one period which is then reversed in the next period (the equipment is sold, with neither a capital gain nor a loss).⁴ If the investment is financed from existing assets (and is thus ‘equity financed’), there is an opportunity cost of investment which depends on the return available from the next-best alternative use. This opportunity cost is referred to as the ‘cost of funds’. If the investment is financed by borrowing (and is thus ‘debt financed’), there is literally a cost of funds reflected in the relevant interest rate paid.⁵

If the return arising from the investment exceeds the cost of funds, there is an incentive to invest in more capital. On the assumption that there are decreasing marginal returns to capital, the return falls as the level of investment increases so that eventually the return is expected to equal the cost of funds. At the profit-maximising situation, the value of an additional unit of investment (costing, say, one dollar) is equal to its cost (equal to the rate of interest) so that additional profits are zero. In the context of marginal productivity theory, the effective capital rental (the equivalent of the wage rate applying to labour inputs) is equal to the marginal revenue product of capital (marginal revenue multiplied by the marginal physical product). It is this capital rental, associated with the profit maximising position, that is called the *user cost* of capital.⁶ *The user cost is thus equal to the rate of interest.*⁷ The idea that user cost relates to a profit-maximising *equilibrium* capital rental is crucial.

³The seminal treatment of user cost was by Hall and Jorgenson (1967), who used a continuous-time framework. Their explanation of the basic result was extremely terse and for this reason further details are given in Appendix A below.

⁴The assumption that there is no uncertainty and the project is reversible – the capital good can be sold at the constant market price – means there is no question regarding the optimal timing of investment. Hence an option value of waiting is not relevant here. A treatment of uncertainty in the simple case considered in this section is contained in Appendix B.

⁵The implications of losses during some periods of a low output price, where there are tax asymmetries in the corporate and personal tax regimes, are also ignored. A pioneering approach to measuring firm-specific marginal tax rates, by Graham (1996), used simplified assumptions about the use of losses and simulation. See also Ramb (2007), Blouin *et al.* (2008). Dwenger and Walch (2011) accounted for losses in looking at the user cost elasticity of investment.

⁶Alternatively, it may be said that the user cost is the minimum pre-tax rate of return (implicit rental) that a project must generate in order to be profitable.

⁷The user cost has the same units as an interest rate. In some treatments the user cost appears to be *defined* in terms of the rate of interest, but this equality is clearly a consequence rather than the definition itself.

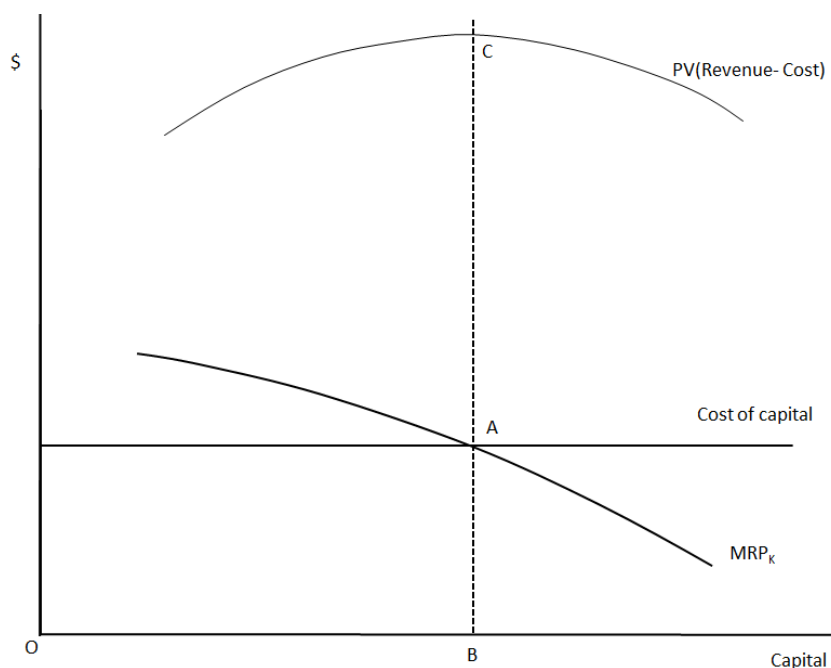


Figure 1: The User Cost: No Taxation

The components are illustrated in Figure 1, which represents the situation for a neoclassical firm with a standard production function involving inputs of capital services. Labour and other inputs are assumed to be fixed. The horizontal axis measures capital. The curve marked $PV(\text{Revenue}-\text{Cost})$ plots the variation in the present value of revenue, net of costs, as the amount of capital used in production is increased. Hence point C, where the curve reaches a turning point, corresponds to the profit-maximising input of capital, B: this is considered to be the desired total capital. At the profit maximising position, a further marginal increase in capital input (a ‘marginal investment project’ of, say, one dollar) necessarily produces no additional profit.

The user cost, c , is the marginal revenue product of capital at the profit maximising position. If the price of the output is assumed to be constant (that is, it is sold in a competitive market), price and marginal revenue are equal and it is possible to normalise the output price to unity, so that c varies just as the marginal physical product varies. In this special context of a small investment in period t which is subsequently reversed, the cost of the project (which is the opportunity cost of the

price of the extra capital) is effectively the interest, r , on the one dollar of investment for one period. The ability to talk in terms simply of a ‘dollar of investment’ arises from an assumption that the supply curve of capital is horizontal.

The equality of the *user cost* of capital, c , with the rate of interest – the *opportunity cost* of capital – is reflected in Figure 1 by the point A, where the marginal revenue product curve, marked MRP_K , intersects the horizontal line reflecting the cost of capital. It is shown below that this cost of capital needs to be modified, effectively shifted upwards in the figure, by the existence of inflation and particularly taxation which affects the real after-tax rate of interest. However, taxation does not affect the MRP_K curve, on the assumption that it does not affect prices. It is shown below how this type of diagram allows marginal and average effective tax rates to be defined in terms of the user cost.

Consider the addition of economic depreciation, at the real geometric rate of δ per period.⁸ In terms of the marginal investment of \$1, considered above, the profit is equal to the gross rental income, c_g , less depreciation over the period, or $c_g - \delta$. The cost of funds is, as before, the real rate of interest available in the market, r . Hence $r = c_g - \delta$ and:

$$c_g = r + \delta \tag{1}$$

This rental, c_g , is the gross user cost of capital.⁹ If the rental is considered to include the cost of depreciation, then the corresponding concept is that of net user cost, c_n . Clearly, $c_n = c_g - \delta$ and $c_n = r$.

The following sections introduce the complications arising from taxation. In view of the many terms introduced, a list of the main variables is provided in Table 1.

3 Allowing for Taxation

The effect of taxation on the user cost is complicated by the fact that it depends both on the investor and the method of finance. The investor may be a corporation or a person, and each case may be subject to the domestic tax regime or may be a foreigner. Under an imputation system, corporation tax is effectively a withholding tax, so that

⁸This involves the use of an effective discount rate of $r + \delta$ instead of r .

⁹Any real capital gain over the period can simply be subtracted from the right hand side of (1), and is thus ignored here.

Table 1: List of Variables Used

Variable	Symbol	
Inflation rate (CPI)	π	
Corporate tax rate	τ	
Tax rate facing domestic investor	m	
Economic depreciation rate	δ	
Fiscal depreciation allowance	δ'	
Nominal interest rate	i	
After-tax nominal interest rate	i^*	$= i(1 - \tau)$
Before-tax real interest rate	r	$= (i - \pi) / (1 + \pi)$
After-tax real interest rate	r^*	$= (i^* - \pi) / (1 + \pi)$
Proportional tax credit	k	
PV of depreciation allowance	Z	$= \delta' / (i + \delta')$
World real interest rate	r_W	
Real rate of return on equity	r_E	
User cost (gross)	c_g	
User cost (net of depreciation)	c_n	$= c - \delta$
Effective marginal (tax exclusive) tax rate	τ^*	

the appropriate tax rate is the marginal rate, say m , faced by an individual investor. If the investment is financed by debt, the interest is generally tax-deductible at the corporate level only. However, equity finance is not eligible for a tax deduction. These and other distinctions give rise to a complex and extensive taxonomy of cases, only a limited number of which are examined here. This section considers the user cost in the context of corporations rather than individuals.

3.1 Taxation of Corporations

The existence of tax credits and fiscal depreciation allowances implies that the cost of a \$1 unit of capital is effectively reduced by an amount, ξ , to $\$(1 - \xi)$. Suppose the statutory marginal corporate tax rate applied to taxable income is τ . The relevant interest rate is now the after-tax real rate, denoted r^* . The equilibrium condition defining the user cost states that the after-tax cost of capital associated with the effective investment of $\$(1 - \xi)$ is equal to the after-tax rate of return. The latter is the after-tax rental, $c_g(1 - \tau)$, arising from the real before-tax gross user cost, c_g ,

minus the depreciation of $\delta(1 - \xi)$. Hence:¹⁰

$$r^*(1 - \xi) = c_g(1 - \tau) - \delta(1 - \xi) \quad (2)$$

and the gross user cost becomes:¹¹

$$c_g = \frac{(r^* + \delta)(1 - \xi)}{1 - \tau} \quad (3)$$

As above, the net user cost, c_n , is obtained simply by subtracting δ from c_g . Separate expressions for c_n are therefore not given below unless they are specifically required.

3.2 Depreciation and Tax Credits

One component of the term, ξ , is a fiscal depreciation allowance at the (geometric) rate, δ' .¹² The question therefore arises of how to value such an allowance. The approach here assumes that only one capital good is used. However, where several capital goods are involved, the fiscal depreciation rate may differ, depending on the type of capital. At the end of the first year, for the \$1 investment, this allowance is simply δ' which is worth $\tau\delta'$. At the end of the second year, the allowance of δ' applies to $(1 - \delta')$ and is worth $\tau\delta'(1 - \delta')$. At the end of the third year depreciation is $\delta'(1 - \delta')^2$, and so on. The present value of the allowance, say Z , discounted at the nominal interest rate, i , is given by:

$$Z = \frac{\delta'}{1 + i} \left\{ 1 + \left(\frac{1 - \delta'}{1 + i} \right) + \left(\frac{1 - \delta'}{1 + i} \right)^2 + \dots \text{etc} \right\} \quad (4)$$

¹⁰If capital gains were non-zero, the appropriate value must be their after-tax value. The above analysis also ignores local property taxes.

¹¹This result corresponds to the original statement by Hall and Jorgenson (1967, p. 393), which is given as $c = q(r + \delta)(1 - k)(1 - uz) / (1 - u)$. Their u is the tax rate, while above q has been normalised to 1. Their r is the real after-tax rate of interest, corresponding to r^* above, and while z is the same as defined in the following subsection, their term $(1 - k)(1 - uz)$ corresponds to $(1 - \xi)$ above (their investment tax credit is specified slightly differently from that in the following subsection). Their z corresponds to Z below.

¹²Downs (1986) modifies the Hall and Jorgenson (1967) result to allow for non-geometric depreciation. He writes the basic Hall and Jorgenson formula as (ignoring the time argument): $c = q(r + \delta - \pi)(1 - \nu - \tau Z) / (1 - \tau)$ where, as above, τ is the tax rate and π is the inflation rate. But his r is the nominal interest rate as he is using the approximation $r - \pi$ for the real rate. His ν is the effective rate of the investment tax credit. Downs shows that, if $h(t)$ is the 'percentage of the asset's original productive capacity lost at the $[t]$ th moment after acquisition', and (using present notation for the real rate of interest) defining $H = \int_0^\infty e^{-r^*t} h(t) dt$, the user cost can be written as (in present notation): $c = r^*(1 - \xi) / \{(1 - H)(1 - \tau)\}$. This allows for any non-geometric form of fiscal depreciation. For geometric depreciation, $h(t) = \delta e^{-dt}$, $H = \delta / (r^* + \delta)$, giving the Hall and Jorgenson result.

Using the standard result for the sum of an infinite geometric progression, $1 + a + a^2 + \dots = 1/(1 - a)$, then:

$$Z = \frac{\delta'}{1+i} \left\{ \frac{1}{1 - \frac{1-\delta'}{1+i}} \right\} = \frac{\delta'}{i + \delta'} \quad (5)$$

A second component of ξ may be a tax credit. This can take several forms, but suppose it is equal to a proportion, k , of the investment (of \$1).¹³ With a tax rate of τ , this is worth τk , so that:

$$\xi = \tau (k + Z) \quad (6)$$

Substituting into (3), the gross user cost is:

$$c_g = (r^* + \delta) \{1 - \tau (k + Z)\} \frac{1}{1 - \tau} \quad (7)$$

with $c_n = c_g - \delta$.

3.3 User Cost in Terms of the After-Tax Nominal Interest Rate

The real after-tax rate of interest, r^* , can be expressed in terms of the nominal after-tax interest rate, i^* . As defined above, the inflation rate is π , and Fisher's equation gives the relationship between nominal and real interest rates as:¹⁴

$$(1 + r^*) (1 + \pi) = 1 + i^* \quad (8)$$

so that:

$$r^* = \frac{i^* - \pi}{1 + \pi} \quad (9)$$

Substitution into (7) gives:

$$c_g = \left(\frac{i^* - \pi}{1 + \pi} + \delta \right) \{1 - \tau (k + Z)\} \frac{1}{1 - \tau} \quad (10)$$

¹³This is the tax allowance value of an investment tax credit. In New Zealand this is captured by 'depreciation loadings', expressed as a percentage, so that k would be replaced by a proportion of Z .

¹⁴As mentioned in the introduction, some authors use the approximation $r^* + \pi = i^*$.

3.4 User Cost in Terms of Before-Tax Real Interest Rate

The user cost can also be related to the before-tax real interest rate. First, the relationship between nominal and real before-tax rates is:

$$(1 + r)(1 + \pi) = 1 + i \quad (11)$$

so that:

$$i = r(1 + \pi) + \pi \quad (12)$$

Using $i^* = i(1 - \tau)$, and substituting into (9), the real after-tax rate of interest is given in terms of i , π and τ by:

$$r^* = \frac{i(1 - \tau) - \pi}{1 + \pi} \quad (13)$$

Finally, substitute (12) into (13) to get:

$$r^* = r(1 - \tau) - \tau \left(\frac{\pi}{1 + \pi} \right) \quad (14)$$

Substituting this expression for r^* into (7) gives:

$$c_g = \left\{ r(1 - \tau) - \tau \left(\frac{\pi}{1 + \pi} \right) + \delta \right\} \{1 - \tau(k + Z)\} \frac{1}{1 - \tau} \quad (15)$$

3.5 User Cost in Terms of Before-Tax Nominal Interest Rate

The user cost of capital in terms of the nominal before-tax interest rate, i , is obtained by substituting for r^* using (13), and for Z using (5), into (7):¹⁵

$$c_g = \left\{ \frac{i(1 - \tau) - \pi}{1 + \pi} + \delta \right\} \left\{ 1 - \tau \left(k + \frac{\delta'}{i + \delta'} \right) \right\} \frac{1}{1 - \tau} \quad (16)$$

Writing $\eta_{a,b}$ to indicate the elasticity of a with respect to changes in b , the following results can be obtained.

$$\eta_{c_g, \pi} = -\frac{\pi}{1 + \pi} \left[\frac{1 + i(1 - \tau)}{\delta(1 + \pi) - \pi + i(1 - \tau)} \right] \quad (17)$$

$$\eta_{c_g, \tau} = -\tau \left[\frac{k + Z}{1 - \tau(k + Z)} + \frac{i}{\delta(1 + \pi) - \pi + i(1 - \tau)} - \frac{1}{1 - \tau} \right] \quad (18)$$

$$\eta_{c_g, i} = i \left[\left\{ i + \frac{\delta(1 + \pi) - \pi}{1 - \tau} \right\}^{-1} + \frac{1}{1 + \delta'} \left\{ \frac{(1 + \delta')(1 - \tau)}{\delta'} - \tau \right\}^{-1} \right] \quad (19)$$

¹⁵This is equivalent to the result in Bengtsson (1997, p. 11), with the rate facing an individual investor, m , replacing the corporate tax rate, τ .

4 The After-Tax Real Interest Rate

In the previous section it was mentioned that distinctions can be drawn between debt and equity financing and the location and identity of the marginal investor. These distinctions can be viewed in terms of the determination of the appropriate value of r^* , the cost of funds. Alternatives are briefly examined in this section.

4.1 Foreign Investors

For foreign-sourced equity funds, the investor is not usually subject to domestic taxation on the real rate of return on equity, r_E .¹⁶ Hence:

$$r^* = r_E \quad (20)$$

Alternatively, suppose foreign-source debt finance is available at a world real interest rate of r_W . Then:

$$r = r_W \quad (21)$$

Nominal interest rate expenses are tax deductible, so the required real after-tax rate of return is lower than r_W . Substituting for $r = r_W$ in equation (14) gives the required after-tax rate of return with foreign debt financing.

4.2 Domestic Residents and Imputation

In an imputation system, the appropriate tax rate depends on the personal tax status of the individual investor. Imputation is now less common than formerly, but applies for example in New Zealand and Australia. If all profits are distributed as dividends, with associated imputation credits, the corporate tax acts as a withholding tax only. Whether the investment is debt or equity financed, the appropriate rate is thus the effective rate on the investor's investment income, say m . Hence, m simply replaces τ in the above expressions.¹⁷ Even without imputation, it may also be argued that, to the extent that investment decisions of firms depend on their marginal investors, the relevant rate is the (marginal) investor's effective rate. Nevertheless analyses of

¹⁶This assumes the existence of double-tax agreements.

¹⁷However, in practice it is not clear whether the top marginal personal income tax rate or the trust rate, or some other rate is appropriate for the investor.

non-imputation regimes often ignore personal-level taxation and concentrate on the corporate rate.

5 The Effective Marginal Tax Rate

In general, the effective marginal tax rate can be defined in terms of the proportional difference between relevant before- and after-tax rates of interest. Define \tilde{p} as the required equilibrium pre-tax rate of return that is necessary to produce a post-tax rate of return of r^* . Denote the tax-inclusive effective rate (the rate applied to the return that includes the tax paid) by $EMTR_I$ and the equal-revenue tax-exclusive rate (the rate applied to the return that excludes the tax paid) by $EMTR_E$. These are marginal rates since the context is of a marginal investment. Thus:

$$r^* = \tilde{p} - \tilde{p}(EMTR_I) \quad (22)$$

so that:

$$EMTR_I = 1 - \frac{r^*}{\tilde{p}} \quad (23)$$

Furthermore:

$$r^* = \tilde{p} - r^*(EMTR_E) \quad (24)$$

giving:

$$EMTR_E = \frac{\tilde{p}}{r^*} - 1 \quad (25)$$

The inclusive and exclusive rates are therefore related by $EMTR_E = EMTR_I / (1 - EMTR_I)$ and $EMTR_I = EMTR_E / (1 + EMTR_E)$.¹⁸ The definitions given here make no reference to the user cost. The following subsection shows how the effective marginal rate is related to the user cost concept. In what follows, any reference to the effective marginal rate is to the tax-inclusive rate, as in (23); this is the rate that compares most closely with statutory rates such as the corporate rate, τ , and the personal income tax rate, m .

¹⁸In the case of a goods and services tax imposed at the tax-exclusive rate of τ , the pre- and post-tax goods prices, p_0 and p_1 , are related by $p_1 = p_0(1 + \tau)$. In this case, the tax-inclusive tax rate is $\tau / (1 + \tau)$.

5.1 The User Cost and the EMTR

The direct link between the EMTR and the concept of user cost is provided by considering the equilibrium condition that defines the user cost.¹⁹ As defined earlier, the user cost net of depreciation, c_n , is the before-tax rental which ensures that the after-tax and depreciation return from the marginal investment is equal to the after-tax real rate of return, r^* . Hence c_n can be interpreted as being equivalent to the real before-tax rate of return, \tilde{p} . Hence from (23):

$$EMTR_I = 1 - \frac{r^*}{c_n} \quad (26)$$

The connection between the user cost and the effective marginal tax rate is illustrated in Figure 2. As in Figure 1, the profit maximising position in the absence of taxation is at point A: in that case the user cost is equal to the real rate of interest. But in the presence of taxation, which is assumed to be fully shifted, the firm now needs a before-tax real rate of interest, \tilde{p} , that generates an after-tax real rate equal to r^* , which in turn is equal to the cost of capital in the absence of taxation.

The ‘tax wedge’ between r^* and \tilde{p} , represented by the height $CD = \tilde{p} - r^*$, determines – along with the shape of the MRP_K curve – the desired amount of capital, OE, in the presence of taxation. Given the downward sloping nature of MRP_K , a larger tax wedge reduces the desired capital further below the profit maximising position that would arise in the absence of taxation.²⁰

The net user cost was defined earlier as the before-tax and after-depreciation capital rental, c_n , which ensures that the after-tax and depreciation return from the marginal investment is equal to the after-tax real rate of return, r^* . Hence the net user cost is represented in Figure 2 by the height $EC = c_n = \tilde{p}$. For a marginal investment, starting from point E, the tax paid is the height CD, which is $c_n - r^*$. The effective marginal rate is thus CD divided by the tax base, where the latter is either the length DE or CE depending on whether the tax-exclusive or inclusive rate is required.

If the tax structure is proportional, as assumed here, then the effective average tax-inclusive rate, given by the area $\tilde{p}CDr^*$ divided by the area $\tilde{p}CEO$, is clearly equal

¹⁹On effective marginal tax rates, see King and Fullerton (1984), McKenzie and Mintz (1992), McKenzie *et al.* (1997), Egger *et al.* (2009) and Fabling *et al.* (2013).

²⁰The marginal revenue product curve can, in its earlier stages, slope upwards but only the downward sloping range is relevant.

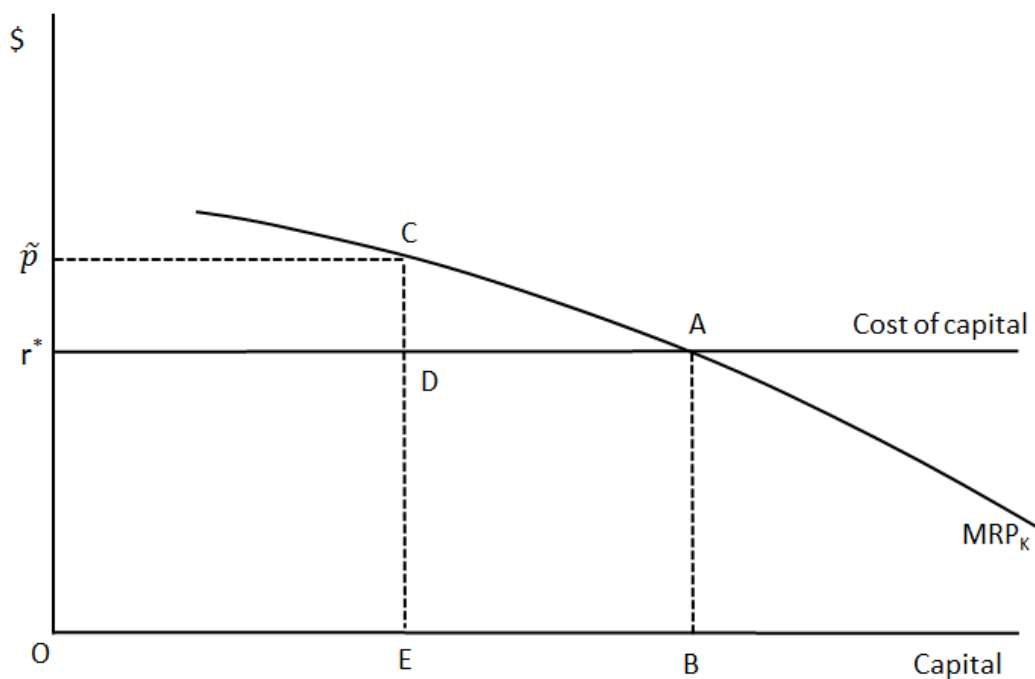


Figure 2: User Cost and The Effective Marginal Tax Rate

to the effective marginal tax-inclusive rate, CD/CE . However, in a different situation in which the use of capital does not correspond to the profit-maximising equilibrium, and where economic rents are thereby obtained, it has been suggested that a slightly different definition of effective tax rates is useful, for which the effective average tax rate does not equal the effective marginal rate.²¹ As is often the case in this literature, care is needed using terms: there is an important distinction between ‘*economic rent*’ and ‘*capital rental*’. The analysis is extended in section 6 to deal with the average effective tax rate in cases where rents are obtained, but first it is useful to consider the properties of the marginal rate in more detail.

²¹However, a desirable property is that as the economic rent tends to zero, the average rate tends towards the marginal rate.

5.2 A Formal Statement

To express the effective marginal rate in terms of the various components used above, first use (7) and $c_n = c_g - \delta$ to get the user cost net of depreciation:

$$c_n = \frac{(r^* + \delta)(1 - \xi) - \delta(1 - \tau)}{1 - \tau} \quad (27)$$

Second, substituting for $\tilde{p} = c_n$ into (23) gives the tax-inclusive effective marginal tax rate as:

$$EMTR_I = 1 - \frac{r^*(1 - \tau)}{(r^* + \delta)(1 - \xi) - \delta(1 - \tau)} \quad (28)$$

In view of the fact that r^* can be negative, the effective marginal tax rate can exceed 1, just as it can exceed 100% for individuals who are subject to the means-testing of benefits in addition to income taxation.

Defining the tax component, $T = (1 - \xi) / (1 - \tau)$, (28) can be expressed as:²²

$$EMTR_I = 1 - \frac{r^*}{(r^* + \delta)T - \delta} \quad (29)$$

or:

$$EMTR_I = \frac{T - 1}{T - \frac{\delta}{r^* + \delta}} \quad (30)$$

Another way to write (29) is:

$$EMTR_I = \tau + \frac{\left(\frac{\delta}{r^* + \delta}\right)\tau - \xi}{\left(\frac{1 - \xi}{1 - \tau}\right) - \left(\frac{\delta}{r^* + \delta}\right)} \quad (31)$$

Which indicates how the effective rate differs from the statutory rate. When $\delta = 0$ and $k = 0$, so that $\xi = 0$, the effective marginal rate is equal to the statutory rate, τ . For domestic shareholder-level taxation, the corporate marginal rate is replaced by the appropriate shareholder rate, m .

The effective marginal rate can be zero under a number of circumstances. From (29), $EMTR_I = 0$ when $r^* = -\delta$, which requires the real after-tax rate of interest to be negative. Substituting for r^* from (13) shows that this requires the nominal interest rate to equal $\{\pi - (1 + \pi)\delta\} / (1 - \tau)$. The lower bound for the nominal rate is zero, so that an $EMTR_I$ of zero also requires $\delta < \pi / (1 + \pi)$. Alternatively (29) shows that

²²Using this definition of T , it can be seen that $c_n = (r^* + \delta)T - \delta$.

the marginal tax rate can be zero if $T = 1$, which arises when $k + Z = 1$: this is the case when $i = \delta'k/(1 - k)$. A further possibility is where $k = 0$ and $Z = 1$, which requires $i = 0$ and hence $r^* = -\pi/(1 + \pi)$.²³

It is useful to consider the variation in $EMTR_I$ further. For example, the net user cost, c_n , is a linear increasing function of the nominal interest rate, i , and a decreasing function of the inflation rate, π . The tax component, T , is a nonlinear increasing function of i , and is independent of π . The variation in the effective marginal tax rate is complicated in the present context by the fact that c_n can become negative for some values of i and π . Hence the expression for $EMTR_I$ in (29) can have a singularity when $c_n = 0$: it is subject to positive and negative asymptotes. Numerical examples are given below: all cases examined are for the tax-inclusive effective marginal rate.

5.3 Variation in $EMTR_I$ with Interest and Inflation Rates

An example of the variation in $EMTR_I$ with the nominal interest rate, i , is shown in Figure 3 for two values of the inflation rate, $\pi = 0.02$ and $\pi = 0.04$. The values are obtained for $\tau = 0.3$, $k = 0.2$ and $\delta = 0.15 = \delta'$. For low values of i , and the low inflation rate, the $EMTR_I$ is increasing and above the statutory rate of $\tau = 0.3$, while at higher nominal interest rates the $EMTR_I$ is increasing but below the statutory tax rate. This relationship is highly sensitive to the inflation rate, as can be seen by a comparison with the profile for $\pi = 0.04$, where the nature of the variation is reversed.²⁴ The $EMTR_I$ declines as i increases: at low values of i the effective tax rate moves below the statutory rate, but for higher values of i , the $EMTR_I$ moves towards τ . In both the cases, the lower ranges of i are associated with negative real rates of interest, r^* , and negative user costs, c_n . The higher ranges of i are associated with positive r^* and c_n .²⁵

²³The extreme case where $k = 1$ and $z = 0$ is too unrealistic to be of interest.

²⁴For examples of profiles with similar characteristics, see King and Fullerton (1984, p. 288).

²⁵In both cases the value of z falls from 0.94 to 0.52 as i rises from 0.01 to 0.14.

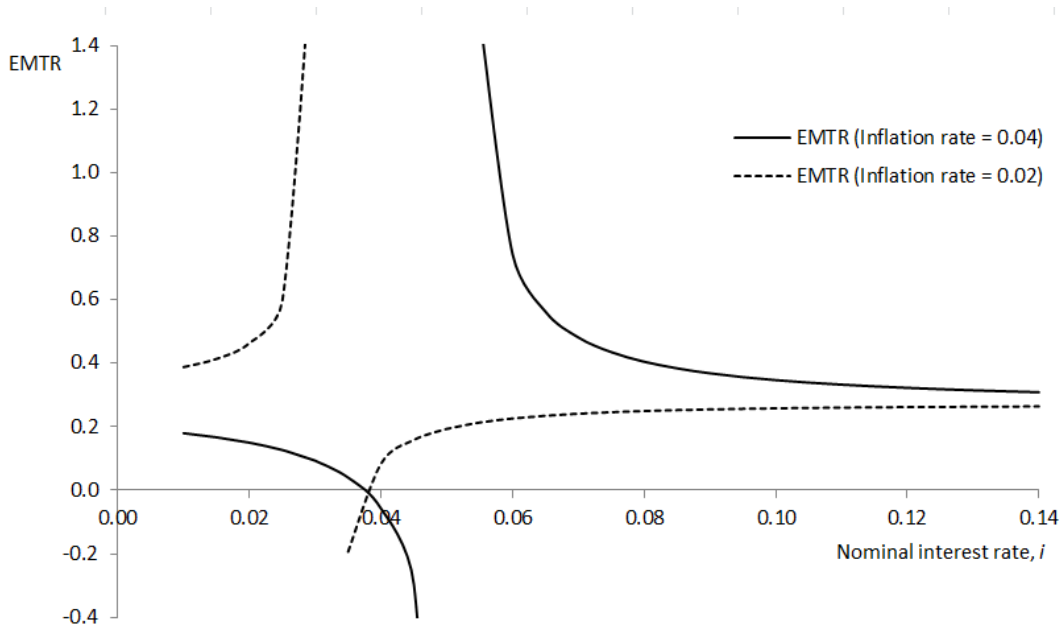


Figure 3: Variation in Effective Marginal Tax Rate with Nominal Interest Rate

5.4 Variation in $EMTR_I$ with the Statutory Tax Rate

It is perhaps tempting to think that the $EMTR_I$ increases systematically as the statutory marginal tax rate increases. However, the variation is again complicated by the existence of the singularity in the expression for the effective rate, combined with the fact that this can arise for relevant ranges of the statutory rate (in combination with other variables).

5.4.1 The Role of the Nominal Interest Rate

Figure 4 shows the variations in the effective marginal tax rate and the net user cost of capital as the statutory rate, τ , varies, when the nominal interest rate, i , is held constant at 0.03 (the dashed line) and at 0.05 (the solid line). These results are obtained for depreciation rates of $\delta = \delta' = 0.15$ and an inflation rate of $\pi = 0.02$. For the lower interest rate, the net user cost is positive at low statutory rates and $EMTR_I$ is negative and declines as τ increases. But at higher values of τ the value of c_n turns negative and $EMTR_I$, now positive, decreases with increasing τ . The $EMTR_I$ relationship is

substantially modified for a higher nominal interest rate of 0.05.²⁶ However, the net user cost decreases steadily, as before, although it remains positive.

5.4.2 The Role of Fiscal Depreciation and Tax Credit Rates

The variation in $EMTR_I$ and c_n with τ , as the fiscal depreciation rate, δ' , varies is illustrated in Figure 5. These profiles are for an economic depreciation rate of $\delta = 0.15$. In each case the inflation rate is $\pi = 0.015$, the nominal interest rate is $i = 0.03$. Hence in the first two examples, the fiscal depreciation rate is below the economic depreciation rate. In these cases the $EMTR_I$ is non-negative and increases systematically as τ increases, but the nature of the variation is very different. For the higher fiscal depreciation rate of 0.15, the singularity is in the relevant ranges of τ and $EMTR_I$ is negative and falling as τ increases over low values of τ .²⁷ The variation in the net user cost of capital with the statutory rate, shown in the lower diagram in Figure 5, is much more systematic. For higher fiscal depreciation rates the $EMTR_I$ falls with τ and the gradient increases as δ' falls, becoming quite steep for the case of zero fiscal depreciation.

The effect of the investment tax credit rate, k , on the way in which the $EMTR_I$ and c_n vary with τ are illustrated in Figure 6. These are obtained for $\pi = 0.02$, $i = 0.03$ and $\delta = \delta' = 0.15$. The sensitivity of these relationships to variations in the nominal rate of interest is shown by comparison with Figure 7, where the nominal interest rate is 0.05.

One implication of the variations shown in these diagrams is that the user cost measure, and its relationship with the statutory tax rate, often provides a more reliable indication of the effects of taxation on the incentives facing firms.

²⁶The singularity arises for a much higher and unrealistic statutory rate.

²⁷For higher values of τ the $EMTR_I$ is positive and again declines with further increase in τ , while the net user cost is negative in this range.

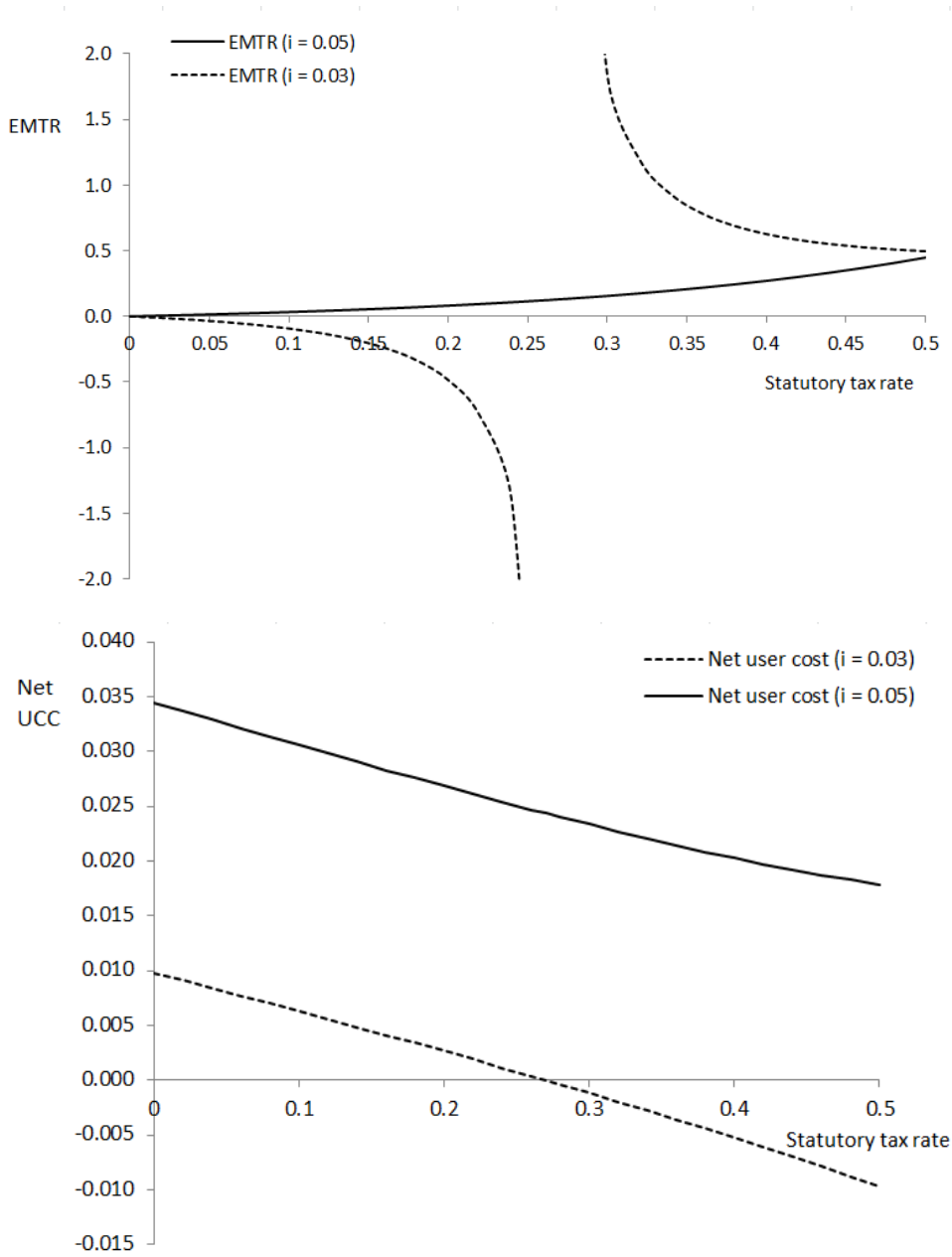


Figure 4: Variation in EMTR and User Cost with Statutory Tax Rate: Nominal Interest Rates of 0.03 and 0.05

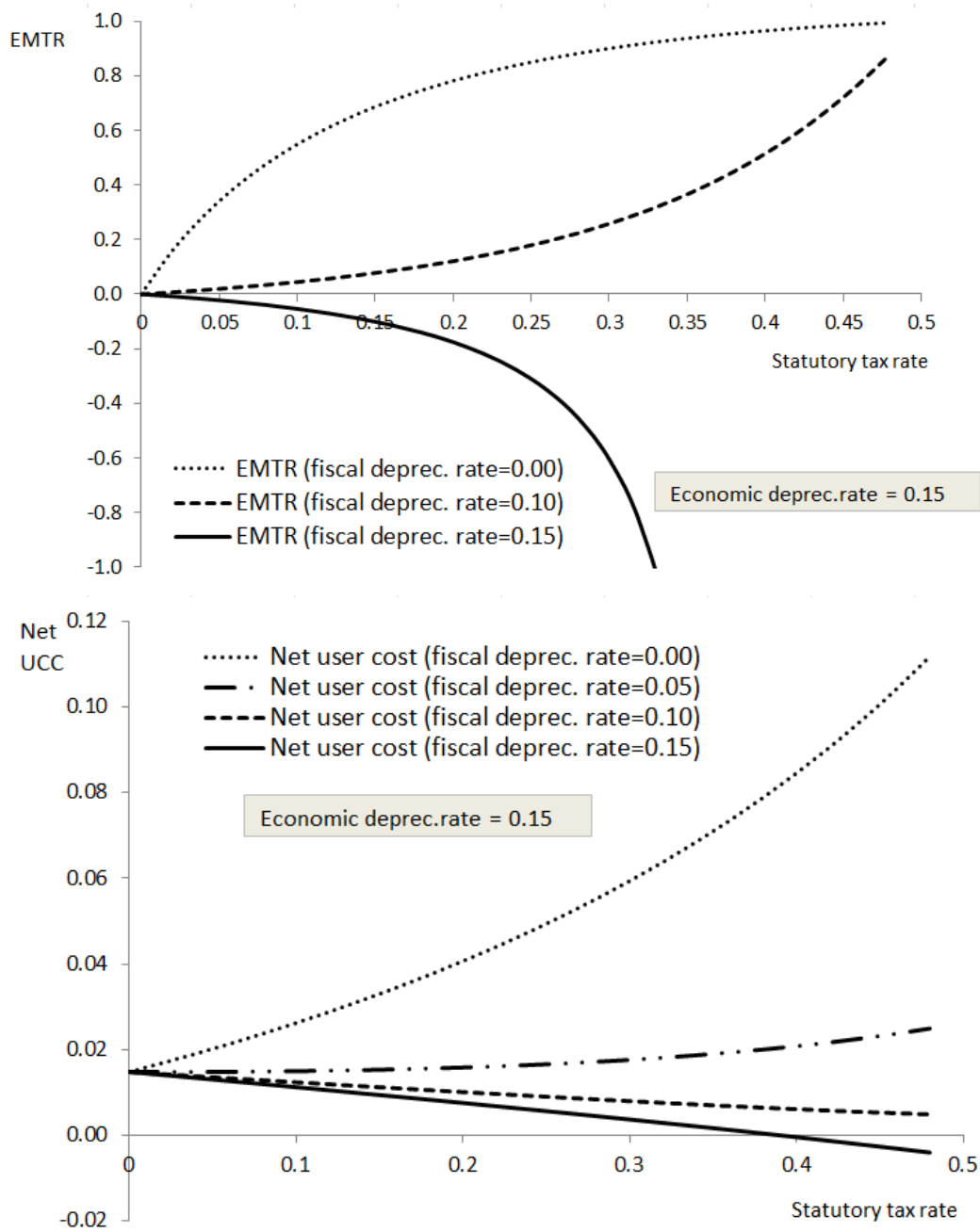


Figure 5: Variation in EMTR and Net User Cost with Statutory Rate: Alternative Fiscal Depreciation Rates

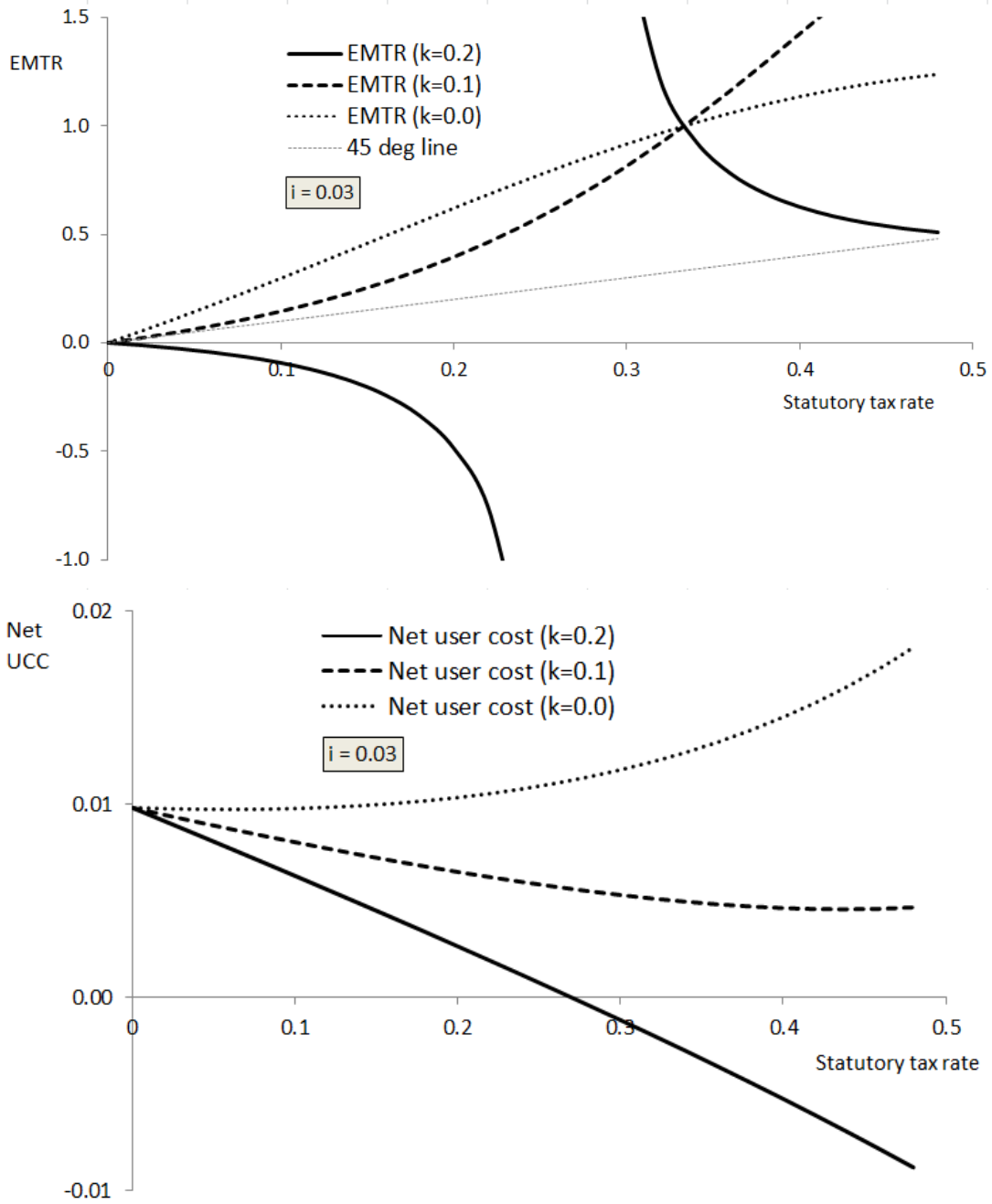


Figure 6: Variation in EMTR and Net User Cost with Statutory Tax Rate for Alternative Tax Credit Rates: Nominal Interest Rate of 0.03

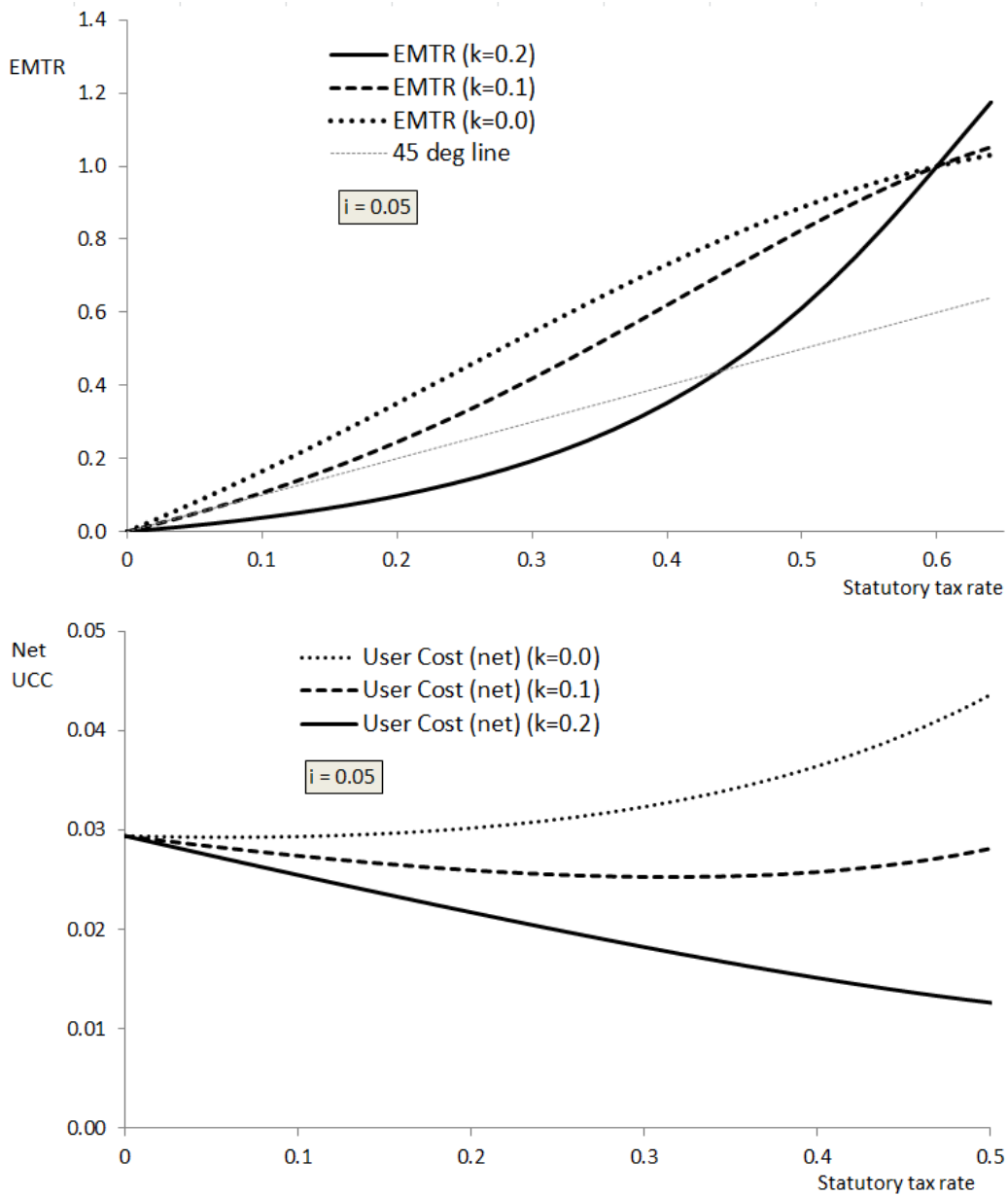


Figure 7: Variation in EMTR and Net User Cost with Statutory Tax Rate for Alternative Tax Credit Rates: Nominal Interest Rate of 0,05

6 An Effective Average Tax Rate

It has been stressed that the effective tax rates considered in the previous section are necessarily marginal rates, given that the basic concept of user cost refers to a marginal investment. However, in the profit-maximising situation for which the concept of user cost applies, and where the statutory rate is constant (that is, the tax is proportional), the effective average tax rate is equal to the effective marginal rate. The present section explains how the analysis can be extended to deal with situations in which the firm is in the position of earning economic rents.²⁸

An important context in which this may be relevant is that of foreign direct investment by multinational corporations. The effective marginal rate is not always the relevant rate in considering investment incentives, since it is expected to apply to the ‘intensive margin’ (concerning variations in the amount invested). In examining discrete decisions at the ‘extensive margin’, regarding for example the international location of investment, where rents may be obtained, a somewhat different concept of an effective average tax rate may be thought to be more appropriate. An approach to defining an average tax rate is by Devereux and Griffith (2003), who took as their starting point a discrete-time variant of the present value of the returns from a non-marginal investment, V , and the change in the present value of that income stream resulting from the investment.²⁹ They describe this change in the capital stock as a ‘perturbation’. The thought experiment here is of a one-period increase in investment, followed by a decrease in investment in the next period to return the real capital stock to its previous value. An important assumption of this approach is that the incentive for the non-marginal investment is an expected return from the investment that exceeds the required return to make the investment marginally profitable. Hence, this non-marginal investment is assumed to earn an economic rent, in the presence of taxation, of R^* .³⁰

In this approach, as in the marginal investment case described above, interest fo-

²⁸As stressed earlier, the fundamental distinction between the concept of *economic rent* and that of the *capital rental* is crucial.

²⁹See also Devereux and Sørensen (2006), Sørensen (2008), Krzepkowski (2013).

³⁰The notation involving * superscripts in the present section is therefore consistent with earlier sections (and most of the earlier literature) where r^* and r respectively define real interest rates after allowing for taxation and before tax. However, it differs from Devereux and Griffith, who use R^* to denote values in the absence of taxation.

cuses on the change, ΔV , in the present value of the returns, after deducting all costs, arising from the investment. Although the perturbation takes place in period t only, the returns arise over multiple subsequent periods. However, in this non-marginal case, the return, net of costs, takes the form of economic rent, R^* , from the investment. Hence:³¹

$$R^* = \Delta V = \sum_{s=0}^{\infty} \frac{\Delta \Omega_{t+s}}{(1+i)^{1+s}} \quad (32)$$

where Ω_t represents the after-tax profit in period t , and i is the firm's nominal discount rate.

In other words, the present value of the perturbation results from an increase in the capital stock in period t , leaving all future periods' capital stocks unchanged. By focussing on these returns from a non-marginal investment, they are thus not increasing the use of capital from an initial profit maximising position, which is the focus of the user cost concept. Rather, the context could be thought of as a decision over the discrete choice between two mutually exclusive investments, such as when, or where, to make a given investment. However, the profit maximising case corresponds to setting $R^* = 0$ and solving for the rental rate (the rate of return), which gives the user cost, c_n , as above.³²

In the marginal investment case in section 5, the $EMTR_I$ was shown to depend on the difference between the before-tax marginal rate of return, \tilde{p} , and the after-tax return, r^* ; see Figure 2. For the non-marginal case here, a further component is required, namely the before-tax return on this non-marginal investment, denoted p , which includes any economic rent associated with the investment. Devereux and Griffith refer to this as the 'real financial return' from a non-marginal perturbation (as distinct from the real financial rate of return, \tilde{p} , from a *marginal* investment; see Devereux and Griffith, 2003, p. 111). Investment projects must necessarily involve using an amount of capital that is less than (or equal to) the profit maximising total amount such that the real financial return exceeds the user cost. Thus, for $R^* > 0$, $p \geq \tilde{p} = c_n$.

³¹Devereux and Griffith (2003, p. 110) write R^* (or R in their notation) in terms of the shareholders' nominal discount rate rather than taking the point of view of the firm, as in Hall and Jorgensen (1967). In considering Ω , any new equity used to fund the investment must be repaid out of the income stream. There is a small printing error, as the discount factor in their equation (2003, p. 110, equation 3) is given an exponent of s , instead of $s + 1$.

³²In their notation c_n is denoted by \tilde{p} .

To define the effective average tax rate for this case, it is also necessary to define R , the net present value of the income stream from the non-marginal perturbation in the absence of taxation.³³ The tax liability associated with the investment can then be defined as $R - R^*$, being the difference between the before-tax and after-tax rates of return. However, the definition of an associated average tax *rate* is not straightforward.

From Devereux and Griffith (2003), one concept of an average tax rate applying to this non-marginal investment is the tax-inclusive rate, $EATR_{I-DG}$, defined as:³⁴

$$EATR_{I-DG} = \frac{R - R^*}{R} \quad (33)$$

This is effectively the tax paid divided by the before-tax present value of the income stream (the before-tax economic rent).³⁵ Devereux and Griffith define an alternative average rate, $EATR_{DG}$, in which the tax liability, $R - R^*$, is ‘scaled’ by the net present value of the before-tax income stream, net of depreciation. The latter is simply p , discounted using the before-tax real interest rate, r , to make it comparable in present value terms to $R - R^*$. Hence:

$$EATR_{DG} = \frac{R - R^*}{p / (1 + r)} \quad (34)$$

Considering the terms in the numerator of (34), R is the net present value of the rent associated with the perturbation in the absence of taxation. Remembering that this also needs discounting only one period, R is given by:

$$R = \frac{p - r}{1 + r} \quad (35)$$

The term, R^* , the equivalent net present value with taxation, is given by:

$$R^* = \frac{(p - \tilde{p})(1 - \tau)}{1 + r} \quad (36)$$

The term $p - \tilde{p}$, rather than $p - r$, appears in (36) because, with taxation, \tilde{p} is the required return (the user cost) in the absence of rents being earned by the investment.

³³This rate of return in the absence of taxation is also assumed to be equal to the after-tax rate of return when tax is introduced. That is, the introduction of the tax does not alter the ‘initial’ required rate of return received by the investor.

³⁴The notation here differs somewhat from that used by Devereux and Griffith.

³⁵These present values actually involve discounting for only one period (values are received at the end of the first period), given the nature of the experiment considered.

With taxation, those rents, $p - \tilde{p}$, are taxed at rate τ , leaving rent net-of-tax but before discounting, of $(p - \tilde{p})(1 - \tau)$. Substituting these expressions into (34) gives:

$$\begin{aligned} EATR_{DG} &= \frac{\tilde{p} - r + (p - \tilde{p})\tau}{p} \\ &= \frac{c_n - r + (p - c_n)\tau}{p} \end{aligned} \quad (37)$$

Devereux and Griffith also define the tax-inclusive marginal tax rate, referred to here as $EMTR_{DG}$, using $EMTR_{DG} = (c_n - r)/c_n$, which can be seen to be similar to the marginal rate in (23), where $\tilde{p} = c_n$. However, whereas (23) follows standard practice and uses the after-tax interest rate r^* , Devereux and Griffith instead use the before-tax rate, r , but do not discuss their reasons for this choice.

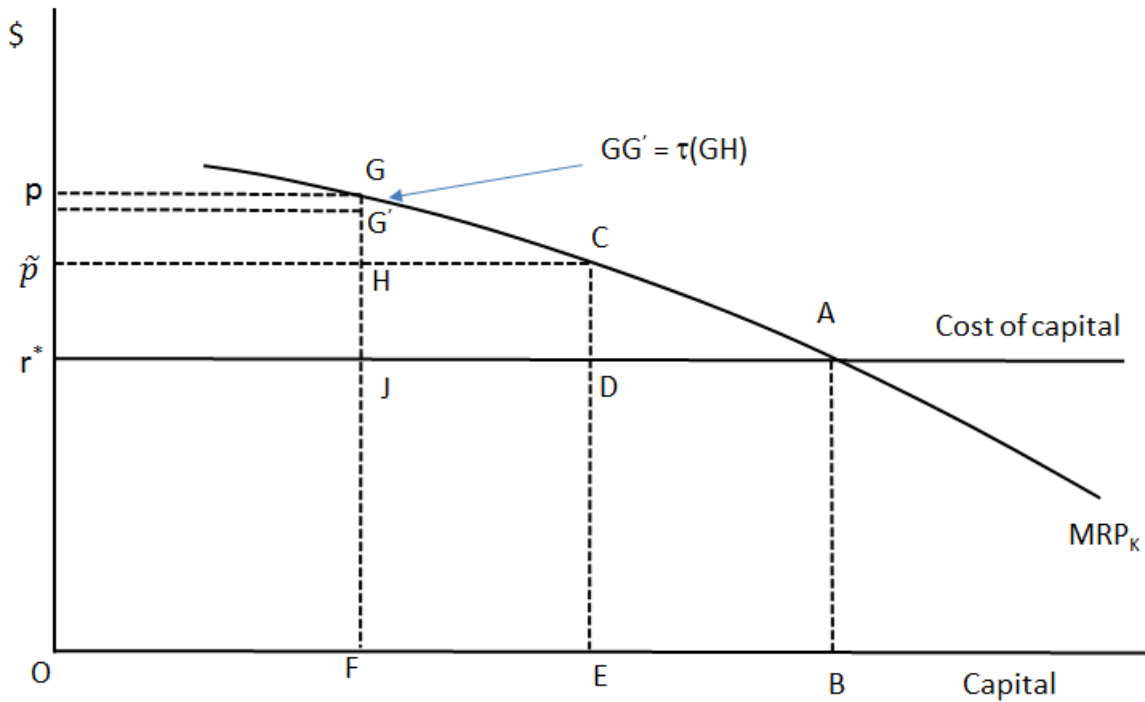


Figure 8: Effective Average Tax Rate

These alternative tax rate definitions can be illustrated by redrawing Figure 2 for this non-marginal case. In Figure 8, consider an investment of capital of amount OF with a marginal revenue product of p per unit, and an after-tax real interest rate,

r^* . As in Figure 2, for a *marginal* project the before-tax return, required to yield an after-tax return of r^* , is \tilde{p} . As argued above, for a non-marginal investment to earn some positive economic rent it is required that OF is less than OE, such that $p > \tilde{p}$. Measured in per unit of investment terms, and before discounting, the before-tax rent is therefore given by $p - \tilde{p}$, or the distance GH in Figure 8. It follows that the after-tax rent is given by $(p - \tilde{p})(1 - \tau)$ as shown; namely in (36), and in Figure 8 is a proportion $(1 - \tau)$ of the distance GH; namely GG' . However, total tax on the income stream includes tax on the non-rent component; that is: $\tilde{p} - r^*$ (= HJ). Hence total tax liability is equal to the sum of those two components, $HJ + GG'$, or $(\tilde{p} - r^*) + (p - \tilde{p})\tau$. This can be seen to be equivalent to the numerator of (37), except that r^* replaces r , as discussed above.

Figure 8 also illustrates the alternative methods of defining the tax *rate*, which should be measured using the corresponding tax base. If the tax base is defined as the *total* return, then the tax rate could be defined as a fraction of the tax base, p , such as $EATR_{DG}$ in (37). However the tax could be thought of as applying to a base measured as the return in excess of a measure of the cost of capital, r or r^* in Figure 8, where this cost is regarded as deductible from the total tax base in determining tax liability.³⁶ In this case the tax base would be, for example, $p - r$, which, suitably discounted, is shown in (35) to be equal to R . Devereux and Griffith (2003) argue in favour of the *total* tax base, p , in part because it facilitates comparison with backward-looking average tax rates based on actual tax and capital income (profit) data, rather than economic rents, and because a tax rate based on R , such as $EATR_{I-DG}$ in (33), is undefined for a marginal investment where $R = 0$.

Average and marginal tax rates therefore differ for an investment involving economic rents. The relationship between the two rates can be seen by rearranging (37). Using the definition of $EMTR_{DG}$ above, the two effective rates can be shown to be related as follows:

$$EATR_{DG} = EMTR_{DG} \left(\frac{c_n}{p} \right) + \tau \left(1 - \frac{c_n}{p} \right) \quad (38)$$

This is equivalent to the decomposition given by Devereux and Griffith (2003, p. 112). Their effective average rate is thus a weighted average of their effective marginal tax

³⁶Of course, the extent to which this cost is deductible, typically depends on a number of conditions such as whether the investment is equity or debt financed.

rate and the statutory tax rate, with weights depending on the ratio of the user cost to the capital rental associated with the non-marginal investment.

For small projects, well below the profit maximising scale, p is likely to be much larger than c_n , so that c_n/p is small (except where investment returns decline only slowly with the scale of an investment; that is, if the MRP_K curve in Figure 8 is relatively flat). For this ‘small c_n/p ’ case the effective average rate is relatively close to the statutory tax rate. For larger projects, as p approaches c_n the ratio moves closer to 1 and the average effective tax rate is closer to the marginal rate. For $p = c_n$, average and marginal rates are equal, and this clearly corresponds to the case discussed at the end of subsection 5.1.

Devereux and Griffith further show that, where it is desired to allow for differences between personal and corporate-level taxation, the statutory rate, τ , is replaced by an ‘adjusted statutory tax rate’, τ' . In particular, this adjustment takes account of any differences in the personal tax treatment of new equity and distributions, and discounting uses the shareholders’ nominal discount rate, ρ , rather than the nominal interest rate, i , faced by the firm. Devereux and Griffith (2003, p. 113) show that the appropriate adjusted statutory rate is:

$$\tau' = 1 - \gamma(1 - \tau) \frac{(1 + r)(1 + \pi)}{(1 + \rho)} \quad (39)$$

where γ reflects the differential personal treatment of new equity and dividend distributions. The term $(1 + r)(1 + \pi)$ is, by Fisher’s equation, equal to $(1 + i)$, so that if $\rho = i$ and $\gamma = 1$, $\tau' = \tau$.³⁷

7 Investment and the User Cost

It has been seen that the user cost concept is intimately related to optimal investment by a profit maximising firm. The firm invests, that is adjusts its capital stock, to the point where the returns match the cost of capital. This gives rise to an equimarginal condition which can be used to express the user cost in terms of the interest rate, the inflation rate, depreciation, taxation and so on. Only values at the time of investment are relevant because of the assumption that investment is reversible and there are no

³⁷Devereux and Griffith differ from most other literature in using the shareholders’ discount rate to derive present values of returns.

adjustment costs. In this case there are clear implications for the optimal capital stock in terms of the user cost.

The gross user cost, as a pre-tax rental, is the capital rental associated with profit maximisation. Expressing the production function as a function of only capital, K , output is $Y = F(K)$ and the capital rental is the marginal revenue product, equal to the product of marginal revenue and the marginal physical product of capital. In a competitive market, marginal revenue and price are equal, and the latter can be normalised to unity. Hence $c_g = \partial F(K) / \partial K = F'_K$. Consider the Cobb-Douglas production function, with an exponent of α on capital services. Then $F'_K = \alpha Y / K$. If K^* represents the desired capital stock, it is simply given by rearranging $\alpha F(K^*) / K^* = c_g$, so that:

$$K^* = \frac{\alpha Y}{c_g} \quad (40)$$

Hence the logarithm of the desired capital stock is a linear function of the logarithm of output and the logarithm of the user cost.

This can easily be extended to deal with imperfect output markets and, say, the constant elasticity of substitution production function. For inputs of labour and capital of L and K , and normalising the efficiency term to unity, the Constant Elasticity of Substitution (CES) function is (with a re-definition of α):

$$Y = (\alpha K^\rho + \beta L^\rho)^{1/\rho} \quad (41)$$

where $\rho = 1 - \frac{1}{\sigma}$ and σ is the elasticity of substitution between labour and capital. The marginal physical product of capital is:

$$\frac{\partial Y}{\partial K} = \alpha K^{\rho-1} Y^{1-\rho} \quad (42)$$

If the price of the good per unit is p and the elasticity of demand is η , then using the well-known property that marginal revenue, $MR = p \left(1 - \frac{1}{\eta}\right)$, the capital rental is given by:

$$c_g = (MR) \frac{\partial Y}{\partial K} \quad (43)$$

$$= \alpha p K^{\rho-1} Y^{1-\rho} \left(1 - \frac{1}{\eta}\right) \quad (44)$$

and using $1 - \rho = 1/\sigma$, desired capital stock is:

$$K^* = Y \left(\frac{\alpha}{1 - \frac{1}{\rho}} \right)^\sigma \left(\frac{c_g}{p} \right)^{-\sigma} \quad (45)$$

As with the simple Cobb-Douglas case, the desired capital stock is a loglinear function of output and user cost, but the coefficient on the logarithm of user cost is $-\sigma$ rather than -1 .

Sometimes the expression for desired capital stock is used along with a specified adjustment process in order to produce an investment function. However, this necessarily involves a serious conflict, since the fundamental user cost derivation discussed earlier explicitly assumes there are no adjustment costs (the cost of capital is fixed independent of the amount of investment). However, from the basic relationship relating capital at time, t , and $t - 1$, and investment, I_t :

$$K_t = K_{t-1} + I_t - \delta K_{t-1} \quad (46)$$

Rearrangement gives the following expression for the growth rate of capital:

$$\frac{K_t - K_{t-1}}{K_{t-1}} = \frac{I_t}{K_{t-1}} - \delta \quad (47)$$

This proportional change can be approximated by the change in logarithms, Δk_t . For example, a simple partial adjustment specification has $\Delta k_t = \theta (k_t^* - k_{t-1})$. Alternatively, error-correction or distributed lag models can be applied.³⁸ The effect on investment of changes in tax regulations or rates can therefore be traced via the effect on user cost.

8 Conclusions

The aim of this paper has been to provide an introductory review of the concept of user cost and its determinants. The concept of user cost was seen to relate to the rental, the rate of return to capital, that arises in a profit maximising situation in which further investment in capital produces no additional profit. Despite this apparently simple statement it has been seen that the concept gives rise to a complex taxonomy or range of cases which need to be distinguished. This paper sets out in some detail, using a consistent terminology, the range of assumptions involved in obtaining alternative expressions for the user cost.

The user cost refers to a before-tax capital rental, the rate of return that ensures that the (after-tax) cost of capital is equal to the post-tax returns. Hence, associated

³⁸On alternative specifications, see Bond and Van Reenen (2003).

with the user cost measure is an effective marginal tax rate. This can differ substantially from the statutory marginal rate applicable to the investor. Particular attention was given to the properties of the effective marginal tax rate in different circumstances, drawing attention to the difference between tax-inclusive and exclusive rates.

A related effective average tax rate was also defined for the context in which the firm obtains economic rents. This may be important in the context of multinational investment where the firm is operating below its profit maximising output. The link between the user cost, effective tax rates and investment was only briefly discussed as this warrants separate extensive treatment.

Appendix A: Derivation of the Basic Hall and Jorgensen Result

In their seminal paper, Hall and Jorgenson (1967) used a continuous-time approach. This appendix explains how their first result is derived. They began by taking the simplest case of no taxation, no depreciation and no inflation. A firm makes a marginal increase in its input of capital by obtaining a new capital good at time, t , with a supply price of capital of $q(t)$. Capital services at time, $s \geq t$, are valued at $c(s)$. Hence $c(s)$ measures the marginal revenue product, and this clearly depends on the price of the good produced by the firm as well as the productivity of the equipment. Strictly, this rental depends on the marginal revenue, but on the assumption that the good is sold in a competitive market, price and marginal revenue are equal. Investment continues up to the point where the supply price, $q(t)$ is equal to the present value of additional returns. Hence, at the profit maximising position, and with continuous discounting at the rate, r , Hall and Jorgenson (1967) write:

$$q(t) = \int_t^{\infty} c(s) e^{-r(s-t)} ds \quad (\text{A.1})$$

An implication is that a further marginal increase in capital, made in only period t , involving an increase in $q(t)$, is exactly matched by the change in the present value of returns, measured by the right-hand side of (A.1). Writing $\dot{q}(t) = \frac{\partial q(t)}{\partial t}$, a further marginal increase in capital, made in only period t , involving an increase in $q(t)$, is exactly matched by the change in the present value of returns, measured by the right-hand side of (A.1). Hence:

$$\frac{\partial q(t)}{\partial t} = \frac{\partial}{\partial t} \int_t^{\infty} c(s) e^{-r(s-t)} ds \quad (\text{A.2})$$

This equation therefore expresses the kind of investment that was discussed above. There is a marginal investment in one period which is reversed in the subsequent period. The right-hand side can be obtained by using the Leibniz Integral Rule. This states, for the general function $f(s, t)$ and limits of integration given by $a(t)$ and $b(t)$, that:

$$\frac{\partial}{\partial t} \int_{a(t)}^{b(t)} f(s, t) ds = \int_{a(t)}^{b(t)} \frac{\partial f(s, t)}{\partial t} ds + f(b(t), t) \frac{\partial b(t)}{\partial t} - f(a(t), t) \frac{\partial a(t)}{\partial t} \quad (\text{A.3})$$

Consider the term in (A.2) corresponding to the first term in (A.3). Then:

$$\frac{\partial}{\partial t} \{c(s) e^{-r(s-t)}\} = c(s) r e^{-r(s-t)} \quad (\text{A.4})$$

so that:

$$\int_t^\infty \frac{\partial}{\partial t} \{c(s) e^{-r(s-t)}\} ds = r \int_t^\infty c(s) e^{-r(s-t)} ds \quad (\text{A.5})$$

which is equal to $r q(t)$. Furthermore, it can be seen that the second term in (A.5) is zero and the third term is simply $c(t)$: this is because the term $f(a(t), t)$ is equal to $c(t) e^{-0} = c(t)$ and $\frac{\partial a(t)}{\partial t} = 1$. Hence, writing $\dot{q}(t) = \frac{\partial q(t)}{\partial t}$, (A.2) becomes:

$$\dot{q}(t) = r q(t) - c(t) \quad (\text{A.6})$$

This is the Hall and Jorgensen result. The fact that only period- t values are relevant in (A.6) arises from the strong assumption that the project is reversible. Furthermore, assume that the price of the capital good does not depend on the amount already invested, so that there are no adjustment costs: the supply curve is essentially horizontal. This means that $\dot{q}(t)$ is assumed to be zero. With constant consumer prices, and dropping the time subscript, t , (A.6) becomes:

$$\frac{c}{q} = r \quad (\text{A.7})$$

The rental per unit of capital is thus equal to the rate of interest. It is conventional to normalise the price of a unit of the capital good, so that setting $q = 1$ gives the simple result that $c = r$. The rental associated with profit maximisation is, by definition, the *user cost* of capital. Hence the user cost is equal to the interest rate.

Appendix B: Allowing for Uncertainty

Modification of the above results to allow for uncertainty and risk aversion rapidly becomes very complicated. However, some insight can be obtained by first considering the simplest possible case where there is no taxation, no depreciation and no capital gain. In the deterministic case, the user cost of a dollar invested in capital is simply $c_g = r$. Suppose now that r is uncertain, although the nature of the distribution is known. Risk aversion is modelled by supposing that there is a concave utility function, $U(c_g)$ associated with the user cost. If the investor is assumed to have constant relative risk aversion of $\varepsilon \neq 1$, then utility takes the form $U(c_g) = c_g^{1-\varepsilon} / (1-\varepsilon)$.

The certainty-equivalent user cost is that rental which, if received with certainty, gives the same utility as the expected utility from the distribution. Hence if r has the distribution function, $F(r)$, the user cost is given by:

$$\frac{c_g^{1-\varepsilon}}{1-\varepsilon} = \int \frac{r^{1-\varepsilon}}{1-\varepsilon} dF(r) \quad (\text{B.1})$$

so that:

$$c_g = \left[\int r^{1-\varepsilon} dF(r) \right]^{1/(1-\varepsilon)} \quad (\text{B.2})$$

Thus c_g is the power mean of order ε of the distribution of r . Some insight may be obtained by assuming that r is lognormally distributed, so that $F(r) = \Lambda(r|\mu, \sigma^2)$, with mean and variance of logarithms of μ and σ^2 respectively. From the moment generating function of the lognormal distribution, it is known that:

$$\int r^{1-\varepsilon} d\Lambda(r) = \exp \left\{ (1-\varepsilon)\mu + \frac{1}{2}(1-\varepsilon)^2\sigma^2 \right\} \quad (\text{B.3})$$

and therefore (supposing that the range of r is $0 < r \leq \infty$):

$$c_g = \exp \left(\mu + \frac{(1-\varepsilon)}{2}\sigma^2 \right) \quad (\text{B.4})$$

Consider the effect of an increase in uncertainty. If this is assumed to result simply from an increase in σ^2 , the arithmetic mean as well as the variance of r changes, since the arithmetic mean is given by $E(r) = \exp(\mu + \frac{1}{2}\sigma^2)$. However, increasing risk can be modelled as a mean-preserving spread of the distribution. Hence, when σ^2 increases,

the value of μ must fall to maintain a constant arithmetic mean. From the total differential:

$$dE(r) = E(r) d\mu + \frac{1}{2} E(r) d\sigma^2 \quad (\text{B.5})$$

It can be seen that:

$$\left. \frac{d\mu}{d\sigma^2} \right|_{E(r)} = -\frac{1}{2} \quad (\text{B.6})$$

Thus the effect on the user cost of a mean-preserving increase in uncertainty is given by totally differentiating (B.4):

$$\frac{dc_g}{d\sigma^2} = c_g \left. \frac{d\mu}{d\sigma^2} \right|_{E(r)} + \frac{c_g(1-\varepsilon)}{2} = -\frac{c_g}{2} + \frac{c_g(1-\varepsilon)}{2} = -\frac{c_g\varepsilon}{2} \quad (\text{B.7})$$

and the elasticity of c_g with respect to a mean-preserving increase in σ^2 is:

$$\eta_{c,\sigma^2} = \frac{\sigma^2}{c_g} \frac{dc_g}{d\sigma^2} = -\frac{\varepsilon\sigma^2}{2} \quad (\text{B.8})$$

The elasticity of user cost with respect to a mean-preserving spread in r is thus (minus) half the product of the degree of relative risk aversion and the variance of logarithms of r . In the risk-neutral case, increased uncertainty (as a mean-preserving spread) has no effect since only expected values matter.

References

- [1] Auerbach, A.J. (1979) Wealth maximization and the cost of capital. *Quarterly Journal of Economics*, 93, 433–446.
- [2] Auerbach, A.J. (1983) Taxation, corporate financial policy and the cost of capital. *Journal of Economic Literature*, 21, 905-962.
- [3] Auerbach, A.J. (2002) Taxation and corporate financial policy. In *Handbook of Public Economics*, Volume 3 (Edited by A.J. Auerbach and M. Feldstein), pp. 1251-1292. Amsterdam: Elsevier.
- [4] Benge, M. (1997) Taxes, corporate financial policy and investment decisions in Australia. *Economic Record*, 73, 1-15.
- [5] Benge, M. (1998) Depreciation provisions and investment incentives under full imputation. *Economic Record*, 74, 329-345.
- [6] Blouin, J., Core, J.E. and Guay, W. (2008) Improved estimates of marginal tax rates: why they are needed, approach and implications. Wharton School, University of Pennsylvania.
- [7] Bond, S.R. and Van Reenen, J. (2007) Microeconomic models of investment and employment. In *Handbook of Econometrics*, Volume 6A (Edited by J.J. Heckman and E.E. Leamer). Amsterdam: Elsevier.
- [8] Bond, S. and Xing, J. (2010) Corporate Taxation and Capital Accumulation. *Oxford University Centre for Business Taxation. Working Paper No. WP10/15*.
- [9] Devereux, M.P and Griffith, R. (2003) Evaluating tax policy for location decisions. *International Tax and Public Finance*, 10, 107-126.
- [10] Devereux, M.P. and Sørensen, P.B. (2006) The corporate income tax: international trends and options for fundamental reform. *European Commission Directorate General for Economic and Financial Affairs Economic Paper*, no. 264.
- [11] Downs, T.W. (1986) Using the user cost. *Journal of Economics and Business*, 297-305.

- [12] Dwenger, N. and Walch, F. (2011) Tax losses and firm investment: evidence from tax statistics. German Institute for Economic Research Department of Public Economics.
- [13] Egger, P., Loretz, S., Pfaffermayr, M. and Winner, H. (2009) Firm-specific forward-looking tax rates. *International Tax and Public Finance*, 16, 850-870.
- [14] Egger, P. and Loretz, P. (2010) Homogeneous Profit Tax Effects for Heterogeneous Firms? *The World Economy*, 33, 1023-1041.
- [15] Fabling, R., Gemmell, N., Kneller, R. and Sanderson, L. (2013) Estimating firm-level effective marginal tax rates and the user cost of capital in New Zealand. *New Zealand Treasury Working Paper*, WP 13/29.
- [16] Graham, J.R. (1996) Debt and the marginal tax rate. *Journal of Financial Economics*, 41, 41-73.
- [17] Gravelle, J.G. (1982) Effects of the 1981 depreciation revisions on the taxation of income from business capital. *National Tax Journal*, 35, 1-20.
- [18] Hall, R.E. and Jorgensen, D.W. (1967) Tax policy and investment behaviour. *American Economic Review*, 57, 391-414.
- [19] Kemsley, D. (1998) The effect of taxes on production location. *Journal of Accounting Research*, 36, 321-341.
- [20] King, M. A. and Fullerton, D. (1984) *The Taxation of Income from Capital*. Chicago: University of Chicago Press.
- [21] Krzepkowski, M. (2013) Marginal versus average effective tax rates and foreign direct investment. University of Calgary Department of Economics.
- [22] McKenzie, K.J. and Mintz, J. (1992) Tax effects on the cost of capital. In *Canada-US Tax Comparisons* (Edited by J.B. Shoven and J. Whalley), pp. 189-216. Chicago: University of Chicago.
- [23] McKenzie, K.J., Mintz, J. and Scharf, K.A. (1997) Measuring effective tax rates in the presence of multiple inputs: a production based approach. *International Tax and Public Finance*, 4, 337-359.

- [24] McKenzie, K.J., Mansour, M. and Brule, A. (1998) The Calculation of Marginal Effective Tax Rates. Working Paper No. 97-15, Department of Finance, Ottawa, Canada.
- [25] Ramb, F. (2007) Corporate marginal tax rate, tax loss carryforwards and investment functions – empirical analysis using a large German panel data set. *Deutsche Bundesbank Discussion Paper*, Series 1, no. 21/2007.
- [26] Sørensen, P.B. (2004) Measuring taxes on capital and labour - An overview of methods and issues. In *Measuring the Tax Burden on Capital and Labour* (Edited by P.B. Sørensen). Cambridge, Mass: MIT Press.
- [27] Sørensen, P.B. (2008) Estimating effective tax rates on corporate income. University of Copenhagen Department of Economics.

About the Authors

John Creedy is Professor of Public Economics and Taxation at Victoria Business School, Victoria University of Wellington, New Zealand, and a Principal Advisor at the New Zealand Treasury.

Email: john.creedy@vuw.ac.nz

Norman Gemmell is Professor of Public Finance at Victoria Business School, Victoria University of Wellington, New Zealand.

Email: norman.gemmell@vuw.ac.nz

