Fixed come hell or high water? Selection and prepayment of fixed rate mortgages outside the US

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Abstract

We examine the decision to prepay a fixed rate mortgage in the UK, Canada, Ireland, Australia and New Zealand. These countries are characterised by having substantial fees which are associated with breaking a fixed rate mortgage. We develop a model which allows for fluctuations both in banks' wholesale rates and credit spreads. We find that households can achieve economically significant benefits both from following an optimal prepayment strategy contingent on the break fee used by their bank, and also by selection of fixed interest rate term and (where available) break fee structure.

Refinancing of mortgages is a well understood phenomenon in the US, both by households and academics. Theoretical work (such as Kau, Keenan, Muller and Epperson (1987, 1992, 1993) and

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Stanton (1995)) has been supplemented and supported by empirical work (such as Deng, Quigley and van Order (2000)) suggesting that households do indeed manage their mortgages in an optimal fashion.

In contrast, however, refinancing outside the US is relatively unexplored territory. Countries such as the UK, Canada, Australia, New Zealand and Ireland, contrary to the US, do not have widespread securitisation of mortgages (see Murphy (1996)). As a result, individual banks are left bearing all risk concerned with their lending. To mitigate this risk, contracts offered in these countries tend to be simpler than their US counterparts, rendering them easier instruments to hedge.

Firstly, mortgages are recourse loans. If a borrower fails to make payments on his loan, the bank can first seize the house securing the loan, and can then hold the borrower responsible for any remaining liability. This largely eliminates the first option available to many US households with respect to their debt: the default option.[1]

As well as differing from a default perspective, most loans only allow borrowers to fix their interest rates for fairly short periods of time (rarely exceeding five years) relative to the mortgage's overall life. Hence these fixed rate loans are more similar to hybrid adjustable rate mortgages (ARMs) in the US. Borrowers also have a predilection for variable rate loans, in contrast to US mortgagors, who generally choose to fix their loans. Daniel (2008) notes that in Australia approximately 80% of borrowers choose a variable rate loan over a fixed rate loan.[2] When a bank makes a fixed rate loan, it typically handles its interest rate risk by entering into an amortising swap so as to convert the borrower's fixed payments into a floating payment, which better matches the bank's funding sources (being mostly deposits or short maturity bills).

The last major distinction from US mortgages is the main focus of this paper. In all the countries considered here, prepayment of a fixed rate loan is associated with a financial penalty, which we refer to as a *break fee*.

In the case of the UK, as discussed in Azevedo-Pereira, Newton and Paxson (2002), this break fee is generally a constant proportion of the principal outstanding on the loan. The analysis of refinancing options is thus fairly similar to that in the US. Even after paying a fee to refinance, Azevedo-Pereira et al. (2002) find that households can have a valuable refinancing option. More recently, a number of UK banks have begun offering break fees which decline as the mortgage's fixed term declines.[3]

Canada and Ireland, in contrast to the UK, both feature break fees which are calculated more consistently with the bank's actual loss from the borrower's prepayment. Canadian and Irish banks will calculate a break fee for a household based on the income lost if the bank relent the money prepaid for a fixed term equal to the remaining fixed term of the original mortgage, but at interest rates prevailing at the time of the mortgage being broken. Since in most cases, the household is choosing to prepay their loan because rates have fallen, the notional new loan would be at a lower rate than the prepaid loan, and a fee is levied. We refer to such a break fee as a *retail* break fee, since its size will be critically contingent on movements in retail interest rates. In Canada, this fee is normally subject to a minimum break fee of three months interest. Canadian banks also often offer customers the choice of a break fee free mortgage (an *open* mortgage as opposed to the conventional *closed* mortgage) but at a substantial spread over the usual fixed rate, and generally for a short fixed term (one year or shorter).

A retail break fee accurately represents a lender's loss in the event that a new customer is found to replace the prepaying borrower. However, if no new customer can be found, an alternative response would be to unwind the swap which was used to align the mortgage's payments with the bank's funding sources. If a bank calculates its break fee to compensate for this loss, we refer to this as a *wholesale* break fee, since the profit or loss on the swap will be determined by wholesale rates.[4] This form of break fee is most prevalent in Australia and New Zealand. In these two countries, the market is divided between banks using a wholesale break fee policy, and those using a retail break fee policy. Table 1 summarises the different break fee regimes across countries.

[Table 1 about here.]

We note that retail and wholesale break fees can vary wildly as interest rates change. Figure 1 shows a history of five year retail and wholesale rates in New Zealand over the last ten years, along with proportional break fees for a five year fixed rate loan prepaid after one year. In many instances, rates were higher at the break date than at inception, resulting in no break fee. However, during 2009, retail break fees rose to be as high as 8.7%, and wholesale break fees peaked at 15.2% of principal prepaid. We further note that in spite of this, there was substantial prepayment activity during 2008, with at least 6.9% of all five year fixed term mortgages being prepaid in December 2008.

[Figure 1 about here.]

Much empirical work has been undertaken on mortgage behaviour in these countries.[5] However, with the exception of the work by Azevedo-Pereira, Newton and Paxson (2002, 2003) on the UK mortgage system, very little theoretical work has been done to understand the effect of the refinancing option for households. In particular, it is unclear the extent to which mortgages subject to retail or wholesale break fees contain embedded refinancing options, if at all. The stylised fact (see Breslaw, Irvine and Rahman (1996) and Miles (2004)) has been one wherein households make a decision between uncertain payments and a fixed payment - a decision which is largely contingent on risk preferences.[6]

This paper aims to perform a similar option theoretic analysis to that offered for US mortgages. We develop a two-factor model for mortgage interest rates, where retail rates are composed of a wholesale rate plus a spread. We explore the optimal exercise policy for mortgage refinancing under the various break fee regimes prevalent in our collection of countries. We find scope for mortgagors to profit by managing their refinancing decision particularly under mechanical break fees (as found in the UK) and under wholesale break fees. We then explore, in the spirit of Stanton and Wallace (1998), whether there exists an optimal mortgage for a household, based on interest rates and the household's expected tenure in the property. We find that mortgagors can, in some circumstances, achieve substantial economic gains through mortgage selection. We also find that in the UK and in Australasia, mortgagors may have an optimal bank to approach, based on break fee policies.

The layout of the remainder of the paper is as follows: the next section discusses our theoretical model for mortgage prepayment decision, the following section discusses the estimation of model parameters, while the subsequent section presents the findings of our work, both in terms of exercise strategies and mortgage selection. The final section is the conclusion.

Theory

As noted in the introduction, loans in the collection of countries considered in this paper are recourse loans, eliminating the possibility of profitable default. Although default can occur (and we will allow for this in our modelling), it will in general leave the household worse off. We therefore model default as an involuntary activity which may occur with some probability, and result in immediate prepayment of the mortgage. Since the payoff in this state does not depend on the underlying house's price, a mortgage's value becomes independent of house prices, and depends purely on interest rates.

When dealing with wholesale break fees (as encountered in Australia and New Zealand) the fee paid by a prepaying household will depend on the level of wholesale swap rates between banks, while the rate they face on a new loan will depend on retail rates. As a result, our model will require two state variables, one to capture movements in wholesale rates, and one to capture credit spreads between wholesale rates and retail rates. From these state variables, we will infer a term structure of mortgage rates.

For each mortgage, we will also infer a *hedge rate*. The hedge rate is defined as being the wholesale fixed rate which a bank would pay if it agreed to receive a (wholesale) floating rate payment based on a principal which declines according to the mortgage's principal. Entering into such a swap allows the bank to eliminate the risk of wholesale rate fluctuations caused by financing a customer's fixed rate loan with short term (floating rate) debt. We will also be interested in valuing such a swap (given that interest rates have changed since its inception) as this will be critical to evaluating wholesale break fees.

Mortgage and hedge rates

We assume that mortgage rates are determined by two state variables: an instantaneous wholesale rate r_t, and an instantaneous credit spread s_t. Under the real world probability measure ($\mathbb{P}$), the two variables follow the processes:

$$
\begin{aligned}
dr_t &= a_r(\mu_r - r_t)dt + \sigma_r r_t^{\gamma_r} d\tilde{W}_{rt} && (1)\\
ds_t &= a_s(\mu_s - s_t)dt + \sigma_s s_t^{\gamma_s}(\rho d\tilde{W}_{rt} + \sqrt{1-\rho^2}d\tilde{W}_{st}) && (2)
\end{aligned}
$$

where a_r, a_s, μ_r, μ_s, γ_r, γ_s, ρ, σ_r and σ_s are constants, and $d\tilde{W}_{rt}$ and $d\tilde{W}_{st}$ are independent Brownian motions. This model allows for r and s to evolve randomly over time (t) with a magnitude of volatility given by σ_r and σ_s. The variables γ_r and γ_s allows for absolute volatility to have varying levels of proportionality with the level of rates and spreads (respectively). When γ_r or γ_s equals zero, rate/spread absolute volatility does not depend on the level of rates/spreads (the Vasicek model) while higher levels of γ_r and γ_s will result in higher rates/spreads leading to greater variability in rate/spread innovations. In addition to volatility specification, the model allows for mean reversion in rates/spreads, so that the wholesale rates and credit spreads tend to return to levels μ_r and μ_s respectively. The speed of mean-reversion can also be varied by varying a_r and a_s. Broze, Scaillet and Zakoïan (1994) show that the individual processes are stable (unconditional moments exist) provided γ_r and γ_s are both less than one.

Although the dynamics of interest rates in the real world are interesting for empirical work, in order to price securities or make decisions regarding refinancing, we must work in the risk neutral probability measure. We assume constant market prices of risk (ξ_r and ξ_s for the two Brownian motions respectively) so that under the risk-neutral probability measure ($\mathbb{Q}$), we have $d\tilde{W}_{rt} = dW_{rt} - \xi_r dt$ and $d\tilde{W}_{st} = dW_{st} - \xi_s dt$ where dW_{rt} and dW_{st} are the increments of independent Brownian motions under $\mathbb{Q}$, so that:

$$dr_t = \left[a_r(\mu_r - r_t) - \xi_r \sigma_r r_t^{\gamma_r}\right] dt + \sigma_r r_t^{\gamma_r} dW_{rt} \tag{3}$$

$$ds_t = \left[a_s(\mu_s - s_t) - \hat{\xi}_s \sigma_s s_t^{\gamma_s}\right] dt + \sigma_s s_t^{\gamma_s}(\rho dW_{rt} + \sqrt{1-\rho^2} dW_{st}) \tag{4}$$

where $\hat{\xi}_s \equiv \rho \xi_r + \sqrt{1-\rho^2}\xi_s$.

The *instantaneous* retail mortgage rate is given by $r_t + s_t$. Longer term discount factors can be inferred from these processes. We denote the *retail discount factor* for time T, observed at time t as $d_t^R(T) = E_{\mathbb{Q}}\left(e^{-\int_t^T r_\theta + s_\theta d\theta}\right)$, and the *wholesale discount factor* as $d_t^W(T) = E_{\mathbb{Q}}\left(e^{-\int_t^T r_\theta d\theta}\right)$.[7] Discount factors represent the values of zero coupon bonds maturing at future dates, discounted at either retail (d_t^R) or wholesale (d_t^W) rates. Closed form solutions are available for wholesale discount factors, provided $\gamma_r = 0$ or $\gamma_r = 0.5$, and for retail discount factors under the condition $\rho = 0, \gamma_r = 0$ or $0.5, \gamma_s = 0$ or $0.5, \gamma_r \xi_r = \gamma_s \xi_s = 0$, since in these cases the model reduces to the affine framework (see Duffie and Kan (1996)). For more general parameter cases (as explored in this paper) numerical solution is required.[8]

Having established a set of dynamics for short term interest rates, we now show how longer term interest rates can be derived from this framework. Specifically, as mentioned earlier, we are interested in two sets of rates: mortgage rates themselves, and the rate appropriate for an amortising swap which a bank might use to hedge its position in a mortgage (the corresponding hedge rate).

Given (3-4), standard asset pricing results allow us to characterise the value of securities as the solutions of partial differential equations. Denoting the value of a security as g per unit of principal, and the contract's time t principal as P_t, we can write:

$$
\begin{aligned}
& \frac{1}{P_t}\frac{\partial}{\partial t}\left[gP_t\right] + \left[a_r(\mu_r - r_t) - \xi_r\sigma_r r_t^{\gamma_r}\right]\frac{\partial g}{\partial r_t} + \left[a_s(\mu_s - s_t) - \hat{\xi}_s\sigma_s s_t^{\gamma_s}\right]\frac{\partial g}{\partial s_t} \\
& + \frac{1}{2}\frac{\partial^2 g}{\partial r_t^2}\sigma_r^2 r_t^{2\gamma_r} + \frac{1}{2}\frac{\partial^2 g}{\partial s_t^2}\sigma_s^2 s_t^{2\gamma_s} + \frac{\partial^2 g}{\partial r_t \partial s_t}\rho\sigma_r r_t^{\gamma_r}\sigma_s s_t^{\gamma_s} \\
= \; & A(r_t, s_t)g - K_t. \qquad (5)
\end{aligned}
$$

Here $A(r_t, s_t) = r_t$ for a security which is valued by discounting at wholesale rates (such as the fixed of floating legs of a swap) and $A(r_t, s_t) = r_t + s_t$ for an instrument whose value should be discounted at retail rates (such as when valuing the remaining payments of a mortgage). K_t is the payment which the security makes at time t (per unit of principal) which may itself be a function of r_t and s_t (as would be the case in valuing the floating leg of a swap).

We denote the time t_0 mortgage rate for a new mortgage with maturity date T and end of fixed period $\tau < T$ as $R_{t_0}(\tau, T)$. Mortgage rates must be consistent with the process for retail rates. This implies that the present value of the outstanding principal at the end of the mortgage's fixed period, plus the value of the mortgage's fixed payments must be equal to the initial principal of the loan (here assumed to be one). Mathematically, assuming continuous payment of interest and principal, for a mortgage incepted at time t_0 and observed at a later time t, the continuous coupon payment on the mortgage per unit of principal (and unit of time) is given by:

$$
C_t(t_0, \tau, T, R_{t_0}(\tau, T)) = \frac{R_{t_0}(\tau, T)}{1 - e^{-R_{t_0}(\tau, T)(T-t)}}. \qquad (6)
$$

Outstanding principal at time t is given by

$$P_t = \frac{1 - e^{-R_{t_0}(\tau,T)(T-t)}}{1 - e^{-R_{t_0}(\tau,T)(T-t_0)}}. \tag{7}$$

Given (6) and (7), the mortgage's rate $R_{t_0}(\tau, T)$ must satisfy

$$\frac{1 - e^{-R_{t_0}(\tau,T)(T-\tau)}}{1 - e^{-R_{t_0}(\tau,T)(T-t_0)}} d^R_{t_0}(\tau) + \frac{R_{t_0}(\tau, T)}{1 - e^{-R_{t_0}(\tau,T)(T-t_0)}} \int_{t_0}^{\tau} d^R_{t_0}(\theta) d\theta = 1. \tag{8}$$

If we know the value of a zero coupon bond maturing at the end of the mortgage's fixed term, and the annuity factor ($\int_{t_0}^{\tau} d_{t_0}(t)dt$) we can find the mortgage rate $R_{t_0}(\tau, T)$ by solving (8). These two components can be priced by solving (5). In the case of the zero coupon bond, we set $P_t = 1$, $A(r_t, s_t) = r_t + s_t$ and $K_t = 0$, with terminal condition $g(T) = 1$. For the annuity factor, we set $P_t = 1$, $A(r_t, s_t) = r_t + s_t$, $K_t = 1$, and set the terminal condition $g(T) = 0$.

We also define the associated *hedge rate* for a new mortgage set up at time t_0, with maturity T and fixed period ending at time τ as $r_{t_0}(\tau, T)$. The hedge rate is defined as the fixed rate which would be paid on a $\tau - t_0$ year fixed-floating swap which has declining principal identical to that of the associated mortgage. To calculate the hedge rate, we solve

$$E_{\mathbb{Q}} \left(\int_{t_0}^{\tau} P_\theta r_\theta e^{-\int_{t_0}^{\theta} r_z dz} d\theta \right) - r_{t_0}(\tau, T) \int_{t_0}^{\tau} P_\theta d^r_{t_0}(\theta) d\theta = 0. \tag{9}$$

The first term of (9) represents the value of the floating leg of the swap, while the second term represents the value of the fixed leg. We can evaluate the first integral by solving (5), with $A(r_t, s_t) = r_t$ and $K_t = r_t$. The second integral is a wholesale annuity factor (albeit with declining principal) and can be solved similarly to the retail case, with $A(r_t, s_t) = r_t$ and $K_t = 1$. For both valuing the floating leg of the swap, and for valuing the annuity factor, the terminal condition is $g(T) = 0$.

Mortgage valuation

We assume that a borrower may need to sell their house for reasons exogenous to the model (examples of these could be an inability to service the loan due to income loss, lifestyle considerations causing the borrower to need a larger/smaller home, or moving locations for employment reasons). In this case, the borrower could either exit the housing market, or could buy a new house and require a new mortgage. For simplicity, we assume that in the event of buying a new house, the new mortgage is of identical (or larger) size to the existing loan, so that we can treat this as a refinancing of existing debt.[9] These two outcomes (exiting the housing market and buying a new house) occur with constant Poisson intensities of $\lambda_1 dt$ and $\lambda_2 dt$ respectively. In both cases, the household must pay any break fees associated with their existing mortgage.[10]

A mortgage's value at any point in time will depend on its structure, the current level of r and s, and also the mortgage's interest rate. For mortgages with wholesale break fees, the mortgage's associated hedge rate will also affect its value, since it will affect break fees, if they occur. Hence we may write the value of a mortgage, observed at time t, created at time $t_0 \leq t$, per unit of principal as being:

$$f(t, r_t, s_t, \tau, T, R_{t_0}(\tau, T), r_{t_0}(\tau, T)).$$

A borrower choosing a new mortgage at time t, where maturities of $t, t+1, \dots t+\bar{\tau}$ are available, solves the problem:

$$\min_{\substack{\tau=t,\ldots,t+\bar{\tau}\\ \tau\leq T}} f(t,r_t,s_t,\tau,T,R_t(\tau,T),r_t(\tau,T)) \equiv f^*(t,T,r_t,s_t).$$

Note that $\tau = t$ is a possibility, since the household could choose a floating rate loan instead of a fixed loan.

Since the mortgage's value depends on the two state variables r_t and s_t, standard no-arbitrage arguments show that f satisfies the differential equation:

$$\begin{aligned}
& \frac{1}{P_t}\frac{\partial}{\partial t}\left[P_t f\right] + \left[a_r(\mu_r - r_t) - \xi_r \sigma_r r_t^{\gamma_r}\right]\frac{\partial f}{\partial r_t} + \left[a_s(\mu_s - s_t) - \hat{\xi}_s \sigma_s s_t^{\gamma_s}\right]\frac{\partial f}{\partial s_t} \\
& +\frac{1}{2}\frac{\partial^2 f}{\partial r_t^2}\sigma_r^2 r_t^{2\gamma_r} + \frac{1}{2}\frac{\partial^2 f}{\partial s_t^2}\sigma_s^2 s_t^{2\gamma_s} + \frac{\partial^2 f}{\partial r_t \partial s_t}\rho\sigma_r r_t^{\gamma_r}\sigma_s s_t^{\gamma_s} \\
& +\lambda_1(1 + B_t - f) + \lambda_2(f^* - f + B_t) \\
\leq\ & (r_t + s_t)f - C_t(t_0,\tau,T,R_{t_0}(\tau,T)) \qquad (10)
\end{aligned}$$

where B_t is the full break fee associated with the mortgage, which potentially depends on the current level of interest rates, as well as the mortgage's characteristics (see below). C_t and P_t are given by equations (6-7).

Equation (10) is defined in conjunction with either one or two complementary slackness conditions, associated with the refinancing options available to the borrower. The first condition applies to all mortgages:

$$f \leq f^* + B_t, \tag{11}$$

meaning that a household can always choose to completely repay their mortgage. However, in some circumstances, a household may be forgiven a certain portion of their loan prepayment, as a fraction of their *initial* principal (denoted ψ). In this case, it is also potentially optimal for the household to prepay only the forgiven portion of their loan, in which case we have the additional complementary slackness condition

$$f \leq \max\left(0, 1 - \frac{\psi P_{t_0}}{P_t}\right) f' + \min\left(1, \frac{\psi P_{t_0}}{P_t}\right) f^* \tag{12}$$

where f' is the value (per unit of principal) of an otherwise identical mortgage where $\psi = 0$. In the presence of a forgiven portion of the loan, we replace the break fee the household would pay for terminating their entire loan (B_t in equations (10) and (11)) with

$$B_t^{(\psi)} = B_t \min\left(1, \frac{\psi P_{t_0}}{P_t}\right).$$

Lastly, the terminal condition for (10-12) is given by $f(\tau) = f^*(\tau)$, since on conclusion of the mortgage's fixed term, the mortgagor can freely choose a new fixed term.

Note that equation (10) is similar to equation (5) with the addition of the λ terms (handling *suboptimal* prepayment) and the replacement of equality with an inequality coupled with the complementary slackness conditions (handling *optimal* prepayment).

Calculating break fees

As discussed in the introduction, we consider three types of break fee: those which do not depend on interest rates, those based on retail rates, and those based on the cost of breaking the swap a bank has entered into. We focus here on a household who prepays their entire principal outstanding. Similar calculations hold for partial prepayments, scaled accordingly. We express break fees relative to current principal outstanding, as per (10).

Non-rate dependent break fees

The break fees found in the United Kingdom are the most straightforward to describe. We consider two possible break fees here. The first break fee is simply a flat portion of the principal outstanding, as considered by Azevedo-Pereira et al. (2002) and Chen, Connolly, Tang and Su (2009).

Break fee (flat) $= \zeta_1$

where ζ_1 is a constant. The second type of break fee considered is a *declining* fee. Here the fee is proportional to the time remaining of the fixed term:

Break fee (declining) $= \zeta_2(\tau - t)$

where ζ_2 is a constant. In general $\zeta_2 > \zeta_1$. The rationale for charging a declining fee is that the bank's loss, either due to having to relend the prepaid principal at lower rates, or break a hedging swap, declines as the mortgage approaches the end of its fixed term. We can thus see the declining fee as a (very) rough approximation of the retail/wholesale break fee schemes.

Retail break fees

Here the fee is calculated according to the assumption that once the household pays off its loan, the bank will proceed to relend the money, under the same terms (τ and T) as the existing loan. We outline two possible ways in which this could be implemented.

Single discount rate (SDR) In this case, the bank calculates a rate for a new loan set up to mature at time T, and conclude its fixed period at time τ. The household's break fee is the value of their remaining payments discounted at this rate ($R_t(\tau, T)$), less the principal they are repaying:

$$\begin{aligned}
\text{Break fee (SDR. retail)} &= \frac{P_\tau}{P_t} e^{-R_t(\tau,T)(\tau-t)} \\
&\quad + C_t(t_0, \tau, T, R_{t_0}(\tau, T)) \int_t^\tau e^{-R_t(\tau,T)(\theta-t)} d\theta - 1 \\
&= \frac{1 - e^{-R_{t_0}(\tau,T)(T-\tau)}}{1 - e^{-R_{t_0}(\tau,T)(T-t)}} e^{-R_t(\tau,T)(\tau-t)} \\
&\quad + \frac{1 - e^{-R_t(\tau,T)(\tau-t)}}{1 - e^{-R_{t_0}(\tau,T)(T-t)}} \frac{R_{t_0}(\tau, T)}{R_t(\tau, T)} - 1. \qquad (13)
\end{aligned}$$

Note that for the case where $R_t(\tau, T) = R_{t_0}(\tau, T)$, the rate the bank will (notionally) relend the money at is identical to the borrower's current rate, and the break fee will be zero. If the break fee is positive (the new rate is lower than the existing rate) the borrower will pay the fee. If the break fee is negative, the borrower will pay nothing.

Full valuation (FV) In this case, the bank values the household's remaining payments, discounting each at the appropriate zero coupon rate for that date, using (8). Given knowledge of the retail annuity factor, and of the retail discount factor for the maturity date of the mortgage, we can easily calculate the value of the mortgage's remaining payments. The break fee is then given by the difference between this market value and the principal the household repays:

$$\begin{aligned}\text{Break fee (FV. retail)} &= \frac{1-e^{-R_{t_0}(\tau,T)(T-\tau)}}{1-e^{-R_{t_0}(\tau,T)(T-t)}}d_t^R(\tau) \\ &+\frac{R_{t_0}(\tau,T)}{1-e^{-R_{t_0}(\tau,T)(T-t)}}\int_t^\tau d_t^R(\theta)d\theta-1. \end{aligned} \tag{14}$$

Note that the only differences between (13) and(14) are the substitution of $d_t^R(\theta)$ for $e^{-R_t(\tau,T)(\theta-t)}$ as the integrand of the second term and $d_t^R(\tau)$ for $e^{-R_t(\tau,T)(\tau-t)}$ in the first term. The FV and SDR break fees will not exactly match (except for the case where $R_t(\tau,T)=R_{t_0}(\tau,T)$), since the new retail rate is calculated based upon amortising occurring consistent with the new rate, whereas the existing cash flows are amortising consistent with the mortgage's original rate. The difference between these two numbers are generally small, except in situations where the change in interest rates has been very large. In our analyses in this paper, for brevity, we use only the full valuation technique in our retail rate calculations.[11]

Wholesale break fee

In this case, the lender assumes that in the event of prepayment, no new loan is entered into, so that the mortgagee's hedge position must be unwound. The swap in question would have a notional principal on any date equal to the amortised principal of the actual mortgage. The hedge swap can be valued after its initial creation, at time t, per unit of principal, by evaluating the difference between the floating leg (which the bank receives) value and the fixed leg (which the bank pays) value:

$$E_{\mathbb{Q}}\left(\int_t^\tau \frac{P_\theta}{P_t} r_\theta e^{-\int_t^\theta r_z dz} dt\right) - r_{t_0}(\tau, T) \int_t^\tau \frac{P_\theta}{P_t} d_t^r(\theta) d\theta. \tag{15}$$

Note that (15) is simply the left hand side of (9).

There are certain similarities between the retail break fee and the wholesale break fee. In both cases, if interest rates decline, the household will be penalised (either because the household's remaining payments will be more valuable, or because the bank will have lost money on its swap). The sensitivity of this penalty will also decline as the loan approaches the end of its fixed term. However the critical difference between the two fees is the *rate* which determines the break fee. In the case of the wholesale break fee, the rate which is critical for the household's valuation of the mortgage is the *retail* rate, while its break fee is being calculated based on *wholesale* rates. As such, the household may profit substantially from refinancing in situations where wholesale rates have risen (the bank has made a profit on its swap) but retail rates have fallen (presumably due to a decline in s).

Solution technique

We proceed by first deriving term structures of interest rates for different levels of state variables r and s. Using these, we are then able to solve for mortgage values, conditional on the mortgage's remaining life, the initial fixed term, the current fixed term, and term structures of interest rates. We price these in tandem, allowing us to consider the refinancing options available to the household.

Further details are given in the appendix.

Parameter estimation

Data for one month wholesale interest rates are obtained from Datastream interbank rates. Floating mortgage rates are generally available from the central bank of each country. We augment these with longer maturity swap rates and mortgage rates. For Australia and New Zealand, we add 3 year rates, for Canada and the UK 5 year rates, and for Ireland we use the reported rate for loans from 1-5 year fixed term, which we take to represent a 3 year rate. Data cover the period January 1999 to December 2009, except for Ireland, where data are only available from January 2003.

For the UK floating rate, we use standard variable rate (SVR) data. Banks in the UK generally offer two different types of floating rate loan. The first rate, a "standard variable rate" loan, has payments based on the bank's own floating rate. A "tracker" loan, in contrast, has payments directly pegged to the Bank of England's interest rates. Banks often "buffer" their customers against changes in the Bank of England rates, lowering their SVR rates less in response to a cut, and raising SVR rates less in response to a rate hike by the central bank. The tracker loan is thus more similar to an American ARM rate. However, the standard variable rate loan is more comparable to floating rate loans in the other countries we consider here, so we use SVR data to estimate our model.

Stochastic processes (1-2) could be estimated using General Method of Moments (GMM) as described in Chan, Karolyi, Longstaff and Sanders (1992), however this approach would encounter two problems. Firstly, we are interested in not only estimating the real world ($\mathbb{P}$) process followed by interest rates, but also require estimates of the market prices of risk (ξ_r and ξ_s) in order to implement our model (using the $\mathbb{Q}$ probability measure). Secondly, as shown by Ball and Torous (1996) and Faff and Gray (2006), GMM estimates of short rate processes exhibit bias, particularly in the estimates of mean reversion parameters.

In light of these shortcomings, following Ball and Torous (1996) we implement an extended Kalman filter estimation of the short rate processes. We treat the short rates (r and s) as state variables and our floating mortgage rates, interbank interest rates, swap rates and longer term mortgage rates as observed variables. Since our observed rates (as discussed in the first section) depend on the risk-neutral processes followed by r and s, this approach also allows us to extract the market prices of risk.[12]

[Table 2 about here.]

Table 2 contains the results of our estimation. Perhaps the most striking observation is the negative correlation between spreads and the wholesale rate. This suggests that in most of our countries, as is the case in the UK, a rise in wholesale rates is not completely passed through to customers, nor is a decline. This effect, in New Zealand, is documented by Liu, Margaritis and Tourani-Rad (2008).[13]

The second empirical observation is the marked difference between a_r and a_s. With the exception of Ireland, all the countries exhibit a much more rapid mean reversion for credit spreads than is the case for wholesale rates. The implication of this for our model is that fluctuations in credit spreads will lead to marked changes in the slope of the retail mortgage yield curve, whereas changes in wholesale rates will generally result in more parallel shifts in the curve.

We note a reasonable dispersion in γ_r and γ_s. The Canadian and UK wholesale rate processes have very low values for γ_r showing some similarity to the Vasicek model ($\gamma = 0$). In contrast, both

Australian interest rate processes, the Irish credit spread process and the New Zealand wholesale rate process have values of γ between 0.5 and 1, lying somewhere between a Cox-Ingersoll-Ross process ($\gamma = 0.5$) and a Gaussian process ($\gamma = 1$). Evaluating the volatility terms ($\sigma_r r^{\gamma_r}$ and $\sigma_s s^{\gamma_s}$) at the long run levels of r and s (μ_r and μ_s respectively) gives an idea of the absolute volatility of wholesale rates and spreads, respectively. We find that this absolute volatility varies between 0.0075 (Ireland) and 0.0121 (New Zealand) for wholesale rates, and between 0.0022 (Ireland) and 0.0077 (Canada) for credit spreads. The lack of volatility in Ireland is not surprising, given that Ireland uses the Euro as a currency, and therefore interest rates in Ireland will be consistent with Eurozone rates.

Lastly, the estimates of ξ_r are universally negative. The result of this is that the steady state for wholesale rates is higher in the risk-neutral world ($\bar{r}$) than in the real world (μ_r). Hence the model will generate persistent upward sloping wholesale curves, as observed empirically. For credit spreads, with the exception of Canada, we observe the opposite behaviour, with the risk-neutral steady-state credit spread ($\bar{s}$) being less than the real world steady-state (μ_s).

Results

To understand the optimal selection of mortgages, it is important to begin by understanding when a mortgage should be prepaid. Since our model, by construction, offers borrowers long term rates which are consistent with the short interest rate processes, a borrower who takes out a fixed rate loan and services it until the end of the fixed term will have created a debt instrument whose value is identical to the initial principal of the loan. Similarly, a borrower who takes out a floating rate loan will also create a debt instrument which is valued at par. However, a borrower who exercises their option to prepay optimally, will produce an instrument whose value is below par. By examining the extent to which this is the case for a particular borrower for different mortgage structures, we can ascertain both the best choice of mortgage for the borrower, and also measure the economic gain from selection and timing of mortgage prepayment.

We thus begin by discussing the optimal prepayment of mortgages, exploring how this behaviour changes as the fee structure of the loan changes. Next, we explore the optimal selection of mortgages, both in terms of fixed horizon and break fee, and show how this varies across countries and levels of the state variables of our model. Lastly, we examine the economic significance of our results, both for borrowers and lenders.

Optimal prepayment strategies

For this section, we focus our attention on the United Kingdom, Canada and New Zealand. Since Irish banks make use of retail break fees, and Australian banks use retail and wholesale break fees, the results for New Zealand provide most of the intuition for these two countries. The United Kingdom case explores the effect of formulaic break fees, while the Canadian case highlights the effect of open versus closed mortgages. We return attention to Ireland and Australia in our subsequent discussion of mortgage selection and value gains.[14]

For each country, we consider two cases. In the first, the borrower is *intransient*, with $\lambda_1 = \lambda_2 = 0$. This borrower is assumed, therefore, to remain in their house for the entire life of the mortgage in question. The second borrower has $\lambda_1 = \lambda_2 = 0.1$. This borrower thus has a 20% probability of selling their house each year. Conditional on selling the house, with probability 50%, the borrower buys a new house (and therefore takes out a new mortgage), while with probability 50%, they do not buy a new house, and therefore do not require a new mortgage. Both borrowers are assumed to have taken on a 30 year amortising loan, and to have fixed their loan for five years. In all cases, the first 20% of prepayments are taken to be forgiven ($\psi = 0.2$ in inequality (12)).

Our goal in this section is to explore how a borrower with a five year fixed term mortgage should manage their prepayment decision. To evaluate the optimal exercise frontier, we require details of the borrowers' interest payments. For each country, we perturb the short rates/spreads generated by the extended Kalman filter for December 2009 so that selection of a five year fixed rate mortgage (with declining break fee in the UK, wholesale break fee in New Zealand and closed structure in Canada) is optimal for the intransient borrower, when compared with other maturities (see figures 5-7).[15] Our mortgage and hedge rates considered for each country are given in table 3.

A household with a mortgage can be in one of two circumstances. In the first circumstance, they have a mortgage which has not had any portion prepaid. In this case, they have two options available for them in terms of prepayment. The first option is to prepay the forgiven portion of their loan. This will entail no break fee. The second option is to prepay the entirety of their mortgage. This latter option will entail paying a break fee on the unforgiven portion. The second situation a household could find itself in would be where they have already prepaid the forgiven portion of their loan, and now their only option is to prepay the remaining portion of their loan, incurring a break fee in the process. Our graphs thus present two exercise frontiers for the options, the first being the combination of interest rates which would cause a household to pay off its initial forgiven portion, and the second being the set of interest rates (subsequent to this) which would cause prepayment of the unforgiven portion of the loan. In some circumstances, this second region of interest rates will be a superset of the first region, indicating that once the forgiven portion of the loan is prepaid, the household should immediately prepay the unforgiven portion.

[Table 3 about here.]

[Figure 2 about here.]

Before presenting results, we outline some over-arching principles. The household's option to refinance can be thought of as a call option, written on a bond which the household has issued to the bank. The household will exercise this option when the bond is valuable relative to the costs of prepaying and the time value of the option, which in turn depends on the volatility of interest rates and the remaining time to maturity of the option. For a US mortgage, with no break fee, this unambiguously occurs when retail rates are low. In our analyses, this relationship will be complicated by the fact that break fees may themselves depend on the current level of interest rates.

Results for the UK are given in figure 2. We assume $\zeta_1 = 0.03$ while $\zeta_2 = 0.01$ so that the household either pays a flat 3% break fee (fixed fee) or a 1% fee per year remaining in the loan (declining fee). Each line represents the locus of points which would cause the borrower to prepay their mortgage. Hence points below and to the left of the line are situations where the mortgage would be refinanced, and those above and to the right are where the mortgage should continue to be serviced. For each mortgage type, there are two lines. The first (solid) line represents the states where the household should prepay the forgiven portion of their loan. The second (dashed) line represents the states where the unforgiven portion of the loan should be prepaid. For each mortgage type and each borrower, we show frontiers for 1 year and 4 years remaining of the mortgage's fixed term.

In each case, there is a negative relationship between the level of wholesale rate and spread which trigger prepayment. This is because new rates the borrower can access depend on a combination of both state variables. For a low wholesale rate, even for quite a high credit spread, the household may be better off to refinance, since retail rates will still be low. Conversely, for a high wholesale rate, even a very low level of credit spread may still render refinancing suboptimal, since retail rates will still be high. The relationship is not *exactly* linear, because the mortgage depends on a combination of longer maturity rates, each of which depend non-linearly on the state variables (due to mean-reversion).

Comparing across the time dimension, we note that for the forgiven portion of the loan, the household becomes more willing to prepay as the end of the fixed term approaches (the exercise frontier shifts upwards). This is caused by the diminishing time value of their option. For the portion of their mortgage which attracts a break fee, we see that for borrowers with a fixed break fee, exercise is (relatively) more attractive with four years of fixed term remaining since the dashed curve is higher, requiring a smaller decline in r or s to trigger prepayment. Conversely, for the declining fee, exercise in the later years of the mortgage's fixed term is more attractive. This dichotomy makes intuitive sense, since with a flat break fee, the *per year* fee is lower during the early part of the mortgage, but higher later on.

Lastly, comparing the two borrowers (transient and intransient) we see very similar behaviour. In our later examples, we note a slight tendency for the transient borrower toward prepayment. Again, this follows from the time value of the option. Since the transient borrower faces the prospect of suboptimal prepayment, their option time value is lower, and they are more inclined to prepay.

[Figure 3 about here.]

In the Canadian case (see figure 3) we see the distinction between a closed and an open mortgage. For the closed mortgage, a retail break fee is calculated, with a minimum of three months interest charged. For the open mortgage, no break fee is charged, however, the borrower pays a 3% interest premium for this privilege.

Focusing first on the closed mortgage, we again note the rising exercise frontiers as the mortgage approaches the end of its fixed term, and a downward sloping frontier, due to the tradeoff between r and s in determining retail rates. We note that for longer maturities, a borrower is liable to break both sections of his mortgage in fairly rapid succession. Once rates have fallen low enough to motivate prepayment of the forgiven section, even a small further decline will trigger prepayment of the remainder of the loan. However, as maturity approaches, the three months interest minimum break fee becomes more important (relative to the amount saved by the household by refinancing) and renders prepayment less attractive.

For the open mortgage, there is no distinction between parts of the loan, since no fee is paid on prepayment. However, since the mortgage pays interest at a rate in excess of the regular retail yield curve, prepayment will be optimal at even higher levels of r and s than is the case for the forgiven portion of a *closed* loan.

[Figure 4 about here.]

Lastly, figure 4 contains the optimal policies for our New Zealand borrowers. For the retail break fee case, we note that the optimal exercise frontiers are actually slightly *lower* for the forgiven portion of the loan than is the case for the fee-bearing portion. The reason for this is that the retail break fee represents a lose-lose situation for the borrower: if the loan is prepaid when rates have fallen, the household loses all their refinancing gains to break fees. However, if rates have *risen*, the household is uncompensated. Once the free portion of the loan has been prepaid, it is therefore *immediately* optimal to prepay the remainder of the loan, since retail rates must be sufficiently low to place the household in the break fee bearing region (by virtue of having just triggered prepayment of the free portion).

The wholesale break fee case generates the most qualitatively different exercise frontiers. In this case, because the break fee is based on the level of *wholesale* rates, whereas the household's profit from refinancing is based upon *retail* rates, we actually see a hump shaped relationship between the levels of r and s which trigger prepayment. For high levels of wholesale rates, the household faces no fee, and therefore prepays the unforgiven portion of the mortgage similarly to the forgiven portion. However, if wholesale rates are low, the household faces a break fee which grows larger, the lower wholesale rates have become. Hence the household requires a much lower level of *retail rate* to trigger prepayment and therefore a lower level of credit spread. Note that if wholesale rates are low, this can result in situations where the household prepays their forgiven portion, and does not prepay the remainder of the loan, in contrast to the behaviour of a retail break fee borrower. Examining the decision of a household to prepay the *forgiven* portion of their loan, we observe two regions. For low wholesale rates (to the left of the figure) the household requires a lower retail rate to trigger prepayment. However for higher wholesale rates (to the right of the figure) the value of the prepayment option on the *unforgiven* portion of the loan rises dramatically, encouraging the household to prepay the *forgiven* portion, hence we see the household's optimal exercise frontier *rise* as wholesale rates increase, becoming consistent with the frontier for the unforgiven portion of the loan.

Optimal selection of mortgage type

We next turn our attention to the optimal selection of a mortgage contract. Here we consider a range of different levels for the instantaneous wholesale rate and spread. The optimal mortgage is the fixed term (and where appropriate, break fee regime) which minimises the value of the household's debt. We focus here on a household whose mortgage will amortise over thirty years.

We again present results for both intransient ($\lambda_1 = \lambda_2 = 0$) and transient ($\lambda_1 = \lambda_2 = 0.1$) borrowers. Clearly, this exercise could be repeated using a different level for λ_1 and λ_2, and would lead to different preferred mortgage types for different short wholesale rates and spreads. Our two sets of results provide a fairly clear indication of the qualitative effects of household transience on mortgage selection. In all cases, we allow the mortgagor to fix their loan for a maximum of five years, consistent with common lending practices. As was the case for analysis of optimal prepayment, we assume $\psi = 0.2$ for all mortgages.

[Figure 5 about here.]

In many ways, the UK mortgage system presents the simplest case of mortgage selection. The household chooses between a situation where early exercise of the prepayment option may be cheap at the expense of late exercise (a flat fee) versus a fee which is liable to be more in line with option exercise values (a declining fee). Figure 5 presents the optimal choice of mortgage. We consider $\zeta_1 = 0.006(\tau - t_0)$ and $\zeta_2 = 0.01$, so that the flat fee is 0.6% per year of fixed term, while the declining fee remains 1% per remaining year of the loan.

For low levels of r and s (which imply low levels of retail rates) the optimal choice of mortgage is to float. Taking on a fixed rate loan (with either type of fee) has little potential for profitable refinancing, since rates are unlikely to decline from present levels. Furthermore, for a transient borrower, there exists the potential for suboptimal prepayment to result in a fee being paid. As retail rates increase (moving up and to the right in the figures) the potential for a profitable refinancing becomes possible, and eventually exceeds the risk due to suboptimal prepayment. As this occurs, longer fixed terms become optimal. For the intransient borrower, there is no risk of suboptimal prepayment, and therefore even for modest levels of r and s, a fixed rate contract becomes the preferred choice.

The two borrower types also have a preference for break fees when fixing for protracted periods. For the transient borrower, uncertainty about their tenure in the loan reduces the value of later optionality, and therefore choosing a flat fee (which offers better prepayment opportunities early in the loan's life) is preferable to a declining fee when choosing a longer fixed term. Conversely, the intransient borrower has a preference for a declining fee, unless retail rates are very high (the top right corner of the figure).

[Figure 6 about here.]

Canada and Ireland (figure 6), in contrast to the UK situation, both face mortgage choices where the fee is based on the gain the household could achieve from refinancing their loan. The Irish case shows the effect of a retail break fee with no minimum charge (similar to those found in New Zealand and Australia). As noted when we examined optimal prepayment, this eradicates any potential for profitable refinancing of the unforgiven portion of the loan, restricting optionality to the forgiven portion. As was the case for the UK, borrowers trade off the risk of suboptimal exercise against optimal exercise. Not surprisingly, the results are qualitatively similar: for low interest rates, floating is optimal, and as rates increase, fixing for longer terms becomes preferred. Again, intransient borrowers have a preference for longer fixed terms.

Examining the case of the intransient Canadian borrower, we see similar behaviour. Low rates encourage fixing for short terms. We note that the open mortgage is only optimal for extremely high interest rates. For the transient borrower, however, results are quite stark. For almost all commonly encountered levels of interest rates, floating is the optimal choice. The enforcement of a lower bound on break fees almost completely eliminates option value for our transient borrower. We note that borrowers with lower levels of λ_1 and λ_2 may find fixing optimal under lower retail rate regimes, since their optimal selections will lie between the two cases considered here.

[Figure 7 about here.]

Figure 7 contains the optimal selection for a household in New Zealand (top panel) and Australia (bottom panel). For both countries, the figures can be essentially broken into three regions. For low interest rates (both low wholesale rates and low spreads), a floating rate mortgage is most valuable, for identical reasons to the UK, Irish and Canadian situation. As retail rates increase, fixing the mortgage becomes valuable, since the possibility of using the refinancing option becomes feasible. Generally speaking, the higher the retail rate, the longer the optimal fixing term becomes, since the refinancing option becomes more and more valuable.

The selection of whether to choose a retail or wholesale break fee is a function of the level of the credit spread and this divides the region where fixing is optimal into two sub-regions. As credit spreads increase, there becomes a possibility of a mortgagor with a wholesale break fee being able to refinance profitably, in a situation where retail rates fall, but wholesale rates rise (i.e. the decline in credit spread exceeds the rise in wholesale rates). This considerably increases the option value for wholesale break fees. In contrast, for retail break fees, the household has no ability to profit in the presence of break fees, and can only gain option value from the portion of their loan which is exempt from break fees.

However, this option value must be weighed against the risk to the household of suboptimal exercise. With a wholesale break fee, the household not only faces a loss if they prepay into a higher retail rate, but also in the case when they prepay and the retail rate has *fallen*, but the wholesale rate has *fallen by more* so that the break fee outweighs the mortgagor's profit.[16] This effect is most likely to occur when credit spreads are low, and so in these situations, the mortgagor finds it optimal to choose a retail based break fee.

Value gains from prepayment

Lastly, we turn our attention to the economic significance of the mortgage selection and optimal refinancing decision. As noted previously, a household who either takes a floating loan or holds a fixed loan until maturity creates a par value debt instrument. In contrast, a household who exercises their prepayment option optimally will reduce the value of their loan. Since floating is always an option, this will result in mortgage values below the loan's principal. By taking the difference between the mortgage's value and the loan's principal, we can ascertain the value gained from optimal prepayment and mortgage selection. Note that this gain to the household is a *loss* to the lender of exactly the same amount. This can be contrasted to the situation in US mortgages, where *default* on a loan could lead to different losses for the two counterparties, if the household may face future credit costs associated with the default. Here the situation is far simpler: a dollar gained by the mortgagor is a dollar lost to the mortgagee.

[Figure 8 about here.]

Figure 8 gives value gains for the United Kingdom. Clearly, the intransient investor achieves greater gains through mortgage timing. In both cases, it is the high interest rate environments, where long term fixing is optimal, where gains are greatest. We further note that in high credit spread environments, the gains for the borrower can be quite substantial.

[Figure 9 about here.]

Figure 9 shows results for Canada and Ireland. Given that break fees in both these countries are based on the retail methodology, the prepayment option is considerably less valuable than in the UK. In both these countries, the household can only profit on prepaying the forgiven portion of the loan. For the case of Canada, where fees have a minimum applied, we find that the gains for transient borrowers are quite small. Even for intransient borrowers, mortgage choice and prepayment is of small significance, rarely exceeding 1% of the loan's principal. Irish results are more substantial, with intransient borrowers frequently able to reduce loan value by 1.5%. Nevertheless, both types of borrowers' gains are far more subdued when compared to the UK numbers in figure 8.

[Figure 10 about here.]

Figure 10 shows results for Australia and New Zealand. For both countries, we observe the largest gains to be achieved in situations where the wholesale break fee loan is optimal (high credit spreads and high wholesale rates). Here the household can profit both from prepaying the forgiven portion of the loan, and also from prepaying the unforgiven portion.

Our results suggest three main lessons. Firstly, high interest rate environments promote fixing loans for longer periods and lead to more value being captured by households, particularly those who use their options optimally. Second, imposing minimum break fees, as in Canada, dramatically reduces option values. Thirdly, break fees which allow households to refinance profitably (as is the case with formulaic fees in the UK or wholesale break fees in Australasia) give the maximum scope for household value capture.

Conclusion

In this paper, we have established a framework for modelling the prepayment timing and mortgage selection problem of borrowers in a collection of countries whose lending regimes are characterised by potentially substantial fees associated with unwinding of a fixed rate contract. Our findings suggest that for all the countries considered, there is economically significant gain to be achieved by borrowers at the expense of lenders.

Two comments regarding why this might occur are in order. First, it is difficult to see how banks can price this out of the market. The problem for the lender is the unobservability of λ_1 and λ_2. Under the US mortgage system, the use of points (see Stanton and Wallace (1998)) allows lenders to generate a finely graded separating equilibrium where borrowers of varying degrees of transience pay different up-front fees in order to reduce their mortgage rates. Without a points system in place, the countries in our sample have only a relatively coarse set of products available, and therefore to price contracts so as to eliminate borrowers' option values is difficult.

The second point relates to credit risk. As mentioned in the introduction of this paper, the banking systems in our collection of countries are characterised by recourse lending regimes, meaning that lenders can prosecute defaulting borrowers for the full value of their loan, rather than being limited to seizing the property which secures the mortgage (as is the case in many US states). This eliminates the default optionality which is common in US mortgages. However, it does not mean that default is costless for the lender. Legal and administrative costs involved with default are still liable to be substantial, resulting in dead-weight losses in the event of default. Our model suggests that the households who best exploit prepayment opportunities are intransient households, who are also probably those households who are *less likely* to default on their loans. Indirectly, these "good risk" borrowers borrow more cheaply, since they extract gains from prepayment and loan selection. Prepayment options thus provide a mechanism for attracting more creditworthy borrowers.

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Appendix: Solving the mortgage valuation problem

Assuming a constant unit P_t, we solve equations (5) and (10) using the Craig and Sneyd (1988) Alternating Direction Implicit (ADI) finite difference algorithm. This technique works by splitting the operator in equation (10) across the dimensions of the problem (r and s). The technique then alternates which dimension is solved for implicitly, so that instead of requiring one huge system of equations to be solved, many smaller systems must be solved. Craig and Sneyd (1988) improve upon the regular Douglas (1962) style splitting by updating the cross-partial term between substeps of the algorithm. This maintains the second order accuracy of the method, allowing us to take larger time steps than would normally be possible when solving a problem with cross partial terms using ADI methods. The finite difference method calculates security values at discrete grid points $t = 0, \Delta t, 2\Delta t, \ldots, T$, $r = r_0, r_0 + \Delta r, \ldots, r_0 + N_r \Delta r$, $s = s_0, s_0 + \Delta s, \ldots, s_0 + N_s \Delta s$, where N_r and N_s are constant integers. Solution proceeds by backward induction, solving for f_t using the solution at $f_{t+\Delta t}$.[17]

When dealing with a security with declining principal (as is the case for mortgage valuation or hedge swap valuation), we value the security by rescaling the subsequent time step to account for the security's declining principal, i.e. working with a rescaled solution $\hat{f}_{t+\Delta t} = \frac{P_{t+\Delta t}}{P_t} f_{t+\Delta t}$ when evaluating the security value at time t (f_t).

We begin by using our algorithm to solve (5) in order to determine, for each point on our finite difference grid, $R_t(\tau, T)$ and $r_t(\tau, T)$ for $\tau = t + \Delta t, t + 1, t + 2, \ldots, \min(t + \bar{\tau}, T)$. This allows us to determine retail rates and associated hedge rates for new mortgages available to the borrower at any date.

Next, we work backwards from time T. At each time step, we solve (10), with $\lambda_1 = \lambda_2 = 0$ for all possible mortgages a household could hold. For a given type of break fee, this consists of: all possible levels of r_{t_0} and s_{t_0}, all possible current fixed terms $(\tau - t)$, and all possible initial fixed terms $(\tau - t_0)$. We repeat this exercise for each type of break fee available to the borrower, and also consider both the case where the mortgage has $\psi > 0$ and $\psi = 0$ (i.e. we consider mortgages who have already had their forgiven portion prepaid).

In tandem with solving for all the mortgages' values, we also solve (5) to value the annuity factors and zero coupon bond prices needed to evaluate equations (13 - 15). For the valuation of the hedge swaps, we use a similar trick to the mortgages, by rescaling the solution to account for declining principal.

Having solved (10) for each possible mortgage, we find f^* for each point in the grid. This allows us to evaluate whether it is optimal to prepay each of our possible mortgages. Since we know the value of mortgages with $\psi = 0$, we can evaluate whether it would be optimal to partially repay, if this option has not been exercised yet. Finally, we adjust the solution to allow probability $\lambda_1 \Delta t$ of prepayment without refinancing and probability $\lambda_2 \Delta t$ of prepayment with refinancing.

Our solution consists of matrices of mortgage values for each time step in our solution. We can also produce matrices of the optimal fixing period to generate f^*. Lastly, by examining the points where the complementary slackness conditions (11-12) hold with equality, we can observe when a household should prepay their mortgage.

[1]This is not to say that default losses do not exist, as even with the right to pursue a household, the household's limited liquid wealth may lead to the mortgagee being unable to recover the shortfall between the house's value and the mortgage's principal.

[2]Miles (2004) finds comparable numbers for the UK, with 25% of borrowers choosing a fixed rate in 2003. As at December 2009, 16% of Irish mortgages (by value) had fixed terms of more than one year. In New Zealand, 36% of loans (by value) were fixed for more than one year (although 38% were fixed for one year or less). Canada is slightly anachronistic: Breslaw et al. (1996) find 72% of Canadian mortgages are fixed rate (in 1988). More recently, the FIRM Residential Mortgage Survey (2009) finds 69% of Canadian loans are fixed rate.

[3]Much of the reason for the simplicity of UK break fees is due to the difficulty of enforcement of a more complex fee (see Miles (2004)). Fees are required by the Banking Ombudsman to be "calculatable in a way which is understandable to an ordinary borrower who has no understanding of how money markets work"(Banking Ombudsman's annual report 1998-1999).

[4]Since the bank's swap position will be paying a fixed rate and receiving a floating rate, the bank will make a loss when wholesale rates fall, and make a profit when wholesale rates rise.

[5]See, for example: Leece (2000) for the UK, Breslaw et al. (1996) and Zorn and Lea (1989) for Canada, and Daniel (2008) for Australia.

[6]See also the discussion of Campbell and Cocco (2003) and Koijen, Van Hemert and Van Nieuwerberger (2009) for the US.

[7]Our treatment of the pricing of default risk can be seen to be similar to that of Duffie and Singleton (1999), where our parameter s is the product of the intensity of default for the security and the expected portion of market value which is lost in the event of default.

[8]Our analysis implicitly assumes that markets are complete, and therefore mortgages are redundant securities. In

so far as households are unable to actually hedge their exposure to wholesale and retail interest rate shocks, this may be incorrect. The alternative, incomplete markets, approach (see for example, Koijen et al. (2009) and Campbell and Cocco (2003)) generally requires strong assumptions regarding household investments, and fairly abstract treatment of the mortgages. Since our analysis is largely focused on the technical details of mortgages, we consider the complete markets approach more suitable here.

[9]The model could potentially be augmented to allow borrowers to downsize by adding a third type of suboptimal prepayment, in which the household's new mortgage has lower principal than the outstanding loan.

[10]We note that in some cases, a household may be able to transfer their existing mortgage to a new house, hence avoiding any break fees. Where this is a possibility, we would regard λ_2 as reflecting the probability of moving with a mismatch between selling one property and buying the new one, which would require creation of a new mortgage.

[11]This exact system of break fee exists in Denmark, where mortgagors can discharge their mortgage by delivering mortgage backed bonds to the mortgagee, effectively purchasing another borrower's mortgage in the secondary market. See Frankel, Gyntelberg, Kjeldsen and Persson (2004).

[12]We use the approximations contained in Nowman (1997) for the first and second moments of the r and s innovations, approximating the covariance as ρ times the product of the standard deviations. We estimate the standard deviation of the noise in the observed interest rates as separate parameters, constraining these to lie between one basis point and one percent. To generate the initial unconditional distribution of the state variables, we use our finite difference scheme to evaluate variances and covariances for the rates over a 30 year timeframe.

[13]Given empirical evidence from the US corporate bond market, we also should not be surprised to find that default intensities and losses (measured by s) are negatively correlated with interest rates (measured by r). See, for example, Duffee (1999).

[14]Results for Ireland and Australia are qualitatively similar to those for New Zealand, and are available on request from the authors.

[15]For the UK case, the transient borrower finds a 5 year flat fee mortgage optimal. For New Zealand, the transient

borrower also finds a 5 year wholesale fee optimal. For Canada, the transient borrower prefers to float. For the Canadian case, setting a rate high enough for both borrowers to find five year fixed rates optimal would result in unrealistically high wholesale rates (see figure 6).

[16]Many households in Australasia found themselves in exactly such a situation during the recent credit crunch. See figure 1.

[17]An accessible reference for PDE solution is Ames (1992). See in particular, pages 349–354.

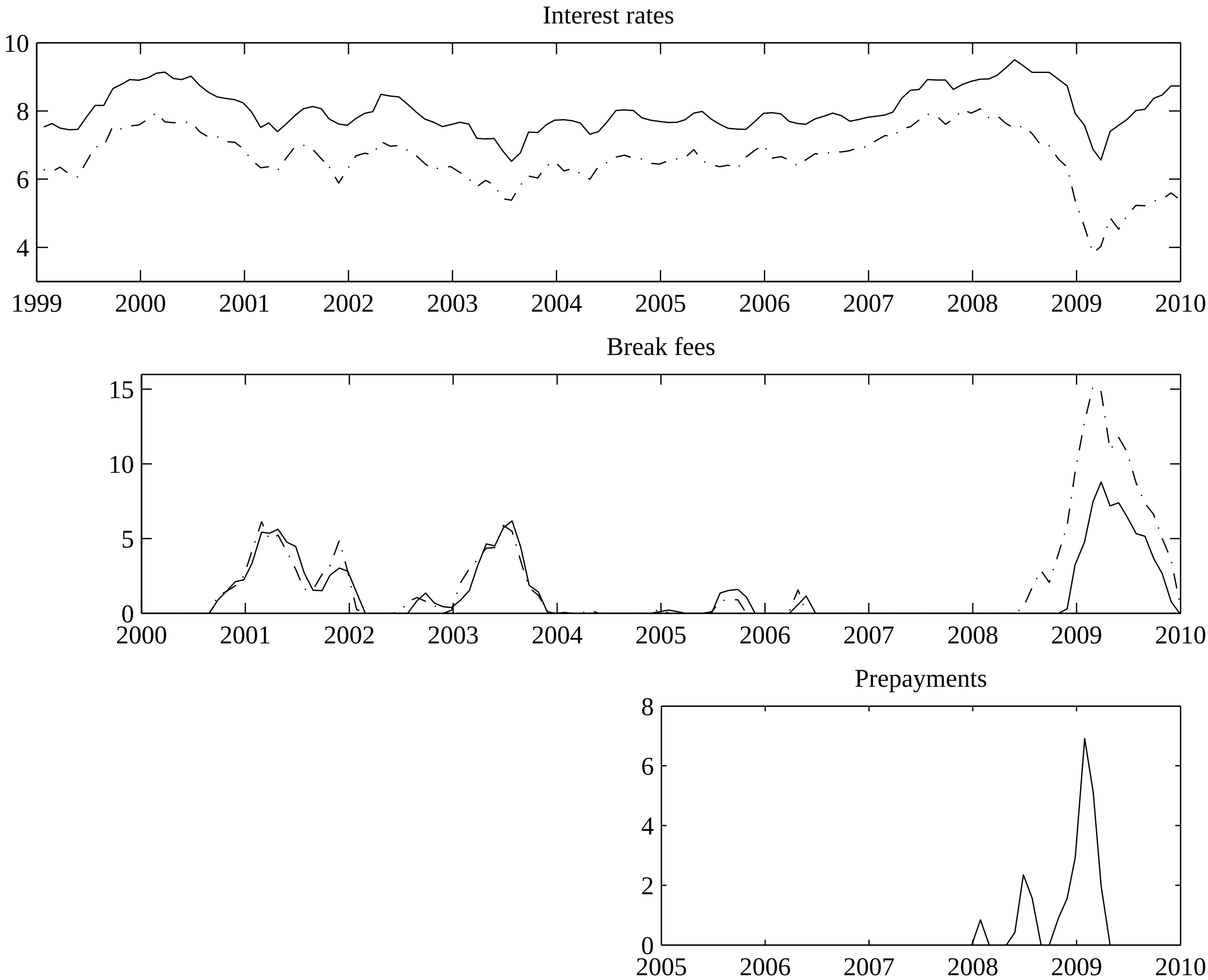


Figure 1: Top graph shows five year mortgage rates (solid) and corresponding hedge swap rates (dashed) for New Zealand. Both are measured in percent. Middle graph shows retail break fees (solid) and wholesale break fees (dashed) for the same period, assuming a mortgagor took out a five year fixed rate loan (with 30 year amortisation) and broke this one year later, as a percentage of principal prepaid. The third graph represents a conservative lower bound on 4-5 year mortgage refinancing activity (measured in percent) based on bank stocks of mortgages sorted by maturity. We assume no new business at the five year fixed term and that mortgages in each age category are uniformly distributed by inception month. Data sourced from Datastream and Reserve Bank of New Zealand.

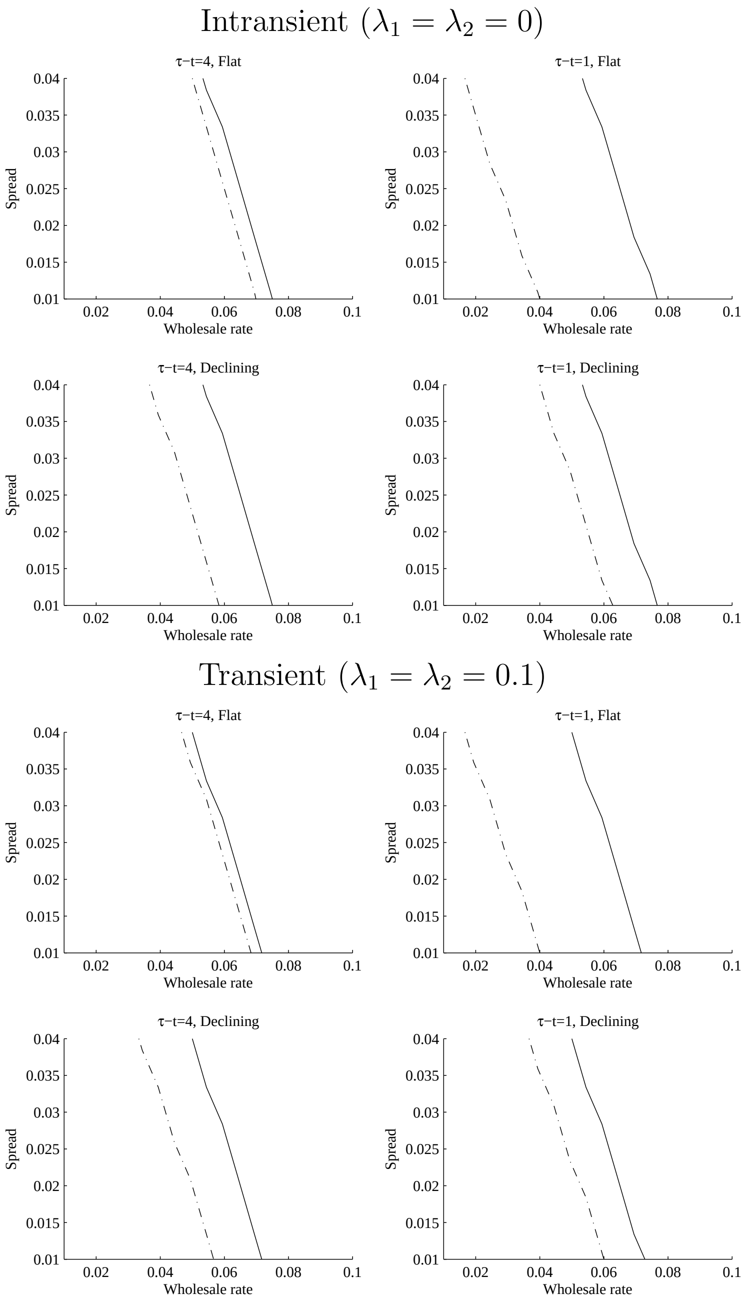


Figure 2: Optimal mortgage refinancing policy for the United Kingdom. Solid line is for break fee exempt part of mortgage, while dashed line denotes exercise frontier for mortgage once forgiven portion has been exhausted. Mortgage is initally a thirty year loan, with five year fixed period. Area above (and to the right of) the curves represents the region in which the mortgage should continue to be serviced, while area below (and to the left of) the curve is that in which the mortgage should be prepaid.

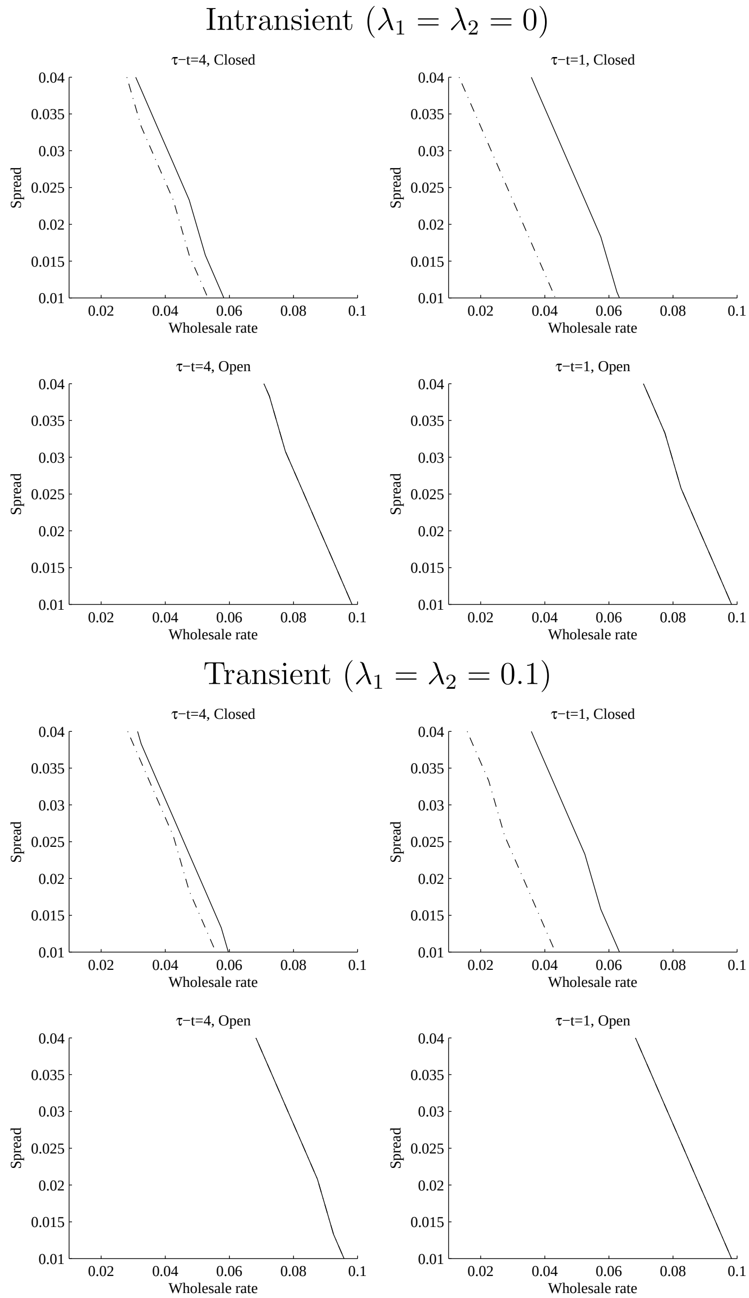
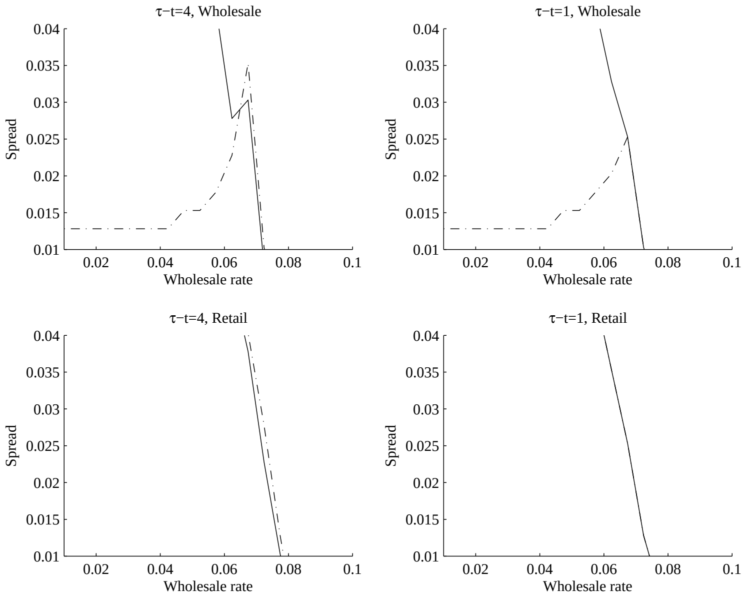


Figure 3: Optimal mortgage refinancing policy for Canada. Solid line is for break fee exempt part of mortgage, while dashed line denotes exercise frontier for mortgage once forgiven portion has been exhausted. Mortgage is initally a thirty year loan, with five year fixed period. Area above (and to the right of) the curves represents the region in which the mortgage should continue to be serviced, while area below (and to the left of) the curve is that in which the mortgage should be prepaid.

Intransient ($\lambda_1 = \lambda_2 = 0$)



Transient ($\lambda_1 = \lambda_2 = 0.1$)

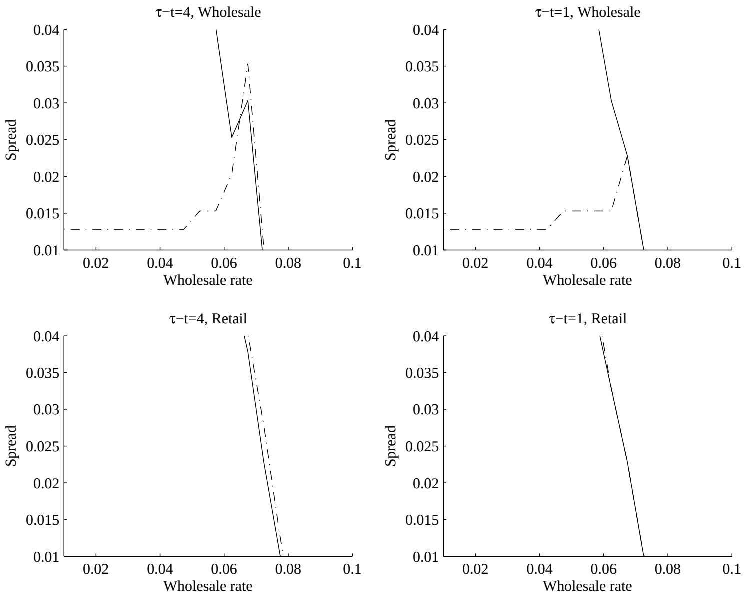


Figure 4: Optimal mortgage refinancing policy for New Zealand. Solid line is for break fee exempt part of mortgage, while dashed line denotes exercise frontier for mortgage once forgiven portion has been exhausted. Mortgage is initally a thirty year loan, with five year fixed period. Area above (and to the right of) the curves represents the region in which the mortgage should continue to be serviced, while area below (and to the left of) the curve is that in which the mortgage should be prepaid.

United Kingdom

Intransient ($\lambda_1 = \lambda_2 = 0$) Transient ($\lambda_1 = \lambda_2 = 0.1$)

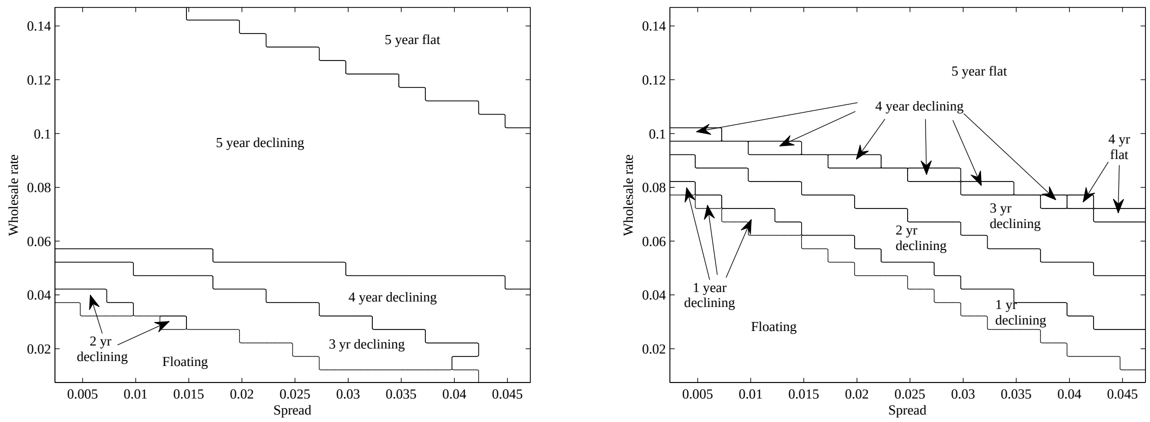


Figure 5: Optimal mortgage selection in the UK, for intransient (left) and transient (right) borrowers. Spread and wholesale rate are instantaneous (s and r respectively). Regions show the mortgage type which minimises the value of the borrower's debt for this combination of wholesale rate and credit spread.

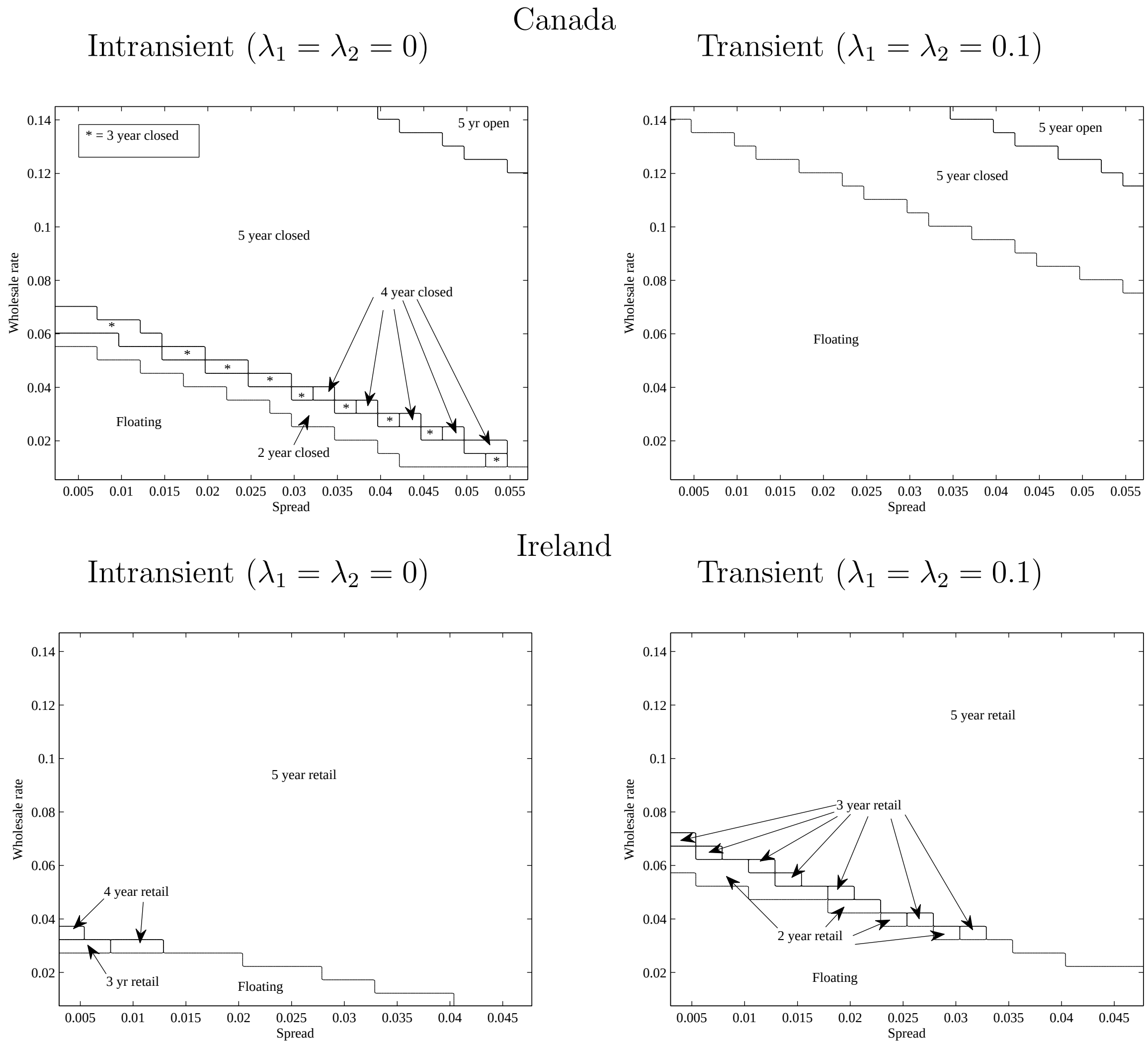


Figure 6: Optimal mortgage selection in Canada (top) and Ireland (bottom), for intransient (left) and transient (right) borrowers. Spread and wholesale rate are instantaneous (s and r respectively). Regions show the mortgage type which minimises the value of the borrower's debt for this combination of wholesale rate and credit spread.

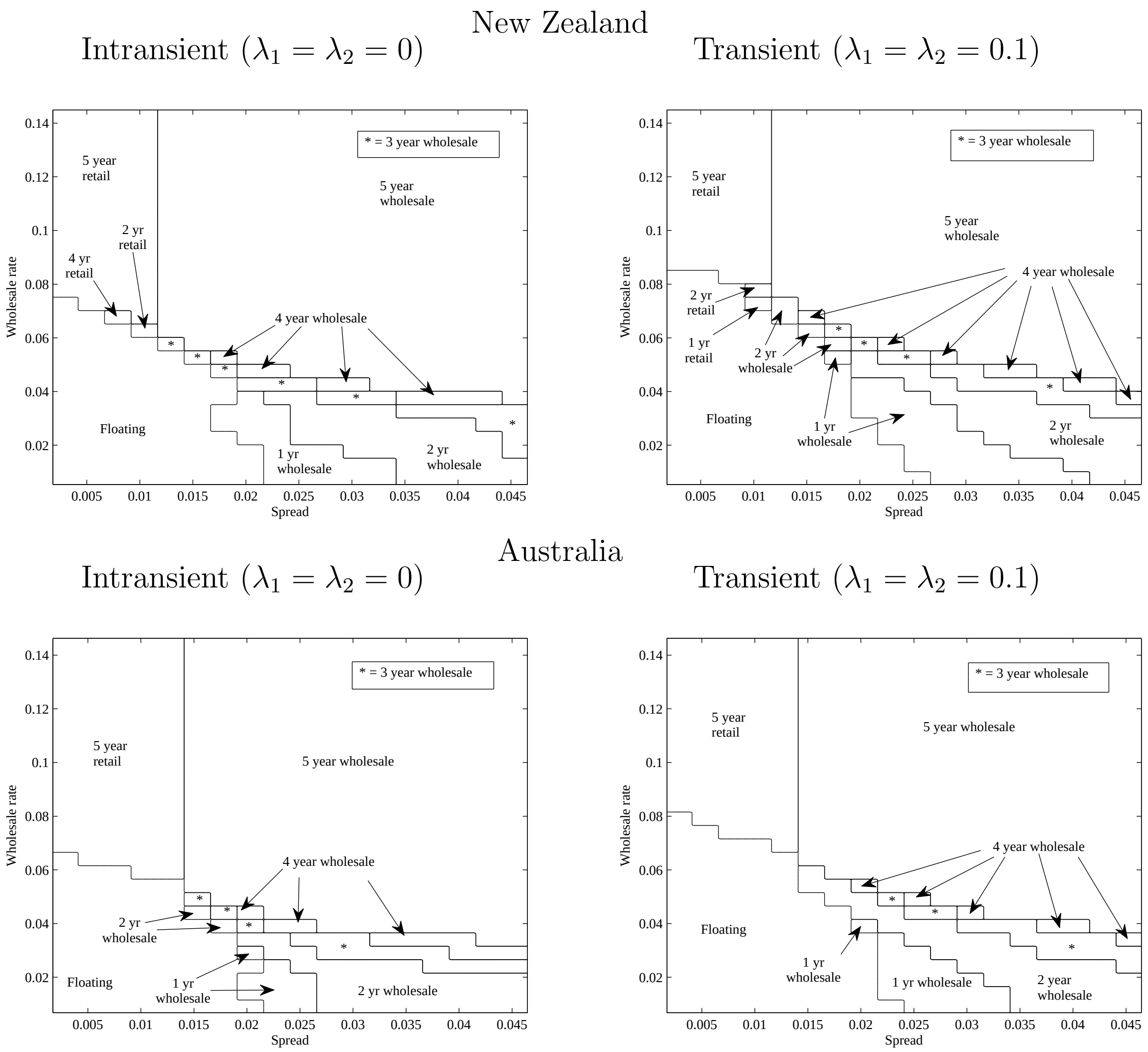


Figure 7: Optimal mortgage selection in New Zealand (top) and Australia (bottom), for intransient (left) and transient (right) borrowers. Spread and wholesale rate are instantaneous (s and r respectively). Regions show the mortgage type which minimises the value of the borrower's debt for this combination of wholesale rate and credit spread.

United Kingdom

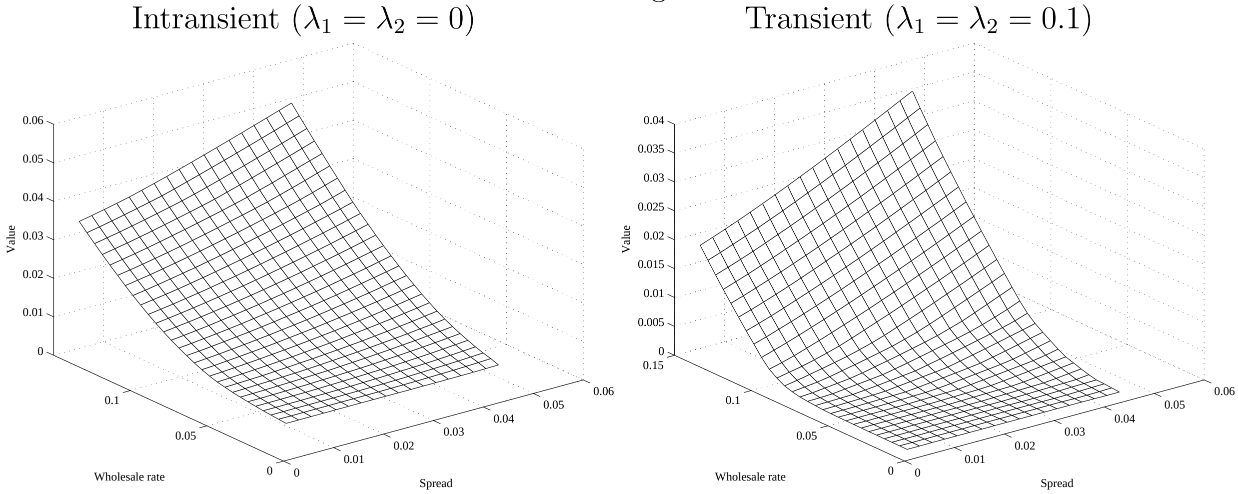


Figure 8: Value gains for investor (loss for bank) from prepayment exercise in the UK. Spread and wholesale rate are instantaneous (s and r respectively). Left graph represents the intransient case, while right graph represents the transient case. The height of the graph shows the relative reduction in mortgage value by selecting the optimal mortgage, and following the optimal refinancing rule for that mortgage, as compared to a borrow and hold strategy.

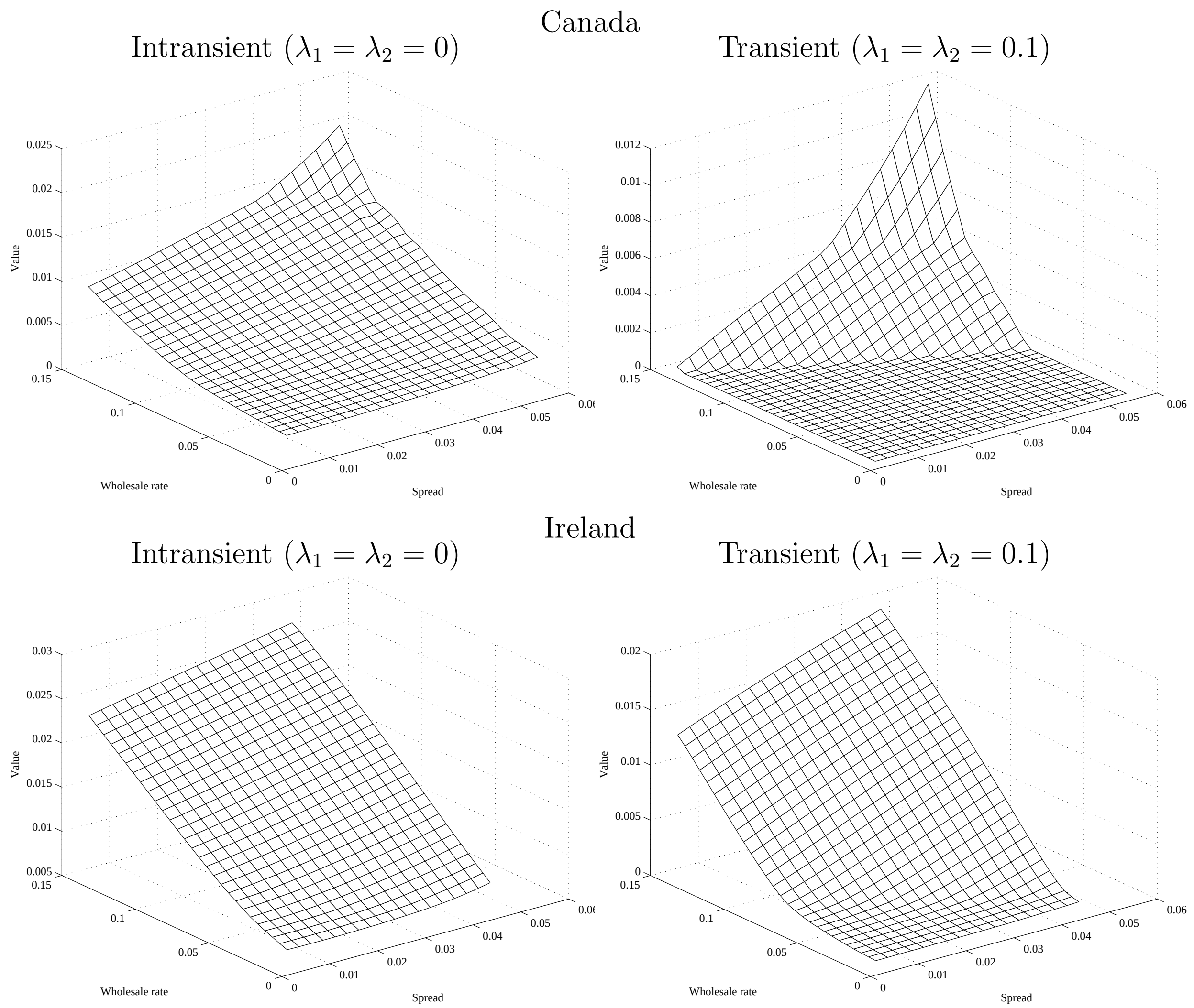


Figure 9: Value gains for investor (loss for bank) from prepayment exercise in Canada (top) and Ireland (bottom). Spread and wholesale rate are instantaneous (s and r respectively). In each case, left graph represents the intransient case, while right graph represents the transient case. The height of the graph shows the relative reduction in mortgage value by selecting the optimal mortgage, and following the optimal refinancing rule for that mortgage, as compared to a borrow and hold strategy.

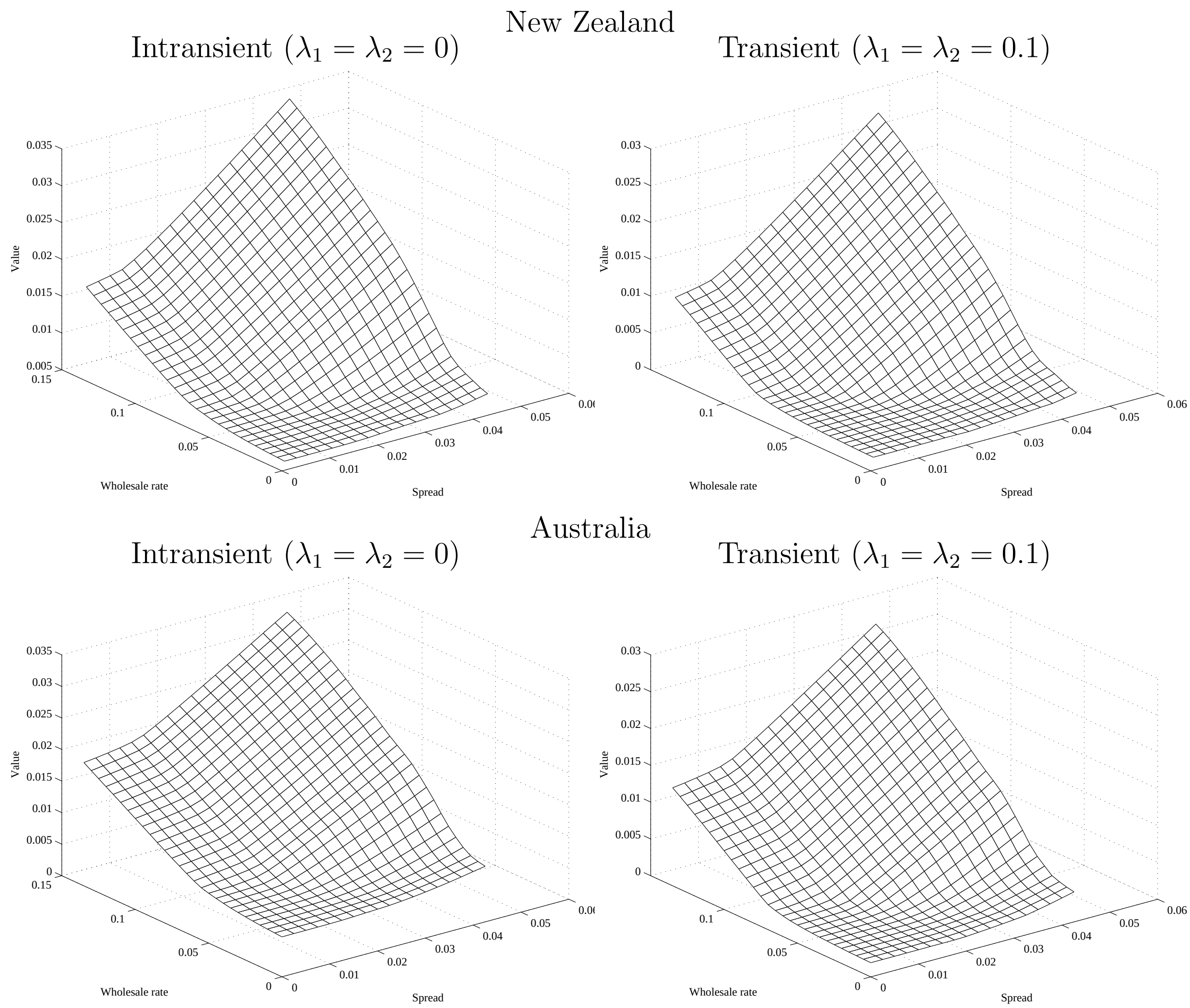


Figure 10: Value gains for investor (loss for bank) from prepayment exercise in New Zealand (top) and Australia (bottom). Spread and wholesale rate are instantaneous (s and r respectively). In each case, left graph represents the intransient case, while right graph represents the transient case. The height of the graph shows the relative reduction in mortgage value by selecting the optimal mortgage, and following the optimal refinancing rule for that mortgage, as compared to a borrow and hold strategy.

Country	Break fee types
UK	Flat portion of principal, *or*
	Declining portion of principal.
Canada	Closed loan: retail break fee (generally with minimum charge), *or*
	Open loan: no break fee.
Ireland	Retail break fee.
New Zealand	Retail break fee, *or*
	Wholesale break fee.
Australia	Retail break fee, *or*
	Wholesale break fee.

Table 1: Summary of break fee regimes for countries in sample.

Parameter	UK	Canada	Ireland	NZ	Australia
μ_r	0.0527	0.0350	0.0263	0.0613	0.0533
a_r	0.3922	0.2910	0.2854	0.6590	0.5927
σ_r	0.0245	0.0134	0.0279	0.0991	0.0937
γ_r	0.2577	0.1199	0.3606	0.7537	0.7659
ξ_r	-0.1249	-0.7035	-0.7451	-0.3923	-0.2817
$\bar{r}$	0.0564	0.0580	0.0513	0.0692	0.0583
Long Wholesale	5	5	3	3	3
μ_s	0.0157	0.0199	0.0111	0.0209	0.0177
a_s	0.3980	0.4177	0.0781	0.8773	1.2541
σ_s	0.0521	0.0346	0.0595	0.0489	0.0683
γ_s	0.6343	0.3834	0.7310	0.5644	0.7125
$\hat{\xi}_s$	4.1055	-0.2359	0.9698	2.5365	1.8345
$\bar{s}$	0.0028	0.0246	0.0023	0.0102	0.0132
Long Retail	5	5	(3)	3	3
ρ	-0.8630	-0.5793	-0.7682	-0.7060	-0.5725

Table 2: Parameter estimates for the wholesale rate and spread processes. Estimates are generated by maximum likelihood using a Kalman filter to back out the latent short rate. Long wholesale refers to the maturity of long-term swap used in conjunction with the 1 month interbank rate for estimation. Long retail is the maturity of fixed term mortgage rate used for estimation (along with floating rate). In the case of Ireland, the long term retail rate is an average of 1-5 year maturity rates, which we take to be representative of a 3 year rate. $\bar{r}$ and $\bar{s}$ are the steady state levels for r and s under the *risk-neutral* probability measure $\mathbb{Q}$.

Rate	UK	Canada	NZ
$R_0(5, 30)$	8.02%	7.77%	8.27%
$r_0(5, 30)$	7.18%	4.92%	6.84%
r_0	8.94%	4.25%	6.74%
s_0	3.09%	3.33%	3.28%

Table 3: Five year retail rates, five year hedge rates, short wholesale rates and short spreads for the analysis of optimal prepayment. Short rates are as generated from the extended Kalman filter for December 2009, with wholesale short rates perturbed so that a five year fixed rate mortgage is optimal for an intransient borrower. Longer term rates are consistent with these short rates and the parameters given in table 2.