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**Basic understanding of social  
inequality dynamics**

Jacek B Krawczyk and Wilbur Townsend

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# BASIC UNDERSTANDING OF SOCIAL INEQUALITY DYNAMICS

JACEK B. KRAWCZYK & WILBUR TOWNSEND

ABSTRACT. We provide an introduction to a model of social inequality dynamics. Because capital is distributed less equally than labour, we propose that that one of the main forces driving income inequality is the ratio of factor shares. In this paper we give an easy proof to show that this ratio is driven by the output elasticity of capital.

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## 1. INTRODUCTION

This working paper reports on some preliminary ideas concerning a mathematical model for income inequality. This corresponds to a situation where workers do not own capital stock – or, more realistically, where the share of capital owned by the top 1% dwarfs the share owned by the rest of the society. In other words, the factor shares ratio should be able to explain the proportion of income of the top 1% (and moreover, the top 0.1%) to the rest of income earners.

This paper comprises the following sections: Section 2 contextualises our research by discussing the recent research in [Piketty \(2014\)](#). Section 3 derives a simple mathematical model for the basic relation between a generalised inequality coefficient and economic development.

## 2. PIKETTY'S CONTEXT

[Piketty \(2014\)](#) aims to explain the significant shifts in income inequality since the Industrial Revolution. His explanation involves both shifts in the shares of income going to different factors of production and shifts in the ownership of those factors. Here we will summarise his explanation as to why income inequality first decreased and then increased uniformly across the developed world. In doing so we'll describe why he believes that capitalist economies tend towards increased inequality and so contextualise our own policy analysis.

Piketty shows that the ratios of capital to income have shifted significantly since the 19th century. In 1870 Britain, capital was worth almost 700% of national income ([Piketty, 2014](#), Technical Appendix Table SI.2). By the end of World War Two it was only worth 313%, and by 2010 it had rebounded to 522%. These shifts are representative of trends across the West because the shocks that caused them – ideological, military, economic – were as well. From our perspective, their importance comes from their relationship to shifts in the proportion of income going to capital ([Piketty, 2014](#), p. 220). While an increasing capital stock will reduce the marginal product of capital and so reduce

interest rates, according to Piketty, this effect will not be sufficiently strong to keep capitalists' share of income constant. In more technical terms, Piketty believes the elasticity of substitution between capital and labour is greater than 1.

In modern developed economies wage income is correlated with capital income – many of the top 1% of wage earners are in the top 1% of capital owners (Piketty, 2014, pp. 254 - 255). However, capital is distributed less equally than labour (Piketty, 2014, p. 244). Thus a greater proportion of income going to capital will increase income inequality. This justifies using the ratio of capital income to labour income as a measure of inequality, linking Piketty's functional argument with his distributive conclusions.

However Piketty doesn't pretend that functional arguments can explain all of the shifts in income inequality. As he shows, the distributions of capital and wages have themselves changed. The increase in wage inequality is difficult to summarise. It differs significantly between Anglo-America and Continental Europe, suggesting it is driven more by labour market institutions and local norms than anything particularly fundamental (Piketty, 2014, ch. 9); also, compare Krawczyk and Shimomura (2003).

He attributes capital inequality to gaps between the return on capital and economic growth rates,  $r - g$  (Piketty, 2014, ch. 10). Infinite-horizon macroeconomic models with endogenous savings imply  $r > g$  – were that not the case, consumers would borrow to shift consumption forward, lifting interest rates as they did so. Piketty, hesitant to place too much trust in dynamic macroeconomic models, prefers to see  $r > g$  as a historical tendency. Regardless, as the gap between  $r$  and  $g$  increases, so too does the importance of old capital over new.<sup>1</sup>

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<sup>1</sup>He puts it more formally in his technical appendix: the distribution of capital follows a Pareto law, and “Pareto laws are generated by dynamic process with multiplicative shocks” (Piketty, 2014, Technical Appendix p. 61). The importance of these multiplicative shocks increases with  $r - g$ . See Piketty and Zucman (2014) for a fuller exposition. Also, see Judd (1985) for a proof that  $r = g$  is optimal long term in a macrodynamic simple perfect foresight model.

In our research we will consider how policy can constrain inequality. We will focus on the functional side, on the ratio of returns to labour with returns to capital. Further, we assume an elasticity of substitution = 1. Piketty would contest this assumption so it deserves some justification. First, elasticities of substitution appear to be lower over the short term than the longer term – see for example [Tipper \(2012\)](#). Thus our paper can be thought of as considering Piketty’s questions over a shorter timeframe. Second, elasticities of substitution change with technology – a high elasticity of substitution suggests that labour and capital fulfill essentially the same task and so corresponds a roboticised economy, whereas a low elasticity of substitution suggests that labour and capital fulfill fundamentally different roles which cannot be substituted. Thus our paper can be thought of examining the economy as it is now, and not as it is will be in a roboticised future. Finally, it’s worth noting that empirical analyses of the elasticity of substitution have consistently found elasticities less than one, making an elasticity = 1 a fairly conservative assumption.<sup>2</sup> It is the elasticity of substitution = 1 that implies the Cobb-Douglas production model we use below.

### 3. INEQUALITY VERSUS ECONOMIC DEVELOPMENT

Our model is based on [Judd \(1985, 1987\)](#); [Krawczyk and Judd \(2015\)](#).

[Judd \(1987\)](#) and [Krawczyk and Judd \(2015\)](#) use a model that comprises a representative producer with a Cobb-Douglas production function ( $y = Ak^\alpha \ell^{1-\alpha}$ ), an infinitely lived representative household who enjoys consumption ( $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ ) and prefers not to work ( $v(\ell) = V \frac{\ell^{1+\eta}}{1+\eta}$ ), and a government which taxes labour ( $\tau_L$ ) and capital ( $\tau_K$ ) to finance its debt ( $B$ ) and exogenous spending ( $g$ ). Depreciation is  $\delta$  and our household’s discount rate is  $\rho$ . Labour supply and savings are both endogenous.

**3.1. System’s dynamics.** The fundamental law of motion for capital  $k$  is determined by net output *i.e.*,  $y - \delta k$ , where  $y$  is output and  $\delta >$

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<sup>2</sup>For example, see [Szeto \(2001\)](#), [Hall and Scobie \(2005\)](#) and [Tipper \(2012\)](#) for New Zealand examples.

0 is the rate of depreciation, diminished by consumption  $c > 0$  and government expenditure is  $g \geq 0$ . If so and assuming a Cobb-Douglas type production function for output, we get, in continuous time,

$$(1) \quad \frac{dk}{dt} = Ak^\alpha \ell^{1-\alpha} - \delta k - c - g.$$

As usual,  $\ell > 0$  is labour,  $A > 0$  is total factor productivity and  $\alpha \in (0, 1)$  is output elasticity of capital. In this model, expenditure  $g$  is assumed constant but several values of  $g$  can be allowed for in the computations.

The utility of consumption and the disutility of labour of a representative agent are described by  $u$  and  $v$  defined above. The coefficients  $V, \gamma, \eta$  are positive. If  $\lambda > 0$  is the private marginal value of capital at time  $t$ , then it follows from maximization of the utility function  $u(c) - v(\ell)$ , on an infinite horizon with the discount rate  $\rho > 0$ , that<sup>3</sup>

$$(2) \quad \frac{d\lambda}{dt} = \lambda(\rho - \bar{r}).$$

Here,  $\bar{r} = (1 - \tau_K) \left( \frac{\partial y}{\partial k} - \delta \right)$  is the after tax marginal product of capital, where  $\tau_K \in [0, 1]$  is capital tax. Expanding  $\bar{r}$  in (2) yields

$$(3) \quad \frac{d\lambda}{dt} = \lambda \left( \rho - (1 - \tau_K) \left( \alpha A \left( \frac{\ell}{k} \right)^{1-\alpha} - \delta \right) \right).$$

To characterize the economy at hand, we will also use government debt  $B$ , which grows in  $g$  and diminishes with tax  $T$  as follows:

$$(4) \quad \frac{dB}{dt} = \bar{r}B - T + g$$

where, as above,  $\bar{r}$  is the net-of-tax interest rate. In this economy, tax rates on capital and labour are  $\tau_K$  and  $\tau_L$  ( $\tau_L, \tau_K \in [0, 1]$ ); if so, the

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<sup>3</sup>Except where stated otherwise, all settings in our model are the same as in [Judd \(1987\)](#), which can also be traced down to [Brock and Turnovsky \(1981\)](#). In particular, the private marginal value of capital  $\lambda$  (or, agent's marginal utility of consumption) is the adjoint state in the perfect-foresight household utility  $u(c) - v(\ell)$  maximisation problem. Part of its specification is a request for the satisfaction of the consumers' transversality condition at infinity. To obtain optimal consumption, it is sufficient to solve the underlying optimal control problem and use (7).



expression for total tax  $T$  in (4) at time  $t$  becomes

$$(5) \quad \begin{aligned} T &= \tau_K \alpha A k^\alpha \ell^{1-\alpha} + \tau_L (1 - \alpha) A k^\alpha \ell^{1-\alpha} \\ &= (\alpha(\tau_K - \tau_L) + \tau_L) A k^\alpha \ell^{1-\alpha}. \end{aligned}$$

Combining the last two expressions results in the following debt dynamics

$$(6) \quad \frac{dB}{dt} = \bar{r}B - (\alpha(\tau_K - \tau_L) + \tau_L) A k^\alpha \ell^{1-\alpha} + g,$$

where  $\bar{r}$  will be expanded later. In simple terms, we see that debt can diminish if output is large or if the tax rates are high (and when output is not too small).

While the private marginal value of capital,  $\lambda$ , can adequately characterize the consumer's behavior, it lacks an easy economic interpretation. We will replace the equation for  $\frac{d\lambda}{dt}$ , (2), by a differential equation for consumption, easily interpretable.

The marginal utility of consumption is  $\frac{du}{dc} = \frac{1}{c^\gamma}$ ; and, on the other hand,  $\lambda$  is the marginal utility of consumption  $\lambda = \frac{du}{dc}$ . Hence,

$$(7) \quad c = \frac{1}{\lambda^{1/\gamma}},$$

which, after differentiation in the time domain, yields

$$(8) \quad \frac{dc}{dt} = \frac{-1}{\gamma} \cdot \frac{1}{\lambda^{1+1/\gamma}} \cdot \frac{d\lambda}{dt} = \frac{-1}{\gamma} c^{1+\gamma} \frac{d\lambda}{dt}.$$

Using (3), after some simplifications, we get

$$(9) \quad \frac{dc}{dt} = -c \cdot \frac{\rho + (\delta - \alpha A k^{\alpha-1} \ell^{1-\alpha}) (1 - \tau_K)}{\gamma}$$

We can see that consumption has a trivial steady state and will grow if  $\rho$  (discount rate) and/or  $\delta$  (depreciation) are “small” to reach a positive steady state.

We now want to express labour  $\ell$  through capital and consumption and thus “close” the dynamic system (1), (9), (6).

Let  $w$  denote (time-dependent) wages; they equal to the marginal product of labour:

$$(10) \quad w = \frac{dy}{d\ell} = \frac{(1-\alpha)k^\alpha A}{\ell^\alpha}$$

In equilibrium, the marginal utility of consumption weighted by the after-tax wages must be equal to the marginal disutility from labour:

$$(11) \quad \frac{(1-\tau_L)w}{c^\gamma} = \ell^\eta V.$$

Substituting wages and solving for labour yields,

$$(12) \quad \ell = \left( \frac{(1-\tau_L)(1-\alpha)Ak^\alpha}{c^\gamma V} \right)^{\frac{1}{\alpha+\eta}},$$

from which we see that labour can be determined by capital and consumption. Here, we can observe that if  $\gamma > \alpha$  then labour decreases in consumption faster than it grows in capital.

We now use (12) to substitute labour in (1), (9), (6) and obtain

$$(13) \quad \frac{dk}{dt} = \left( A^{\eta+1} k^{\alpha(\alpha+\eta-1)+1} \frac{((1-\alpha)(1-\tau_L))^{1-\alpha}}{(V c^\gamma)} \right)^{\frac{1}{\alpha+\eta}} - g - k\delta - c$$

$$(14) \quad \frac{dc}{dt} = -\frac{c}{\gamma} \cdot \left( \rho + \left( \delta - \frac{\alpha((1-\alpha)(1-\tau_L))^{\frac{1-\alpha}{\alpha+\eta}} A^{\frac{\eta+1}{\alpha+\eta}}}{k^{\frac{(\alpha-1)^2}{\alpha+\eta}} (V c^\gamma)^{1-\alpha}} \right) (1-\tau_K) \right)$$

$$(15) \quad \frac{dB}{dt} =$$

$$g - (\alpha\tau_K + \tau_L(1-\alpha)) \left( A^{\eta+1} k^{\alpha(\alpha+\eta-1)+1} \frac{((1-\alpha)(1-\tau_L))^{1-\alpha}}{(V c^\gamma)} \right)^{\frac{1}{\alpha+\eta}}$$

$$+ k\delta\tau_K - B \left( \delta - \frac{A^{\frac{\eta+1}{\alpha+\eta}} \alpha((1-\alpha)(1-\tau_L))^{\frac{1-\alpha}{\alpha+\eta}}}{(V c^\gamma)^{1-\alpha} k^{\frac{(1-\alpha)^2}{\alpha+\eta}}} \right) (1-\tau_K)$$

Allowing for these substitutions informs us, among others, that the sign of the right hand side of (14) can be negative for significant discount and depreciation rates. Hence high consumption levels may quickly diminish. Large consumption will also contribute to a decline of capital and a rise of debt. However, this multiple downturn could be avoided

by an “early” (preemptive) drop of taxes on capital. One can see in [Krawczyk and Judd \(2015\)](#) from which states such a preventive drop can be efficient.

The three equations of motion (13), (14), (15) jointly constitute the basic representation of the economy at hand. For this economy, the loci of economic states (viability kernel) were established in [Krawczyk and Judd \(2015\)](#), from which moderate tax adjustments can guarantee a balanced evolution of the economy.

We recognize that this system is nonlinear with multiple steady states. We can see that, as one would expect, the consumption growth or decline can be moderated by adjusting the capital tax rate while debt will (mainly) depend on the labour tax rate. If the rates were identical ( $\tau_L = \tau_K$ ), then increasing them/it will slow down the consumption rate and diminish debt. With high taxation rate, consumption and debt will naturally diminish and capital will grow. We also notice that debt will grow very fast for large  $B$  and non-excessive capital taxation.

To fully describe the tax model dynamics, the equations (1), (9), (6) (with (12)) need be completed by two differential inclusions for the two tax rates  $\tau_L$  and  $\tau_K$ :

$$(16) \quad \frac{d\tau_L}{dt} = u_L \in [-d_L, d_L] = U_L \quad \text{and} \quad \frac{d\tau_K}{dt} = u_K \in [-d_K, d_K] = U_K$$

where  $d_L, d_K$  are positive numbers. The inclusions represent bounds on the speed at which tax rates can change. This corresponds to the government policy of “smooth” tax rates adjustments determined by  $d_L$  and  $d_K$ .

In the current version of the model we will assume that the only tax is a proportional income tax, so the tax rate on labour and capital are equal *i.e.*,  $\tau_L = \tau_K = \tau$ . Therefore, the two inclusions in (16) collapse to

$$(17) \quad \frac{d\tau}{dt} = u \in [-d, d] = U, \quad d \geq 0.$$

**3.2. Single tax.** Assuming that  $\tau_L = \tau_K = \tau$ , the following can be derived.

Gross wages:

$$(18) \quad w = y_L = \left( \frac{A^\eta (1 - \alpha)^\eta (k^\eta V c^\gamma)^\alpha}{(1 - \tau)^\alpha} \right)^{\frac{1}{\alpha + \eta}}$$

Net wages:

$$(19) \quad \bar{w} = w(1 - \tau_L) = (((1 - \tau)A(1 - \alpha))^\eta (k^\eta V c^\gamma)^\alpha)^{\frac{1}{\alpha + \eta}}$$

Gross labour income share of output:

$$(20) \quad \frac{lw}{y} = 1 - \alpha$$

Net labour income share of output:

$$(21) \quad \frac{l\bar{w}}{y} = (1 - \alpha)(1 - \tau)$$

Gross return on capital:

$$(22) \quad r = y_k = \alpha A^{\frac{\eta+1}{\alpha+\eta}} \left( \frac{(1 - \alpha)(1 - \tau)}{k^\eta V c^\gamma} \right)^{\frac{1-\alpha}{\alpha+\eta}}$$

Net return on capital:

$$(23) \quad \bar{r} = (r - \delta)(1 - \tau) = \alpha (A(1 - \tau))^{\frac{\eta+1}{\alpha+\eta}} \left( \frac{1 - \alpha}{k^\eta V c^\gamma} \right)^{\frac{1-\alpha}{\alpha+\eta}} - \delta(1 - \tau)$$

Gross capital income share of output:

$$(24) \quad \frac{kr}{y} = \alpha$$

Net capital income share of output:

$$(25) \quad \frac{k\bar{r}}{y} = \alpha(1 - \tau) - \left( \frac{k^\eta (1 - \tau)}{A} \right)^{\frac{\eta+1}{\alpha+\eta}} \left( \frac{V c^\gamma}{1 - \alpha} \right)^{\frac{1-\alpha}{\alpha+\eta}} \delta$$

Finally, the ratio of net capital income to net labour income:

$$(26) \quad \chi \equiv \frac{k\bar{r}}{l\bar{w}} = \frac{\alpha}{1 - \alpha} - \delta \left( \left( \frac{V c^\gamma}{1 - \tau} \right)^{1-\alpha} k^{\eta(1-\alpha)} (A(1 - \alpha))^{-(\eta+1)} \right)^{\frac{1}{\alpha+\eta}}$$

As we explained in Section 2, this ratio correlates with income inequality. When  $\delta = 0$  it simplifies to a constant, suggesting that in a one-tax model  $\chi$  depends only on  $\alpha$ , the Cobb-Douglas output elasticity.

3.3. **Two tax.** When we allow the capital tax rate to differ from the wage tax rate we derive slightly more complicated expressions.

Gross wages:

$$(27) \quad w = y_L = \left( \frac{A^\eta (1 - \alpha)^\eta (k^\eta V c^\gamma)^\alpha}{(1 - \tau_L)^\alpha} \right)^{\frac{1}{\alpha + \eta}}$$

Net wages:

$$(28) \quad w(1 - \tau_L) = (((\tau_L - 1)A(\alpha - 1))^\eta (k^\eta V c^\gamma)^\alpha)^{\frac{1}{\alpha + \eta}}$$

Gross labour income share of output:

$$(29) \quad \frac{lw}{y} = 1 - \alpha$$

Net labour income share of output

$$(30) \quad \frac{lw(1 - \tau_L)}{y} = (1 - \alpha)(1 - \tau_L)$$

Gross return on capital:

$$(31) \quad r = y_k = \alpha A^{\frac{\eta+1}{\alpha+\eta}} \left( \frac{k^{-\eta} (1 - \alpha) (1 - \tau_L)}{V c^\gamma} \right)^{\frac{1-\alpha}{\alpha+\eta}}$$

Net return on capital:

$$(32) \quad (r - \delta)(1 - \tau_K) = \alpha A^{\frac{\eta+1}{\alpha+\eta}} \left( \frac{k^{-\eta} (1 - \alpha) (1 - \tau_L)}{V c^\gamma} \right)^{\frac{1-\alpha}{\alpha+\eta}} (1 - \tau_K) - \delta(1 - \tau_K)$$

Gross capital income share of output:

$$(33) \quad \frac{kr}{y} = \alpha$$

Net capital income share of output:

$$(34) \quad \frac{k(r - \delta)(1 - \tau_K)}{y} = \alpha(1 - \tau_K) - \left( \frac{k^\eta}{A} \right)^{\frac{\eta+1}{\alpha+\eta}} \left( \frac{V c^\gamma}{(1 - \alpha)(1 - \tau_L)} \right)^{\frac{1-\alpha}{\alpha+\eta}} \delta(1 - \tau_K)$$

And again finally the ratio of net capital income to net labour income:

$$(35) \quad \chi \equiv \frac{k\bar{r}}{l\bar{w}} = \frac{1 - \tau_K}{1 - \tau_L} \left( \frac{\alpha}{1 - \alpha} - \delta \left( \left( \frac{Vc^\gamma}{1 - \tau_L} \right)^{1-\alpha} k^{\eta(1-\alpha)} (A(1 - \alpha))^{-(\eta+1)} \right)^{\frac{1}{\alpha+\eta}} \right)$$

Note that  $\delta = 0$  no longer forces  $\chi$  to be constant. As one would expect, greater capital taxation reduces  $\chi$  and greater labour taxation increases it.

#### 4. CONCLUDING REMARKS

It is evident from (35) and, in particular, from (26) that output elasticity of capital is largely responsible for  $\chi$ . This would predict that  $\chi$ , and some of the multiple inequality measures, will be higher in countries with a large  $\alpha$ .<sup>4</sup> Obviously, the actual value of  $\chi$  and the level of inequality, can be, and is, in real life modified by the taxation policy and the resulting levels of capital and consumption.

In a companion paper [Krawczyk and Townsend \(2015\)](#) we discuss historical evidence for inequality in New Zealand and its correlation with  $\chi$ , which we will refer to as the *factor ratio* strength or factor ratio for shortness.

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#### REFERENCES

Alvaredo, F., Atkinson, A.B., Piketty, T., Saez, E., 2014. The world top incomes database. URL: <http://topincomes.g-mond.parisschoolofeconomics.eu/>. accessed 08/12/2014.

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<sup>4</sup>We confirmed this comparing gross operating surplus from OECS data to world top income database, see [Alvaredo, Atkinson, Piketty and Saez \(2014\)](#).

- Brock, W.A., Turnovsky, S.J., 1981. The analysis of macroeconomic policies in perfect foresight equilibrium. *International Economic Review* 22, 179–209.
- Hall, J., Scobie, G., 2005. Capital shallowness: A problem for new zealand? URL: <http://www.treasury.govt.nz/publications/research-policy/wp/2005/05-05/>.
- Judd, K.L., 1985. Redistributive taxation in a simple perfect foresight model. *Journal of public Economics* 28, 59–83.
- Judd, K.L., 1987. The welfare cost of factor taxation in a perfect-foresight model. *Journal of Political Economy* 95, No. 4, 675–709.
- Krawczyk, J.B., Judd, K.L., 2015. A note on determining viable economic states in a dynamic model of taxation. *Macroeconomic Dynamics* forthcoming.
- Krawczyk, J.B., Shimomura, K., 2003. Why countries with the same technology and preferences can have different growth rates. *Journal of Economic Dynamics and Control* 27, 1899–1916. URL: [http://www.vuw.ac.nz/staff/jacek\\_krawczyk/somepapers/JEDC\\_Koji\\_J.pdf](http://www.vuw.ac.nz/staff/jacek_krawczyk/somepapers/JEDC_Koji_J.pdf).
- Krawczyk, J.B., Townsend, W., 2015. New Zealand inequality and the struggle between capital and labour. Working Paper Series xxxx. Victoria University of Wellington, School of Economics and Finance.
- Piketty, T., 2014. *Capital in the Twenty-First Century*. Belknap Press.
- Piketty, T., Zucman, G., 2014. Wealth and inheritance in the long run URL: <http://ssrn.com/abstract=2501546>.
- Szeto, K.L., 2001. An econometric analysis of a production function for new zealand URL: <http://www.treasury.govt.nz/publications/research-policy/wp/2001/01-31/>.
- Tipper, A., 2012. Capital-labour substitution elasticities in new zealand: One for all industries? URL: <http://www.stats.govt.nz/methods/research-papers/working-papers-original/capital-labour-substitution-elasticities-nz-12-01.aspx>.