The effect of asset price jumps on consumption and investment decisions

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ABSTRACT

This paper examines the importance of jumps in asset prices for investment problems, potentially incorporating consumption decisions. We present a technique for solving investment-consumption problems when asset prices jump. We also demonstrate how to quantify utility losses using an "optimal fee" approach – measuring how much a portfolio advisor could charge an investor to provide them with the new investment technology. As an application, we consider empirically plausible models for the S&P 500 index. We conclude that while there are some moderate differences in optimal investment behaviour once jumps are accounted for, the actual utility loss, in economic terms, is very low.

Returns distributions have long been known to be fat tailed: large positive and negative returns occur with a far greater frequency than would be expected were they normally distributed. Further, the left tail is significantly fatter than the right tail. Very bad news is more common than very good news. One possible explanation for this is the presence of jumps in asset prices. Certainly, a number of large sudden movements have been observed in recent times: the crash of 1987, the "dead cat bounce" which followed it, the bull rally that took place in January 1991, and the sudden price runups (and subsequent declines) that occurred at the height of the dot-com bubble in 2000.

There are good reasons to suspect that modelling jumps separately from routine, small movements in stock prices may be important for investment decisions. Arditti (1967) and Kraus and Litzenberger (1976) both suggest that investors might have a preference over the third moment of their wealth distribution. For the same mean and variance, an investor will opt to invest less in a risky asset whose return is negatively skewed.

This paper aims to examine the implications of asset price jumps for investment and consumption decisions. Consideration of consumption is important for descriptive and normative reasons. First, any understanding of general equilibrium must account for investors holding of securities as a source of revenue to finance consumption. Secondly, on a more practical front, many investors *do* manage their portfolio to fund a stream of outgoings. While models which explain how working investors should allocate their savings to build a nest-egg for retirement are helpful, the converse problem of how a retiree should manage and run down these savings is equally important.

The existing literature on portfolio allocation in the presence of jumps has largely focused on the problem of an investor who receives utility from their stock of wealth at a future date. Wu (2003) provides some quite elegant results for the case in which investors must make their portfolio decisions subject to a changing market premium. Wu's comparison of performance of different investment strategies showed that from 1993-1997, investors who were circumspect in their investments due to fear of jumps would have underperformed investors who did not incorporate jumps in their decision process. Similarly, Das and Uppal (2004) find that when investing internationally, the benefit of evaluating systematic risk of jumps across markets is very small. In contrast, Liu, Longstaff, and Pan (2003) note that the presence of jumps may make optimal holdings of equities significantly smaller, although they do not put a value on the difference.

One might suspect that an investor who seeks to finance a consumption stream, would be more concerned about jumps than a pure investor. While over a long horizon, large upward or downward movements in the portfolio may average out, over a shorter horizon, a downward jump may force consumers to revise their consumption plans downward. Similarly, an unexpected upward movement may result in consumers having to revise their plans upward. Since most consumers are motivated to smooth income (i.e. to achieve relatively unvolatile consumption) this may make utility losses from failing to account for jumps somewhat larger.

Framstad, Øksendal, and Sulem (1998) present some results for optimal investment and consumption rules in the presence of jumps, again supporting the hypothesis that investors should allocate less wealth to risky assets in their presence. However, their results do not incorporate stochastic volatility, a relatively accepted phenomenon in financial markets. As noted by Bates (2000), Maheu and McCurdy (2001), Bates (2006) and Chernov, Gallant, Ghysels, and Tauchen (1999), among others, volatile market periods tend to see higher incidences of stock price jumps as well as a greater variability of diffusion induced returns.

This paper examines the consumption investment problem in the presence of jumps. Two models are considered, both motivated by Bates (2006): one in which jumps arrive with a fixed intensity, and one in which jumps arrive with an intensity proportional to volatility. To further explore the income smoothing versus risk-aversion issue, we examine, in addition to standard time-additive utility, a recursive utility function which allows risk aversion and consumption smoothing preferences to differ.

The layout of the paper is as follows: section I outlines the portfolio problem, section II evaluates the significance of accounting for jumps in investment/consumption decisions, and section III concludes. Appendices contain technical details of how to solve the portfolio problem numerically, and the calculation of the optimal fee.

I. The portfolio problem

Consider a finite horizon portfolio problem, in which an investor must allocate wealth to a collection of risky securities (whose value at time t is S_{1t}), and a risk-free security (whose value at time t is S_{0t}). The investor's utility derives from consumption and a bequest function based on terminal wealth. The investor may make this decision himself, or may pay a portfolio manager to make the decision for him.

The investor's utility from a stream of consumption is given by:

$$Y_t = E_t\left(\int_t^T U_c(C_\tau, Y_\tau)d\tau + U_b(W_\tau)\right). \tag{1}$$

Here $U_c(C_\tau, Y_\tau)$ is the investor's felicity function, which takes the form

$$U_c(C_t, Y_t) = \frac{\nu_0}{\eta}\frac{(C_t)^\eta - (\zeta Y)^{\eta/\zeta}}{(\zeta Y)^{(\eta/\zeta)-1}}. \tag{2}$$

where ν_0, ζ and η are constants. This form was originally proposed by Duffie and Epstein (1992a) and Duffie and Epstein (1992b). For the specific case where $\eta = \zeta$, (2) collapses to:

$$U_c(C_t, Y_t) = \frac{\nu_0}{\zeta}C_t^\zeta - \nu_0 Y.$$

which can be written in a non-recursive form as:

$$Y_t = E_t\left(\int_t^T \nu_0 e^{-\nu_0\tau}\frac{C_t^\zeta}{\zeta}dt + e^{-\nu_0 T}U_b(W_T)\right).$$

However, this special case forces the investor to have identical preferences over risk and income smoothing: the lower the value assigned to ζ, the more risk-averse the investor is, and also the more concerned they are about maintaining a steady stream of consumption. In general, the parameter ζ measures the investor's preferences for income smoothing (lower values imply a greater desire for smooth income) whereas η measures their risk aversion (again, lower values imply greater risk aversion).

The investor's bequest function is defined as:

$$U_b(W_T) = \nu_1 \frac{W_T^{\zeta}}{\zeta} \tag{3}$$

where ν_1 is a constant. For a lifetime consumption problem, with bequest utility, we set $\nu_1 = \nu_0$. However, setting $\nu_0 = 0$ creates a problem in which the investor's sole source of utility is the bequest function (3) and their optimal consumption strategy is to set $C_t \equiv 0$. For the pure investment problem, the rate at which the investor discounts is unimportant, so without loss of generality, we assume $\nu_1 = 1$.

Define the investor's wealth at time t as W_t, and the current volatility of the risky security at time t as V_t. The investor (or their manager) must choose a proportion of their wealth to invest in the risky asset at each time. We denote their investment policy as $\theta_t = \theta(W_t, V_t, t)$. The investor must also decide on the proportion of their wealth to consume, $c_t = c(W_t, V_t, t)$.

The processes followed by the two assets are:

$$\frac{dS_{1t}}{S_{1t}} = (r + \mu_0 + \mu_1 V_t - \lambda_t k)dt + \sqrt{V_t} dz_{1t} + \Gamma_t dN_t \tag{4}$$

$$\frac{dS_{0t}}{S_{0t}} = rdt \tag{5}$$

$$dV_t = (\alpha_V - \beta_V V_t)dt + \sigma_v \sqrt{V_t}(\rho_V dz_{1t} + \sqrt{1-\rho_V^2} dz_{2t}) \tag{6}$$

$$\lambda_t = \psi_0 + \psi_1 V_t \tag{7}$$

$$\Gamma_t \sim N(\mu_J, \sigma_J) \tag{8}$$

$$k = E(\Gamma_t) = e^{\mu_J + \sigma_J^2/2}. \tag{9}$$

where dN_t is a Poisson jump process with intensity at time t of λ_t, and dz_{1t} and dz_{2t} are two independent Brownian motions. r, the risk-free rate, and the parameters μ_0, μ_1, α_V, β_V, σ_V, ρ_V, ψ_0, ψ_1, μ_J, and σ_J are positive constants.

Risk comes from two sources: diffusion risk (V_t) and jump risk (λ_t). If $\psi_0 > 0$, then the reward for bearing a unit of jump risk (in terms of increased return) is μ_0/ψ_0, while the market price of diffusion risk is $\mu_1 - \mu_0\psi_1/\psi_0$. When $\psi_0 = 0$, this distinction is less clear, since both risks move in lockstep.

Volatility and jump behaviour are modelled in a fashion consistent with that chosen by many empirical studies investigating option prices.[1] Volatility is stochastic, following a process which is potentially correlated with stock price movements (assuming $\rho_V \neq 0$). It also exhibits mean reversion (whose speed is determined by β_V) so that volatility tends to revert towards α_V/β_V.

Jumps occur in the market with an intensity that potentially depends on the level of volatility in the market. Assuming $\psi_1 > 0$ (since $\psi_1 < 0$ could lead to a negative intensity) more volatile periods are characterised by a higher incidence of jumps.

[1] See Bates (2000), Maheu and McCurdy (2001), Bates (2006) and Chernov, Gallant, Ghysels, and Tauchen (1999).

The process followed by the investor's wealth, given $\theta_t = \theta(W_t, V_t, t)$ and $c_t = c(W_t, V_t, t)$ is:

$$
\begin{aligned}
dW_t &= \theta_t W_t \frac{dS_{1t}}{S_{1t}} + (1-\theta_t) W_t \frac{dS_{0t}}{S_{0t}} - c_t W_t dt - \xi_t W_t dt \\
&= (r + \theta_t(\mu_0 + \mu_1 V_t - \lambda k) - c_t - \xi_t) W_t dt \\
&\quad + \theta_t W_t \sqrt{V_t} dz_{1t} + \theta_t W_t \Gamma_t dN_t
\end{aligned}
$$

where the process $\xi_t = \xi(V_t, t)$ is some fee which the investor pays his portfolio manager. ξ_t will enable us to measure the welfare losses from following suboptimal portfolio/consumption rules.

Denote the investor's expected utility at any given time $Y_t = Y(W_t, V_t, t, c, \theta, \xi)$, a function of wealth, volatility, time, the optimal strategies and the management fee (if any). The investor's decision problem is to choose functions θ and c to maximise $E(dY_t)$ for any given W_t, V_t and ξ_t. Using the Ito formula:

$$
\begin{aligned}
dY_t = {} & \frac{\partial Y}{\partial t} dt + \frac{\partial Y}{\partial W} \left[(r + \theta_t(\mu_0 + \mu_1 V_t - \lambda k - c_t - \xi_t)) W_t dt + \theta_t W_t dz_{1t} \right] \\
& + \frac{1}{2} \frac{\partial^2 Y}{\partial W^2} \theta_t^2 W_t^2 V_t dt + \frac{\partial Y}{\partial V} \left[(\alpha_V - \beta_V V_t) dt \right. \\
& \left. + \sigma_V V_t (\rho dz_{1t} + \sqrt{1-\rho^2} dz_{2t}) \right] + \frac{1}{2} \frac{\partial^2 Y}{\partial V^2} \sigma_V^2 V_t dt \\
& + \frac{\partial^2 Y}{\partial V \partial Y} \sigma_V \theta_t W_t V_t dt + \left[Y(W_t + \theta_t W_t \Gamma_t) - Y(W_t) \right] dN_t + U_c(c_t W_t, Y_t)
\end{aligned} \tag{10}
$$

At time T, the investor's indirect utility function will be defined by their bequest function:

$$Y(W_T, V_T, T, c, \theta, \xi) = U_b(W_T). \tag{11}$$

Since Y is an expectation, for a given consumption and investment strategy, (10) must be a martingale and therefore satisfies:

$$
\begin{aligned}
0 &= \frac{\partial Y}{\partial t} + \frac{\partial Y}{\partial W}(r + \theta(\mu_0 + \mu_1 V_t - \lambda k) - c_t - \xi_t)W + \frac{1}{2}\frac{\partial^2 Y}{\partial W^2}\theta_t^2 W_t^2 V_t \\
&\quad + \frac{\partial Y}{\partial V}(\alpha - \beta_V V_t) + \frac{1}{2}\frac{\partial^2 Y}{\partial V^2}\sigma_V^2 V_t + \frac{\partial^2 Y}{\partial V \partial Y}\sigma_V \theta_t W_t V_t \\
&\quad + E(Y(W_t + \theta_t W_t \Gamma_t) - Y(W_t)) + U_c(c_t, W_t, Y_t)
\end{aligned} \tag{12}
$$

which gives a partial integro-differential equation (PIDE) for the investor's expected utility Y. The terminal boundary condition is given by (11).

Any optimal solution of this problem will be of the form

$$
Y(t, W_t, V_t, c, \theta, \xi) = A(t, V_t, c, \theta, \xi) W^{\zeta}, \tag{13}
$$

with consumption and investment functions only of t and V.[2] In the following analyses, the only strategies we consider are those for which c and θ are invariant across W_t, and the indirect utility will take the form (13).

To solve the problem, we make use of the Markov-Chain approximation technique, described in Kushner and Dupuis (2001) and Munk (1997). Munk (2003) presents some results from the application of the technique to solve the infinite horizon Merton problem. The technique discretises the problem in (12) to reduce the problem to a set of discrete-time/state problems which can be solved to give solutions for Y, θ, and c at the chosen points. These solutions can then be interpolated to provide values at points where the problem was not solved. Details are given in appendix A.[3]

[2]To see this, substitute (13) into (12). The resulting PIDE for A is

$$
\begin{aligned}
0 &= A_t + \left[(r + \theta_t(\mu_0 + \mu_1 V_t - \lambda k - c_t - \xi_t))\frac{1}{2}\zeta(\zeta - 1)\theta_t^2 V_t\right] A + (\alpha_V - \beta_V V_t)A_V + \frac{1}{2}\sigma_V^2 V_t A_{VV} \\
&\quad + \zeta \sigma_V \theta_t V_t A_V + E\left((\theta_t \Gamma)^{\zeta}\right) + \frac{\nu_0}{\eta}\left[c_t^{\eta}(\zeta A)^{\frac{\zeta - \eta}{\eta}} - \zeta A\right],
\end{aligned}
$$

which verifies that A is not a function of W_t. Therefore, the first order conditions to maximise A are not be a function of wealth.

[3]This is not the only technique that can be used for a numerical solution of the portfolio optimisation problem.

II. Are jump-diffusion effects economically significant?

In order to determine whether incorporating jumps leads to significant welfare benefits, it is necessary to have an alternative allocation rule to compare with. Outcomes can then be compared both in terms of the difference in allocations and consumption, and the loss in utility due to following the suboptimal scheme.

For this paper, our straw man consists of a portfolio strategy where volatility is stochastic, but no jumps occur in the risky asset's price (i.e. $\psi_0 = \psi_1 = 0$ in equations (4-9)). We choose parameters for this model so as to match the first two conditional moments of returns as closely as possible. Following Das and Uppal (2004), we then calculate the utility losses, measured first as a certainty equivalent, in dollars, and then as a per annum fee. As noted in the introduction, this leaves the critical impact of jumps as being to introduce skewness to the return distribution: an undesirable feature for risk-averse investors.

More formally, given the moments of the jump distribution:

$$\begin{aligned} E(\Gamma_t) &= e^{\mu_J + \frac{\sigma_J^2}{2}} - 1 \\ SD(\Gamma_t) &= (E(\Gamma_t))^2 + e^{2\mu_J + 2\sigma_J^2} - e^{2\mu_J + \sigma_J^2}, \end{aligned}$$

an investor who does *not* account for jumps uses the model:

$$\begin{aligned} \frac{dS_{1t}}{S_{1t}} &= (r + \mu_0 + \mu_1 V_t)dt + \sqrt{\hat{V}_t} dz_{1t} \\ \frac{dS_{0t}}{S_{0t}} &= rdt, \end{aligned}$$

Detemple, Garcia, and Rindisbacher (2003) demonstrate how Monte Carlo simulation can be used to solve these problems.

where

$$
\begin{aligned}
\hat{V}_t &= \phi_0 + \phi_1 V_t \\
dV_t &= (\alpha_v - \beta_V V_t)dt + \sigma_v \sqrt{V_t}(\rho_V dz_{1t} + \sqrt{1-\rho_V^2} dz_{2t}) \\
\phi_1 &= 1 + \psi_1 (E(\Gamma_t)^2 + SD(\Gamma_t)^2) \\
\phi_0 &= \psi_0 (E(\Gamma_t)^2 + SD(\Gamma_t)^2)
\end{aligned}
$$

In the presence of jumps which do not have a bounded distribution, investors will choose their portfolios such that there are no short sales of either asset ($0 \leq \theta_t \leq 1$). However, investors who do *not* incorporate jumps in their portfolio decisions could well choose portfolios which violate this condition. Due to the Inada condition that $\lim_{W_t \to 0} \frac{\partial Y}{\partial W_t} = \infty$, the investor will have an undefined utility if there is any probability at any time that his wealth will fall below zero. One solution to this problem would be to constrain investors failing to incorporate jump risk to consider only those allocations which actually lead to bounded levels of utility. For lower levels of risk-aversion, the optimal allocation when not incorporating jumps would involve short selling the risk-free asset, and investing the proceeds (along with the investor's initial wealth) in the risky asset. Hence, with short-sale constraints on the risk-free asset, investors would select a corner solution, investing all of their wealth in the risky asset. Results in this case would be driven by the investor's short selling constraint. While this analysis could lead to some interesting results, isolating the effect of jumps requires the elimination of this friction. We consider combinations of risk aversion and market risk premia in which this short selling problem does not occur, and where the investors who do not incorporate jumps in their decision "voluntarily" choose a portfolio without any short positions.

A. Measuring differences in performance

To compare the advantages gained from correctly accounting for jump effects, we will measure the loss from using sub-optimal policies for consumption and investment. Suppose that our optimal strategy using the jump model is given by $(\theta^*,\ c^*)$ and we will compare this to the optimal policy under the stochastic volatility model $(\hat{\theta},\ \hat{c})$. Both the strategies are invariant to the level of wealth.[4]

One measure for the cash value of a utility gain is a *certainty equivalent*, as used in Das and Uppal (2004). This dollar value represents the proportion of initial wealth an investor using the suboptimal scheme should be willing to forego in order to have access to the optimal scheme. Formally, this number solves:

$$Y(t, W_t(1-\epsilon), V_t, \theta^*, c^*, 0) = Y(t, W_t, V_t, \hat{\theta}, \hat{c}, 0). \tag{14}$$

ϵ is the portion of wealth the agent would forfeit at time t to have access to the optimal policy rather than be forced to use the suboptimal policy. Since $Y = A(t, V_t, \theta, c, 0)W_t^{\zeta}$ for both schemes, (14) may be written:

$$A(t, V_t, \theta^*, c^*, 0)(W_t(1-\epsilon))^{\zeta} = A(t, V_t, \hat{\theta}, \hat{c}, 0)W_t^{\zeta}$$

which implies

$$\epsilon = 1 - \left(\frac{A(t, V_t, \hat{\theta}, \hat{c}, 0)}{A(t, V_t, \theta^*, c^*, 0)} \right)^{\frac{1}{\zeta}}.$$

so that for any time and volatility, the one off willingness to pay of the investor to transition between schemes can be calculated by comparing the utility levels for the two policies.

This measure leads to monotonically increasing certainty equivalents as the investment horizon increases, since investors following different policies over very short horizons will see little effect on their terminal wealth (or consumption streams).

Although the certainty equivalent is easy to calculate, when measuring the value of a superior

[4]Since the stochastic volatility model is a special case of the more general jump stochastic volatility model, all optimal strategies for it will also be invariant to W_t.

portfolio strategy, it may be more natural to calculate the maximum per-period fee which could be charged by a portfolio manager to provide an investor (who would otherwise use a suboptimal strategy) with the optimal allocation of assets. More formally, this fee is the maximum state-contingent management fee (ξ) which a fund manager could charge the agent to provide them with policy θ^*, when the investor's alternative is to pay no fee and use the policy ($\hat{\theta}$, $\hat{c}$).

For the case of a pure investment problem, the introduction of a fee will not affect the optimal investment policy, θ^*. However, for the case of an investment-consumption problem, introducing a fee will lead to more consumption than a fee-less economy. The maximal fee function will thus be given by ξ^* such that

$$Y(W_t, t, V_t, \theta^*, \bar{c}, \xi^*) = Y(W_t, t, V_t, \hat{\theta}, \hat{c}, 0), \tag{15}$$

where $\bar{c} = \bar{c}(t, V_t, \theta, \xi)$ maximises the investor's utility when they use investment strategy θ^* and are charged fee ξ^*. Appendix B provides details of how equation (15) can be solved to find ξ^* and $\bar{c}$, given θ^*, $\hat{\theta}$, and $\hat{c}$.

B. Base Scenario

[Table 1 about here.]

The parameters for the economy are given in table I. These parameters are taken from the work of Bates (2006), as estimates of a jump-diffusion process for the S&P 500 index. The only parameters which differ from Bates' work are the interest rate, which we fix at three percent, and μ_1, the price of risk, which is set such that investors are rewarded for being subjected to variance in stock returns (whether from diffusion or jumps) at a 3:1 ratio (i.e. for one percent of variance, the risky security has a risk premium of three percent).

B.1. Jump effects for pure investors

We first consider the case where the investor is motivated solely by their bequest function, or in the notation of section I, $\nu_0 = 0$, $\nu_1 = 1$. We might intuitively think of this investor as a younger person, whose consumption is funded externally to the investment problem, and is deciding how to allocate his/her savings to achieve sufficient wealth to fund consumption in their retirement. Since no utility is achieved from consumption, the optimal policy for consumption (assuming no negative consumption is permitted) is $c \equiv 0$.

Figure 1 contains the optimal and suboptimal allocations to the risky security, whereas table II contains the estimates of value gained by following the optimal strategy.

[Figure 1 about here.]

In figure 1, we see that there are two qualitative effects that take place. First, the investor uses the risky asset as a hedge against changes in volatility. As the time horizon increases, they allocate more of their wealth to the risky asset. Investor utility is an increasing function of V_t, so declines in volatility reduce utility.[5] Since the risky asset return is negatively correlated with volatility, the investor can hedge this risk by increasing his/her holding of the risky security. For short investment horizons, hedging has no effect, but as the horizon increases, the investor hedges to a greater degree.

Secondly, investors time the market. For the JSV investor who incorporates jump risk, as volatility increases, the mix of risk entailed in investing in the risky security changes. At low levels of volatility, risk is predominately caused by the (fixed) jump intensity. As volatility increases, the fraction of risk caused by jumps, as opposed to diffusion, decreases. Since the investor has a preference for diffusion risk over jump risk, this results in the investor allocating more wealth to the risky asset as volatility

[5] To see why this would be the case, consider the case where $\mu_0 = 0$, volatility is constant, and there are no jumps. Then the solution to the investor's problem is $\theta = \frac{\mu_1}{1-\zeta}$ and their indirect utility function is given by $Y = \frac{W^\zeta}{\zeta} e^{\zeta\left(\frac{\mu_1^2 V}{2(1-\zeta)}+r\right)(T-t)}$, which is increasing in V.

increases. In contrast, the SV investor *decreases* their exposure as volatility increases, to achieve a fixed level of volatility exposure.

As a result of this mis-interpretation of the volatility states by SV investors, along with a general over-allocation of wealth to the risky asset, welfare is lost compared to the optimal strategy. These welfare losses are documented in table II, along with the differences between investment strategies. Investors with relatively high risk aversion allocate less of their wealth to the risky security, but also change their allocations less in response to jumps. As a result, the investors who suffer the largest utility losses from mis-allocating their portfolios are those with lower risk aversion ($\zeta = -3$ or $\zeta = -4$). The per-year value of correctly accounting for jumps, as given in pane I.(b) of table II is increasing in time.

These numbers are quite small (the largest loss in certainty equivalent terms is 8.94 basis points of the investor's wealth, and the largest fee that could be extracted is 0.9239 basis points). However, the reader should remember that they presume a suboptimal SV investor whose model for security prices is as close as possible to the true, JSV, model. Biased parameter estimates caused by using a mis-specified model could lead to greater welfare differences.

[Table 2 about here.]

B.2. Investment and consumption

We now turn our attention to the case of a "mature" investor, who manages their portfolio to finance their consumption in addition to saving for a bequest (i.e. $\eta_0 = \eta_1 = 1$). The optimal (and suboptimal) strategies for this problem are displayed in figure 2. Again, hedging demand is exhibited, which manifests itself in a decline in demand for the risky security toward the end of the investor's life. In this case, however, the decline in demand is not confined to the last few years, but in fact there is an appreciable increase in hedging demand even over longer (five to thirty year) horizons.

[Figure 2 about here.]

These investors also attempt to time the market. JSV investors again increase their investment in the risky asset as volatility rises, and SV investors decrease their investment.

Table III tabulates the value gained by mature JSV investors over their SV counterparts. The trend is again that less risk averse (less negative ζ) investors suffer a greater welfare loss in certainty equivalent terms. These certainty equivalent quantities are also much less than those faced by investors who do not finance a consumption stream. We thus conclude that older investors are less concerned about the presence of jumps in asset prices.

However, examination of the fee metric reveals a rather different story. In this case, older investors are actually prepared to pay a *higher* fee than their younger counterparts. This result at first seems contradictory to the observation that certainty equivalents are *lower* for older investors. However, it must be remembered that whereas a young investor (absent of losses in the market) expects to see their wealth grow over their investment horizon, an older investor is in fact expecting to see their wealth shrink over time, as they consume it. Thus although the cash equivalent (an up-front value) is lower for older investors, if spread across their average wealth, the amount is in fact higher than for younger investors. Younger investors would be more willing to invest in technical knowledge which would allow them to manage their own money correctly, but older investors would be willing to pay more for someone else to manage their money for them.

[Table 3 about here.]

Lastly, consider the agents with non-time separable utility (those where $\zeta \neq \eta$). These provide us with information as to whether risk-aversion or the desire for consumption smoothing is the more important motivation for concern over jumps. Investment behaviour is predominately determined by the level of ζ (the desire for income smoothing). However for varying risk aversion, the investor's consumption behaviour changes considerably. For $\eta = -3$, $\zeta = -6$, an investor more concerned with income smoothing than risk, consumption is relatively lower. In contrast, an investor with $\eta = -6$,

$\zeta = -3$ (very risk averse, but with little concern about the smoothness of their consumption flow) consumes at a much faster rate.

Investors with low risk aversion, but strong motivation to smooth consumption (those where $\zeta = -3, \eta = -6$) suffer the *largest* welfare losses from not accounting for jumps. Not only do their investment decisions differ considerably when accounting for jumps, but their consumption rates are considerably lower when facing jumps.[6] In stark contrast, investors who are risk averse, but have little desire to smooth income (those where $\zeta = -6, \eta = -3$) face the smallest losses when not factoring jumps into their investment/consumption decisions.

Why do we observe this? The answer can be clearly seen by considering an investor who has strong desires to income smooth and sustains a jump in their portfolio value. If the jump is downward, the investor is worse off, because their decline in wealth will force a sudden decline in their consumption stream. However, even if the jump is positive, the investor does not receive much benefit, as their increase in consumption will violate their desire for consistent consumption. In contrast, for an investor who is neither especially risk averse, nor desires a smooth consumption stream, jumps hold little fear.

C. Volatility dependent intensities

As mentioned in section I, if $\psi_0 = 0$, the economy features jump risk which moves in tandem with regular volatility risk. In this economy, it may be difficult to separate the two, and we may find different results from our previous analysis. We thus propose examination of a second economy, using the parameters in table IV, an alternative model for volatility proposed in Bates (2006). Most of these parameters are similar to those in table I. However, ψ_0 has been set to zero, and jump arrival instead comes through the parameter ψ_1: jump intensity is proportional to volatility. Again, μ_0 and

[6]In thinking about examples of an investor who might have low risk aversion but a strong desire to smooth income, a university's endowment fund immediately comes to mind. A university, as a large institution might well regard itself as risk-neutral. However, fluctuating consumption spending would be highly undesirable, since most expenses (salaries, fixtures etc) are difficult to reassign from year to year.

μ_1 are set so that jump risk is rewarded by an increase in expected returns. In this case, since there is no constant intensity, $\mu_0 = 0$, and μ_1 is set to be positive.

[Table 4 about here.]

Graphs of the investor's optimal allocations to the risky security (θ) are given in figure 3. In this case, we now see no evidence of market timing: investors choose the same portfolio allocation regardless of the volatility level. This is a result of the one-to-one relationship between the intensity of jumps and the level of volatility. Regardless of the volatility level, jump risk makes up the same proportion of risk for an investor who purchases the risky security.

[Figure 3 about here.]

Once again, hedging demand for the risky security sees an increase in exposure as the time horizon increases. The difference between trading strategies here is largely through the hedging demand: investors with short horizons hold very similar portfolios regardless of whether they account for jumps or not. However, for longer horizons, the difference increases.

Table V quantifies the gains from correctly incorporating jumps into the portfolio decision. These are markedly smaller than in the base case scenario. Even for the least risk-averse case, they are less than a quarter of a basis point. In terms of fees, they are less than one hundredth of a basis point. Examining the differences in portfolio allocations (panel III), the difference in allocation is not entirely trivial: for longer horizons, the over-allocation of wealth to the risky asset by SV investors ranges from 0.4 to 0.6 percent.

[Table 5 about here.]

For investors who both invest and consume, a similar story unfolds (see figure 4). Investment patterns are very similar, and there is almost no difference between consumption decisions.

[Figure 4 about here.]

Table VI contains results for the economic significance of jumps for older investors. Both the certainty equivalent and the optimal fee are both exceptionally low. The maximal fee that could be extracted in this case is 0.0145 basis points. This stands in stark contrast to the constant intensity scenario, where an investor with low risk aversion and strong smoothing preferences ($\eta = -6, \zeta = -3$) would be willing to pay almost seventy times as high a fee.

Conclusions as to the investors who place more value on jump-consistent portfolios are, however, robust between the two cases. The effects are most important for investors with low risk aversion and strong smoothing preferences, and lowest for risk averse investors unconcerned by smoothing ($\eta = -3, \zeta = -6$).

[Table 6 about here.]

We conclude that although there are some small changes to portfolio allocations in response to the presence of jumps in a market where the jump intensities are perfectly correlated with volatility, the welfare loss is not economically significant. Much of the welfare loss in the base scenario is due to the market mis-timing which takes place for constant intensity jumps.

III. Conclusion

We examine the portfolio allocation problem of an investor allocating his wealth between a risky asset and a risk-free asset. Two types of investors are considered: a young investor, motivated solely by a bequest function, and an older investor, who must also finance consumption with his portfolio. We also examine two possible scenarios for jump risk: one in which jumps arrive at a constant intensity and the other in which jumps arrive with an intensity proportional to the level of volatility in the market.

When jumps arrive with an intensity directly proportional to volatility, wealth allocations and consumption are almost identical for consumers who approximate assets' jump-stochastic volatility process with a stochastic volatility process. However, when jump intensity is constant, meaning that the mix of jump and diffusion risk is state-dependent, risk-tolerant investors may overallocate their wealth to risky securities by 4% by failing to account for jumps. In addition, investors may mis-time the market – investing less in risky securities as volatility rises, when they should invest more.

Consistent with the findings of Das and Uppal (2004), we find that the welfare losses from failing to incorporate jumps are quite small. An investor who accounts for time varying volatility, but incorrectly models jump risk as increased volatility will achieve very similar levels of utility to someone who formally separates the two. When measured as a fee that an investor might pay to a portfolio manager to correctly advise them how to allocate his wealth, the loss rarely exceeds one basis point. We thus conclude that although there are theoretically interesting differences in optimal investment behaviour when asset prices jump, in practice, the utility gains from accounting for it are not economically significant.

A. The Markov Chain Approximation Technique

As noted in section I, we need to solve (12). This consists of finding Y, θ and c which satisfy (12), but which are also consistent with maximising (12). For computational simplicity we transform the problem by considering $\log(W_t) \equiv w_t$ to obtain:

$$\begin{aligned} -\frac{\partial Y}{\partial t} &= \frac{\partial Y}{\partial w}\left(r - c_t + \theta_t(\mu_0 + \mu_1 V_t - \lambda k) - \frac{1}{2}\theta_t^2 V_t - \xi\right) + \frac{1}{2}\frac{\partial^2 Y}{\partial w^2}\theta_t^2 V_t \\ &\quad + \frac{\partial Y}{\partial V}(\alpha_V - \beta_V V_t) + \frac{1}{2}\frac{\partial^2 Y}{\partial V^2}\sigma_V^2 V_t + \frac{\partial^2 Y}{\partial V \partial Y}\sigma_V \theta_t V_t \\ &\quad + \lambda_t E(Y(w_t + \hat{\Gamma}_t) - Y(w_t)) + U_c(c_t e_t^w, Y_t) \end{aligned} \tag{16}$$

where

$$\hat{\Gamma}_t = \log(1 + \theta(e_t^\Gamma - 1)),$$

with $\Gamma_t \sim N(\mu_J, \sigma_J)$ as before, and the terminal condition Y_T is now

$$Y_T(w_T) = \nu_1 e^{\zeta w_t} \rho$$

with utility from consumption:

$$U_c = \frac{\nu_0}{\eta} \frac{(c_t e_t^w)^\eta - (\zeta Y)^{\eta/\zeta}}{(\zeta Y)^{(\eta/\zeta)-1}}.$$

We begin by defining a grid in the three dimensions: W, V and t, such that

$$\begin{aligned}
t_i &= i\Delta t \quad i = 1, \ldots, \frac{T}{\Delta t} \\
w_j &= w_0 + j\Delta w \quad j = 0, \ldots, J \\
V_k &= V_0 + k\Delta V \quad k = 0, \ldots, K.
\end{aligned}$$

For each point, we then approximate (16) at point (i, j, k) as follows:

$$\begin{aligned}
Y_{i-1,j,k} &= Y_{i,j,k} + \Delta t \Bigg(\delta_{w1} Y_{i,j,k} (r + \theta(\mu_0 + \mu_1 V_k - \lambda k) - \frac{1}{2}\theta_t^2 V_k - \xi) + \frac{1}{2}\delta_{w2} Y_{i,j,k} \theta_t^2 W_t^2 V_k \\
&+ \delta V Y_{i,j,k} (\alpha_V - \beta_V V_k) + \frac{1}{2}\delta_{V2} Y_{i,j,k} \sigma_V^2 V_k + \delta_{wV} Y \sigma_V \theta_t V_k \\
&+ (\psi_0 + \psi_1 V_k) \sum_{l=-M_1}^{M_2} \omega_{j,l}(\theta)(Y(w_{j+l}) - Y(w_j)) + U_c(c_{i,j,k} e_j^w, Y_{i,j,k}) \Bigg) \qquad (17)
\end{aligned}$$

where we define the operators δ_{w1}, δ_{w2}, δ_{V1}, δ_{V2} and δ_{wV} so as to be facing into the interior of the grid when we are on a boundary:

$$
\begin{aligned}
\delta_{w1} Y_{i,j,k} &= \begin{cases} \frac{-3Y_{i,j,k}+4Y_{i,j+1,k}-Y_{i,j+2,k}}{2\Delta w} & \text{if } j=0 \\ \frac{Y_{i,j+1,k}-Y_{i,j-1,k}}{2\Delta w} & \text{if } J>j>0 \\ \frac{3Y_{i,j,k}-4Y_{i,j-1,k}+Y_{i,j-2,k}}{2\Delta w} & \text{if } j=J \end{cases}, \\
\delta_{w2} Y_{i,j,k} &= \begin{cases} \frac{Y_{i,j+2,k}-2Y_{i,j+1,k}+Y_{i,j,k}}{(\Delta w)^2} & \text{if } j=0 \\ \frac{Y_{i,j+1,k}-2Y_{i,j,k}+Y_{i,j-1,k}}{(\Delta w)^2} & \text{if } 0<j<J \\ \frac{Y_{i,j,k}-2Y_{i,j-1,k}+Y_{i,j-2,k}}{(\Delta w)^2} & \text{if } j=J \end{cases}, \\
\delta_{V1} Y_{i,j,k} &= \begin{cases} \frac{-3Y_{i,j,k}+4Y_{i,j,k+1}-Y_{i,j,k+2}}{2\Delta V} & \text{if } j=0 \\ \frac{Y_{i,j,k+1}-Y_{i,j,k-1}}{2\Delta V} & \text{if } K>k>0 \\ \frac{3Y_{i,j,k}-4Y_{i,j,k-1}+Y_{i,j,k-2}}{2\Delta V} & \text{if } k=K \end{cases}, \\
\delta_{V2} Y_{i,j,k} &= \begin{cases} \frac{Y_{i,j,k+2}-2Y_{i,j,k+1}+Y_{i,j,k}}{(\Delta V)^2} & \text{if } k=0 \\ \frac{Y_{i,j,k+1}-2Y_{i,j,k}+Y_{i,j,k-1}}{(\Delta V)^2} & \text{if } 0<k<K \\ \frac{Y_{i,j,k}-2Y_{i,j,k-1}+Y_{i,j,k-2}}{(\Delta V)^2} & \text{if } k=K \end{cases}, \\
\delta_{wV} Y_{i,j,k} &= \delta_w \delta_V Y_{i,j,k}
\end{aligned}
$$

where

$$
\delta_w = \begin{cases} \frac{Y_{i,j+1,k}-Y_{i,j,k}}{\Delta w} & \text{if } j=1 \\ \frac{Y_{i,j+1,k}-Y_{i,j-1,k}}{2\Delta w} & \text{if } 0<j<J \\ \frac{Y_{i,j,k}-Y_{i,j-1,k}}{\Delta w} & \text{if } j=J \end{cases} \quad \delta_V = \begin{cases} \frac{Y_{i,j,k+1}-Y_{i,j,k}}{\Delta V} & \text{if } k=0 \\ \frac{Y_{i,j,k+1}-Y_{i,j,k-1}}{2\Delta V} & \text{if } 0<k<K \\ \frac{Y_{i,j,k}-Y_{i,j,k-1}}{\Delta V} & \text{if } k=K \end{cases}.
$$

Finally, we define

$$\begin{aligned}\omega_{j,l}(\theta) &= P\left(\hat{\Gamma}_t \in \left[\left(l-j-\frac{1}{2}\right)\Delta w, \left(l-j+\frac{1}{2}\right)\Delta w\right]\right) \\ &= \Phi\left(\log\left(\frac{e^{\left(l-j+\frac{1}{2}\right)\Delta w}-1}{\theta}+1\right)\right) - \Phi\left(\log\left(\frac{e^{\left(l-j-\frac{1}{2}\right)\Delta w}-1}{\theta}+1\right)\right),\end{aligned}$$

where $\Phi(x)$ is a cumulative normal distribution with mean μ_J and standard deviation σ_J. If the jump term requires $Y_{i,j,k}$ for $j < 0$, we approximate using a linear extrapolation from $Y_{i,0,k}$ and $Y_{i,1,k}$, while if we require $Y_{i,j,k}$ for $j > J$, we linearly extrapolate using $Y_{i,J-1,k}$ and $Y_{i,J,k}$.

Our approximations allow us to specify all grid points at time step i in terms of combinations of grid points at time $i+1$. Since we know the value of the indirect utility function (Y) for the final time step, this allows us to (for a given policy of c and θ) step backwards through time. Since the utility functions in question display constant relative risk aversion, we know that these policy variables should not be a function of w, and that the indirect utility function will obey 13. We thus follow the following scheme to work backwards through time, finding Y, θ and c for each grid point:

1. Fix $\bar{j}$ as the points where $w_{\bar{j}} = 0$ (i.e. $W_t = 1$). For each point, solve the optimisation problem:

$$\max_{c_{i,k},\theta_{i,k}} Y_{i-1,\bar{j},k}$$

   given $Y_{i,j,k}$, $j = 0,\ldots,J$, $k = 0,\ldots,K$ and using (17).

2. Now set $c_{i,j,k} = c_{i,\bar{j},k}$ for all $j \neq \bar{j}$.

3. Finally, update the remaining indirect utilities such that $Y_{i-1,j,k} = e^{w_j\zeta}Y_{i-1,\bar{j},k}$.

4. Set $i = i-1$ and repeat from step 1.

This process is thus conceptually similar to pricing an option using an explicit finite difference technique, with the addition of the solution of $K+1$ optimisation problems at step 1. Our approximation is first order accurate in time, and second order accurate in the space dimension, although our boundary conditions are only first order accurate. Our choice of a second order approximation for $\frac{\partial f}{\partial w}$ on the $w = w_0$ boundary is important, however, as this more accurately approximates the high curvature that the indirect utility function will have as it approaches the Inada condition at $W = 0$. Discretising in $\log(W)$ instead of W itself also helps minimise the error due to truncating the problem at some $W_0 > 0$.

Because this approximation is explicit (rather than implicit) we must use a sufficiently small time step Δt to maintain stability of the method. We find that setting $\Delta t = 0.005$ works well, with spatial discretisation steps $\Delta w = 0.05$, $\Delta V = 0.005$ and boundaries $w_0 = -2$, $w_J = 2$, $V_0 = 0.0025$ and $V_K = 0.0525$. The problem could also be solved using an implicit discretisation, however for the case of the consumption problem with $\eta \neq \zeta$, this results in a system of non-linear equations which must be solved, in conjunction with the individual optimisation problems of step 1. Since these must be solved simultaneously, a policy iteration technique must be used, which for convergence requires a relatively small time step. The advantages of being able to choose arbitrarily large time step from a stability perspective are thus lost to needing a small enough time step for policy iteration to converge.

B. Calculating an internally consistent fee

We would like to calculate a fee $\xi^* = \xi^*(V_t, t)$ such that an investor is indifferent between following the investment/consumption strategy $(\hat{\theta}, \hat{c})$ with no fee, and following the investment/consumption strategy $(\theta^*, \bar{c})$ with fee where $\bar{c} = \bar{c}(t, V_t, \theta^*, \xi^*)$ is the optimal consumption strategy consistent with investment strategy θ^* and fee ξ^*. We assume that portfolio strategy θ^* is a function of V_t and t, but not W_t.

Indeifference between the two investment strategies can be written as (15). The indirect utility functions for each strategy must obey the partial-integro differential equation (16). This implies a first order condition for optimal consumption which must be solved by $\bar{c}$:

$$\frac{\partial Y}{\partial w} = \frac{\partial U_c}{\partial c} = \nu_0 \frac{c^{\eta-1} e^{w\eta}}{(\zeta Y)^{(\eta/\zeta)-1}}.$$

Now, by (15) and (13),

$$\frac{\partial Y}{\partial W} = \zeta Y$$

so that the optimal consumption strategy can be derived as:

$$\bar{c} = \left[\frac{\left(\zeta A \left(t, V_t, \hat{c}, \hat{\theta}, \xi^* \right) \right)^{\frac{\eta}{\zeta}}}{\nu_0} \right]^{\frac{1}{\eta-1}}. \tag{18}$$

Now, defining

$$\begin{aligned} L(t, V_t, w_t) \;=\; & \frac{\partial Y}{\partial w}(r - \bar{c}_t + \theta_t^*(\mu_0 + \mu_1 V_t - \lambda k) - \frac{1}{2}\theta_t^{*2} V_t) + \frac{1}{2}\frac{\partial^2 Y}{\partial w^2}\theta_t^{*2} V_t \\ & + \frac{\partial Y}{\partial V}(\alpha_V - \beta_V V_t) + \frac{1}{2}\frac{\partial^2 Y}{\partial V^2}\sigma_V^2 V_t + \frac{\partial^2 Y}{\partial V \partial Y}\sigma_V \theta_t^* V_t \\ & + \lambda_t E(Y(w_t + \hat{\Gamma}_t) - Y(w_t)) + U_c(\bar{c}_t e_t^w, Y_t) \end{aligned}$$

we know from (16) that

$$-\frac{\partial Y}{\partial t} = L - \xi \frac{\partial Y}{\partial w},$$

which can be solved to find

$$\xi^* = \frac{L + \frac{\partial Y}{\partial t}}{\frac{\partial Y}{\partial w}}. \tag{19}$$

This calculation is easily implemented in conjunction with the solution method of appendix A:

1. Assume we know Y for time t_i and wish to calculate ξ^* for time t_{i-1}.

2. Calculate $\frac{\partial Y}{\partial t}$ for strategy $(\hat{\theta}, \hat{c})$.

3. Calculate c^* using (18).

4. Evaluate $\frac{\partial Y}{\partial t}$ again, using the strategy (θ^*, c^*) with zero fee. This is $L(t, V_t, w_t)$.

5. Use L to calculate ξ^* using (19).

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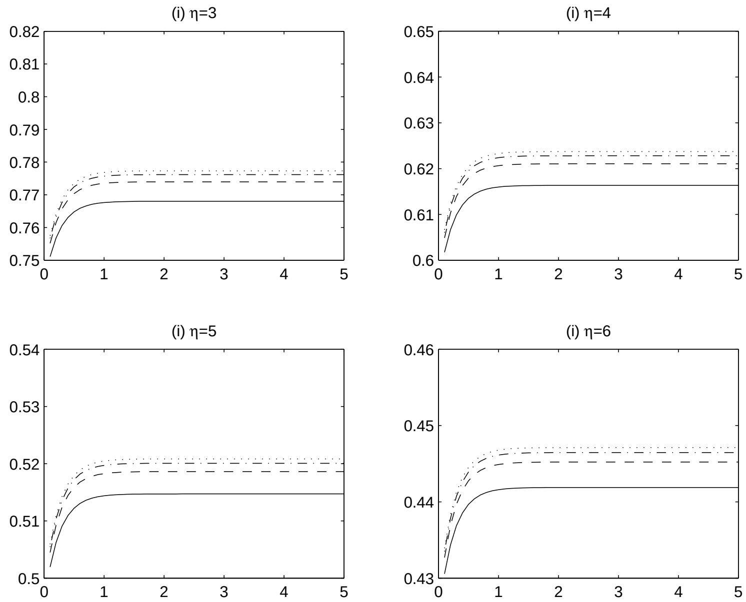
I. Jump consistent allocation.



II. Non-jump consistent allocation.

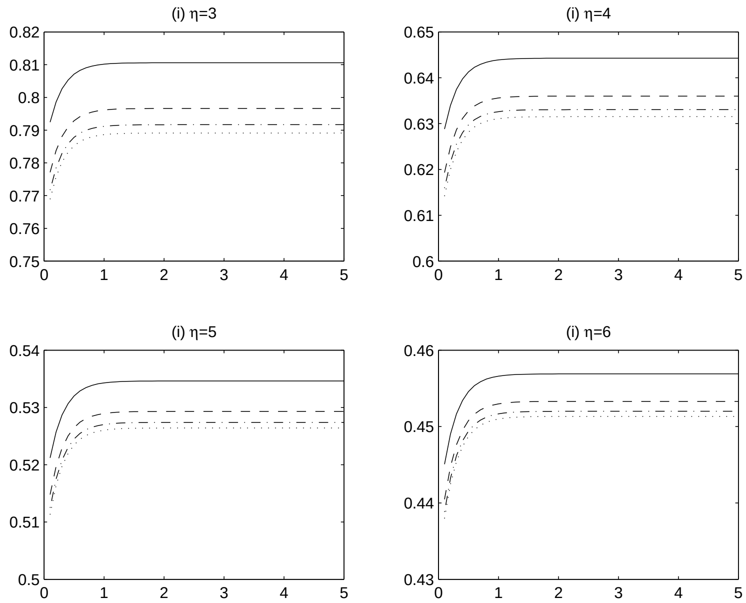
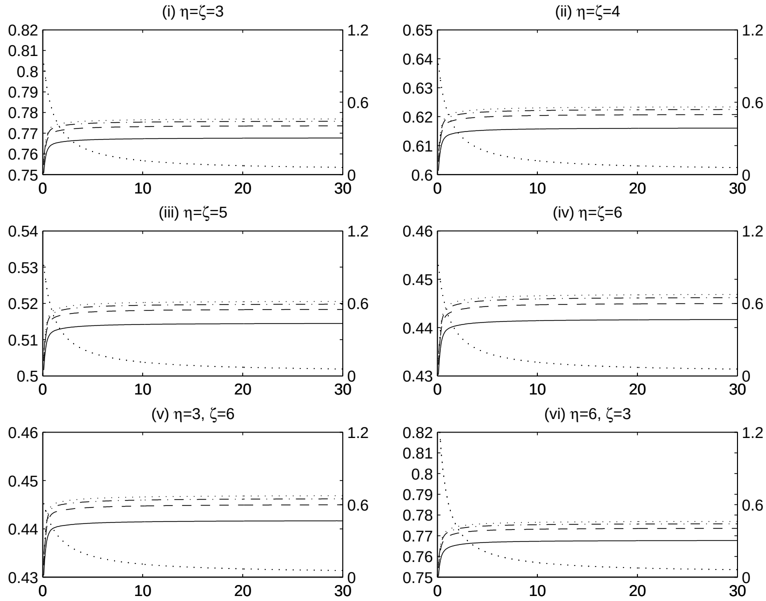


Figure 1: Portfolio allocations to the risky asset for the base scenario. The solid line represents the portfolio allocation when $V = 0.01$, the dashed line represents the allocation when $V = 0.02$, the dash-dotted line the allocation when $V = 0.03$ and the dotted line the allocation when $V = 0.04$. The horizontal axes represent time until bequest date. The vertical axes measure allocation to the risky security.

I. Jump consistent portfolio allocation and consumption strategy.



II. Non-jump consistent portfolio allocation and consumption strategy.

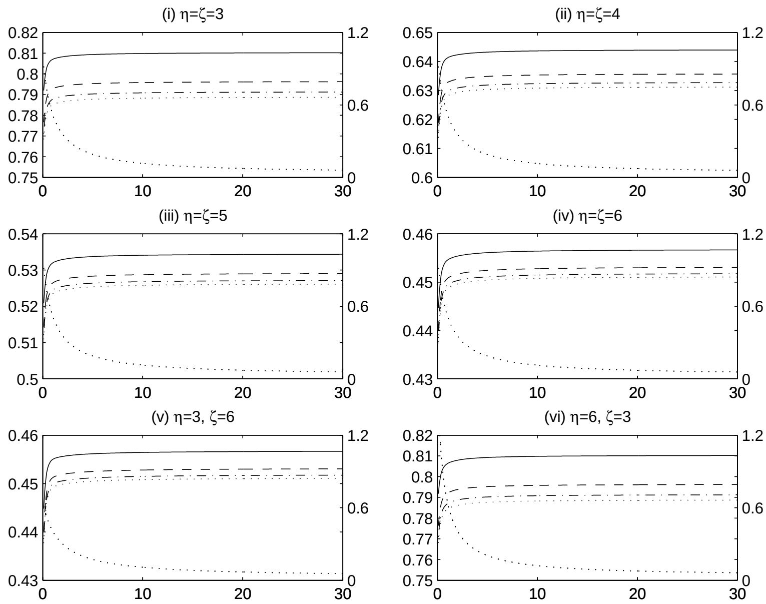


Figure 2: Non jump consistent portfolio allocations to the risky asset for the base scenario. The solid line represents the portfolio allocation when $V = 0.01$, the dashed line represents the allocation when $V = 0.02$, the dash-dotted line the allocation when $V = 0.03$ and the dotted line the allocation when $V = 0.04$. The downward sloping line in each graph is measured on the right hand axis, and denotes the portion of wealth consumed at that time. The horizontal axis measures time until bequest date. The left vertical axes measure allocation to the risky security, while the right vertical axes measure consumption, as a fraction of wealth.

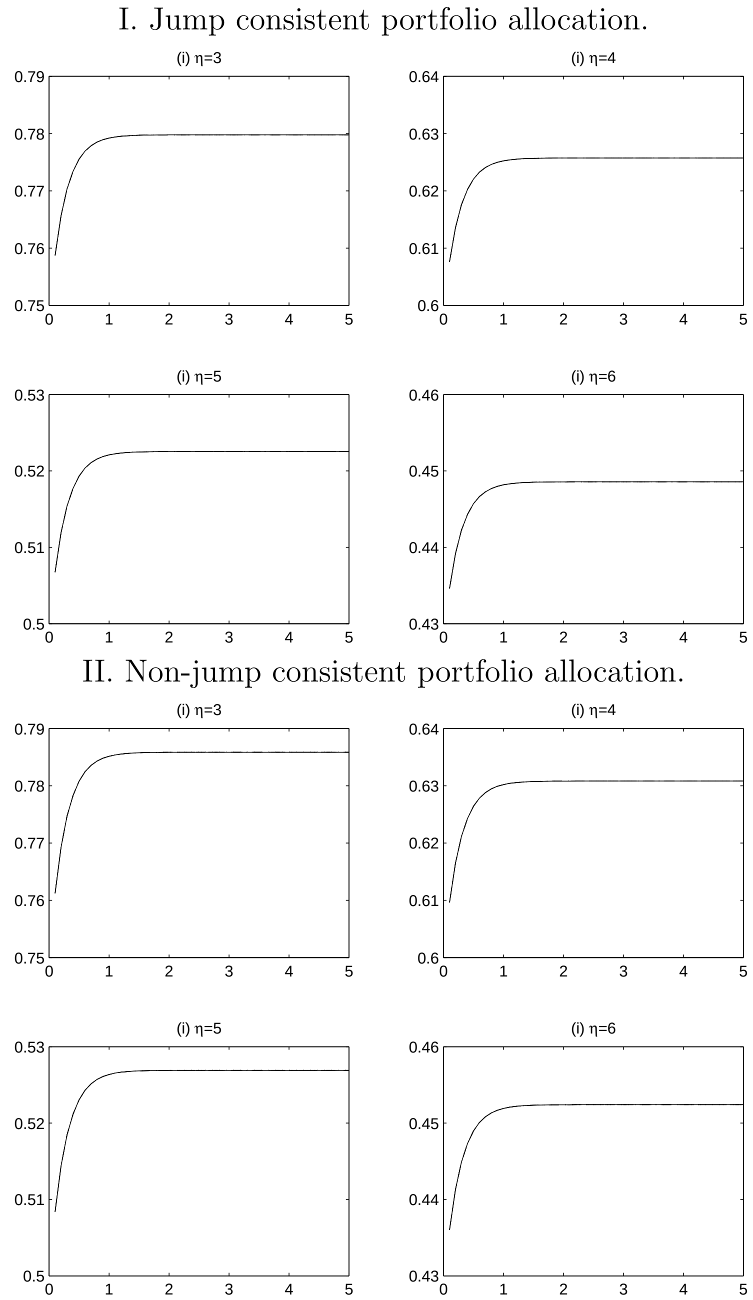
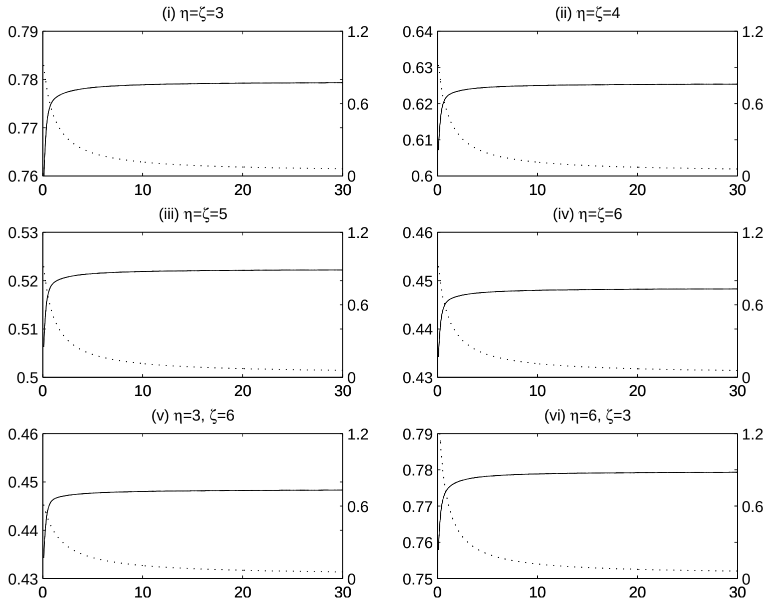


Figure 3: Portfolio allocations to the risky asset for the volatility dependent intensity scenario. The horizontal axis measures time until bequest date. The vertical axes measure allocation to the risky security.

I. Jump consistent portfolio and consumption.



II. Non-jump consistent portfolio and consumption.

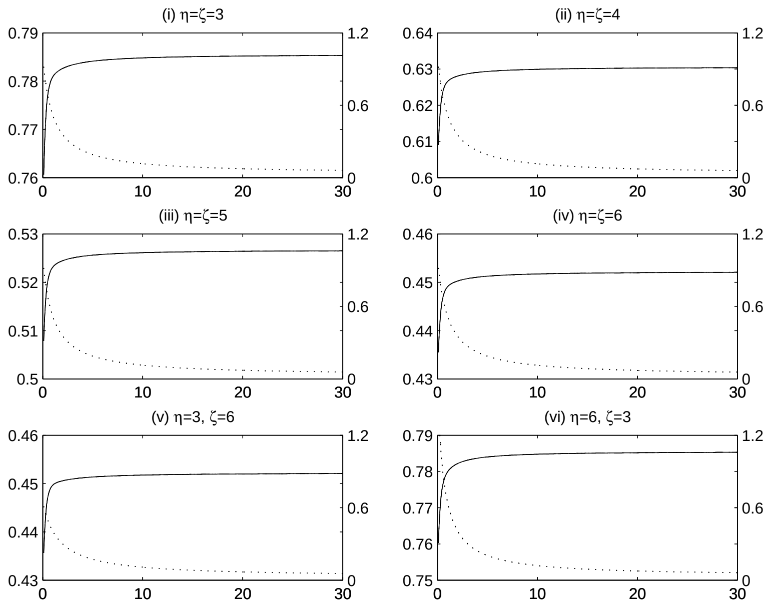


Figure 4: Investment and consumption strategies for investors in the volatility dependent jump intensity scenario. The horizontal axis measures time until bequest date. The left vertical axes measure allocation to the risky security, while the right vertical axes measure consumption, as a fraction of wealth.

Table I: Parameters for the constant intensity test of economic significance. Parameters refer to equations (4)-(9).

Parameter	Value	Parameter	Value
r	0.03	σ_J	0.052
μ_0	6.1054×10^{-3}	α_V	0.063
μ_1	3.0	β_V	4.38
ψ_0	0.744	σ_V	0.244
ψ_1	0.0	ρ_V	-0.612
μ_J	-0.01		

Table II: Value gains for pure investment strategies, for base scenario.

Panel Ia: cash equivalent is the portion of wealth (in percent) which an investor using the non-jump consistent strategy would be prepared to pay to have access to the jump strategy for the remainder of their investment horizon. Panel Ib: the annual fee measure uses the analysis in section A to convert the cash equivalent into an annual fee proportional to the investor's initial wealth (measured in basis points). Panel II presents the optimal allocation to the risky security (θ) for different times to maturity, assuming $V = 0.01$. Panel III presents the difference between the non-jump allocation and the optimal allocation.

I. Value gains due to following jump-consistent strategy.

I.(a) Cash equivalent

	Investment Horizon						
Risk aversion (ζ)	0.1	0.2	0.5	1.0	5.0	10.0	20.0
-3	0.0004	0.0009	0.0023	0.0046	0.0224	0.0448	0.0894
-4	0.0002	0.0005	0.0012	0.0024	0.0119	0.0237	0.0474
-5	0.0001	0.0003	0.0007	0.0015	0.0072	0.0143	0.0286
-6	0.0001	0.0002	0.0005	0.0010	0.0047	0.0094	0.0189

I.(b) Annual fee

	Investment Horizon						
Risk aversion (ζ)	0.1	0.2	0.5	1.0	5.0	10.0	20.0
-3	0.8860	0.8978	0.9165	0.9229	0.9239	0.9239	0.9239
-4	0.4611	0.4685	0.4790	0.4829	0.4834	0.4834	0.4834
-5	0.2750	0.2799	0.2865	0.2891	0.2894	0.2894	0.2894
-6	0.1796	0.1829	0.1876	0.1893	0.1896	0.1896	0.1896

II. Holdings of risky asset for jump-consistent strategy.

	Investment Horizon					
Risk aversion (ζ)	0.1	0.2	0.5	1.0	2.0	5.0
-3	0.7511	0.7568	0.7648	0.7676	0.7680	0.7680
-4	0.6017	0.6066	0.6135	0.6160	0.6163	0.6164
-5	0.5019	0.5062	0.5122	0.5144	0.5147	0.5147
-6	0.4306	0.4344	0.4397	0.4416	0.4419	0.4419

III. Surplus holdings of risky asset for non jump-consistent strategy.

	Investment Horizon					
Risk aversion (ζ)	0.1	0.2	0.5	1.0	2.0	5.0
-3	0.0414	0.0418	0.0423	0.0425	0.0426	0.0426
-4	0.0271	0.0273	0.0278	0.0279	0.0279	0.0279
-5	0.0193	0.0195	0.0198	0.0199	0.0199	0.0199
-6	0.0145	0.0147	0.0149	0.0150	0.0150	0.0150

Table III: Results for the base scenario, with consumption.

Panels I(a), I(b), II and III are as discussed in table V. Panel IV shows consumption levels for $V = 0.01$ for various time horizons. Panel V shows the level of over-consumption for the sub-optimal strategy.

I.(a) Cash Equivalent

Preferences	Investment Horizon					
	0.5	1.0	2.0	5.0	10.0	20.0
$\eta = \zeta = -3$	0.0019	0.0034	0.0059	0.0127	0.0231	0.0420
$\eta = \zeta = -4$	0.0010	0.0018	0.0031	0.0068	0.0123	0.0226
$\eta = \zeta = -5$	0.0006	0.0011	0.0019	0.0041	0.0075	0.0138
$\eta = \zeta = -6$	0.0004	0.0007	0.0012	0.0027	0.0050	0.0092
$\eta = -3, \zeta = -6$	0.0004	0.0008	0.0013	0.0028	0.0050	0.0090
$\eta = -6, \zeta = -3$	0.0018	0.0031	0.0055	0.0123	0.0231	0.0440

I.(b) Annual Fee

Preferences	Investment Horizon					
	0.5	1.0	2.0	5.0	10.0	20.0
$\eta = \zeta = -3$	0.8990	0.9084	0.9160	0.9278	0.9398	0.9608
$\eta = \zeta = -4$	0.4681	0.4739	0.4787	0.4856	0.4929	0.5056
$\eta = \zeta = -5$	0.2789	0.2827	0.2860	0.2908	0.2956	0.3041
$\eta = \zeta = -6$	0.1818	0.1846	0.1870	0.1905	0.1940	0.2001
$\eta = -3, \zeta = -6$	0.1827	0.1849	0.1866	0.1889	0.1909	0.1939
$\eta = -6, \zeta = -3$	0.8955	0.9083	0.9205	0.9396	0.9638	1.0099

II. Holdings of risky security

Preferences	Investment Horizon					
	0.5	1.0	2.0	5.0	10.0	20.0
$\eta = \zeta = -3$	0.7622	0.7647	0.7659	0.7669	0.7673	0.7676
$\eta = \zeta = -4$	0.6113	0.6135	0.6145	0.6154	0.6158	0.6160
$\eta = \zeta = -5$	0.5103	0.5122	0.5131	0.5139	0.5142	0.5144
$\eta = \zeta = -6$	0.4379	0.4396	0.4404	0.4411	0.4414	0.4416
$\eta = -3, \zeta = -6$	0.4384	0.4401	0.4407	0.4412	0.4414	0.4416
$\eta = -6, \zeta = -3$	0.7613	0.7640	0.7655	0.7668	0.7673	0.7676

III. Surplus Holdings of suboptimal plan

Preferences	Investment Horizon					
	0.5	1.0	2.0	5.0	10.0	20.0
$\eta = \zeta = -3$	0.0421	0.0423	0.0424	0.0425	0.0425	0.0425
$\eta = \zeta = -4$	0.0276	0.0278	0.0278	0.0279	0.0279	0.0279
$\eta = \zeta = -5$	0.0197	0.0198	0.0198	0.0199	0.0199	0.0199
$\eta = \zeta = -6$	0.0148	0.0149	0.0149	0.0150	0.0150	0.0150
$\eta = -3, \zeta = -6$	0.0149	0.0149	0.0150	0.0150	0.0150	0.0150
$\eta = -6, \zeta = -3$	0.0421	0.0423	0.0424	0.0425	0.0425	0.0425

IV. Consumption

Preferences	Investment Horizon					
	0.5	1.0	2.0	5.0	10.0	20.0
$\eta = \zeta = -3$	0.6819	0.5189	0.3546	0.1896	0.1149	0.0733
$\eta = \zeta = -4$	0.6818	0.5185	0.3540	0.1887	0.1138	0.0720
$\eta = \zeta = -5$	0.6818	0.5183	0.3535	0.1879	0.1129	0.0710
$\eta = \zeta = -6$	0.6819	0.5182	0.3532	0.1873	0.1122	0.0702
$\eta = -3, \zeta = -6$	0.4979	0.4060	0.2987	0.1721	0.1070	0.0682
$\eta = -6, \zeta = -3$	0.9267	0.6464	0.4078	0.2028	0.1196	0.0756

V. Surplus consumption of suboptimal plan

Preferences	Investment Horizon					
	0.5	1.0	2.0	5.0	10.0	20.0
$\eta = \zeta = -3$	0.0001	0.0002	0.0003	0.0003	0.0003	0.0004
$\eta = \zeta = -4$	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002
$\eta = \zeta = -5$	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002
$\eta = \zeta = -6$	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
$\eta = -3, \zeta = -6$	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
$\eta = -6, \zeta = -3$	0.0002	0.0003	0.0003	0.0003	0.0004	0.0004

Table IV: Parameters for the volatility dependent intensity scenario. The parameters refer to equations (4)-(9).

Parameter	Value	Parameter	Value
r	0.03	σ_J	0.039
μ_0	0.0	α_V	0.061
μ_1	3.425882	β_V	4.25
ψ_0	0.0	σ_V	0.237
ψ_1	93.4	ρ_V	-0.611
μ_J	-0.002		

Table V: Value gains for pure investment strategies, for volatility dependent intensity scenario. Panel Ia: cash equivalent is the portion of wealth (in percent) which an investor using the non-jump consistent strategy would be prepared to pay to have access to the jump strategy for the remainder of their investment horizon. Panel Ib: the annual fee measure uses the analysis in section A to convert the cash equivalent into an annual fee proportional to the investor's initial wealth (measured in basis points). Panel II presents the optimal allocation to the risky security (θ) for different times to maturity, assuming $V = 0.01$. Panel III presents the difference between the non-jump allocation and the optimal allocation.

I. Value gains due to following jump-consistent strategy.

I.(a) Cash equivalent

	Investment Horizon						
Preferences (ζ)	0.1	0.2	0.5	1.0	5.0	10.0	20.0
-3	0.0000	0.0000	0.0000	0.0001	0.0006	0.0012	0.0025
-4	0.0000	0.0000	0.0000	0.0001	0.0005	0.0011	0.0022
-5	0.0000	0.0000	0.0000	0.0001	0.0005	0.0010	0.0020
-6	0.0000	0.0000	0.0000	0.0001	0.0004	0.0009	0.0018

I.(b) Annual fee

	Investment Horizon						
Preferences (ζ)	0.1	0.2	0.5	1.0	5.0	10.0	20.0
-3	0.0007	0.0015	0.0032	0.0041	0.0042	0.0042	0.0042
-4	0.0006	0.0012	0.0027	0.0035	0.0037	0.0037	0.0037
-5	0.0005	0.0010	0.0024	0.0031	0.0033	0.0033	0.0033
-6	0.0004	0.0009	0.0021	0.0028	0.0030	0.0030	0.0030

II. Holdings of risky asset for jump-consistent strategy.

	Investment Horizon					
Preferences (ζ)	0.1	0.2	0.5	1.0	2.0	5.0
-3.0	0.7587	0.7657	0.7755	0.7792	0.7798	0.7798
-4.0	0.6076	0.6136	0.6221	0.6252	0.6257	0.6257
-5.0	0.5067	0.5119	0.5193	0.5221	0.5225	0.5225
-6.0	0.4346	0.4392	0.4457	0.4482	0.4486	0.4486

III. Surplus holdings of risky asset for non jump-consistent strategy.

	Investment Horizon					
Preferences (ζ)	0.1	0.2	0.5	1.0	2.0	5.0
-3.0	0.0025	0.0036	0.0053	0.0059	0.0061	0.0061
-4.0	0.0020	0.0029	0.0044	0.0050	0.0051	0.0051
-5.0	0.0017	0.0025	0.0037	0.0043	0.0044	0.0044
-6.0	0.0014	0.0021	0.0033	0.0037	0.0038	0.0038

Table VI: Results for the volatility dependent intensity scenario, with consumption.

Panels I(a), I(b), II and III are as discussed in table V. Panel IV shows consumption levels for $V = 0.01$ for various time horizons. Panel V shows the level of over-consumption for the sub-optimal strategy.

I.(a) Cash Equivalent

Preferences	Investment Horizon					
	0.5	1.0	2.0	5.0	10.0	20.0
$\eta = \zeta = -3$	0.0000	0.0000	0.0001	0.0003	0.0007	0.0014
$\eta = \zeta = -4$	0.0000	0.0000	0.0001	0.0003	0.0006	0.0013
$\eta = \zeta = -5$	0.0000	0.0000	0.0001	0.0003	0.0005	0.0012
$\eta = \zeta = -6$	0.0000	0.0000	0.0001	0.0002	0.0005	0.0011
$\eta = -3, \zeta = -6$	0.0000	0.0000	0.0001	0.0002	0.0005	0.0010
$\eta = -6, \zeta = -3$	0.0000	0.0001	0.0001	0.0003	0.0007	0.0017

I.(b) Annual Fee

Preferences	Investment Horizon					
	0.5	1.0	2.0	5.0	10.0	20.0
$\eta = \zeta = -3$	0.0025	0.0032	0.0038	0.0048	0.0061	0.0087
$\eta = \zeta = -4$	0.0021	0.0028	0.0034	0.0044	0.0059	0.0087
$\eta = \zeta = -5$	0.0018	0.0024	0.0030	0.0041	0.0057	0.0087
$\eta = \zeta = -6$	0.0015	0.0021	0.0027	0.0039	0.0055	0.0087
$\eta = -3, \zeta = -6$	0.0015	0.0021	0.0025	0.0032	0.0040	0.0055
$\eta = -6, \zeta = -3$	0.0023	0.0032	0.0040	0.0059	0.0088	0.0145

II. Holdings of risky security

Preferences	Investment Horizon					
	0.5	1.0	2.0	5.0	10.0	20.0
$\eta = \zeta = -3$	0.7723	0.7756	0.7771	0.7783	0.7789	0.7792
$\eta = \zeta = -4$	0.6193	0.6221	0.6234	0.6245	0.6250	0.6253
$\eta = \zeta = -5$	0.5169	0.5194	0.5205	0.5215	0.5219	0.5221
$\eta = \zeta = -6$	0.4435	0.4457	0.4468	0.4476	0.4480	0.4482
$\eta = -3, \zeta = -6$	0.4441	0.4463	0.4470	0.4477	0.4480	0.4482
$\eta = -6, \zeta = -3$	0.7712	0.7747	0.7766	0.7782	0.7789	0.7792

III. Surplus Holdings of suboptimal plan

Preferences	Investment Horizon					
	0.5	1.0	2.0	5.0	10.0	20.0
$\eta = \zeta = -3$	0.0047	0.0053	0.0056	0.0058	0.0059	0.0060
$\eta = \zeta = -4$	0.0039	0.0044	0.0047	0.0048	0.0049	0.0050
$\eta = \zeta = -5$	0.0033	0.0038	0.0040	0.0042	0.0042	0.0043
$\eta = \zeta = -6$	0.0029	0.0033	0.0035	0.0037	0.0037	0.0038
$\eta = -3, \zeta = -6$	0.0030	0.0034	0.0035	0.0037	0.0037	0.0038
$\eta = -6, \zeta = -3$	0.0045	0.0052	0.0055	0.0058	0.0059	0.0060

IV. Consumption

Preferences	Investment Horizon					
	0.5	1.0	2.0	5.0	10.0	20.0
$\eta = \zeta = -3$	0.6819	0.5188	0.3546	0.1897	0.1150	0.0734
$\eta = \zeta = -4$	0.6818	0.5185	0.3540	0.1887	0.1138	0.0721
$\eta = \zeta = -5$	0.6818	0.5183	0.3535	0.1879	0.1129	0.0710
$\eta = \zeta = -6$	0.6818	0.5182	0.3532	0.1874	0.1122	0.0702
$\eta = -3, \zeta = -6$	0.4978	0.4060	0.2987	0.1721	0.1071	0.0682
$\eta = -6, \zeta = -3$	0.9266	0.6463	0.4078	0.2029	0.1196	0.0757

V. Surplus consumption of suboptimal plan

Preferences	Investment Horizon					
	0.5	1.0	2.0	5.0	10.0	20.0
$\eta = \zeta = -3$	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
$\eta = \zeta = -4$	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
$\eta = \zeta = -5$	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
$\eta = \zeta = -6$	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
$\eta = -3, \zeta = -6$	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
$\eta = -6, \zeta = -3$	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002