

Hedging the Value of Waiting*

Glenn W. Boyle

New Zealand Institute for the Study of Competition and Regulation

Graeme A. Guthrie[†]

Victoria University of Wellington

March 8, 2004

Abstract

We analyze the optimal hedging policy of a firm that has flexibility in the timing of investment. Conventional wisdom suggests that hedging adds value by alleviating the underinvestment problem associated with capital market frictions. However, our model shows that hedging also adds value by allowing investment to be delayed in circumstances where the same frictions would cause it to commence prematurely. Thus, hedging can have the paradoxical effect of reducing investment. We also show that greater timing flexibility increases the optimal quantity of hedging, but has a non-monotonic effect on the additional value created by hedging. These results may help explain the empirical findings that investment rates do not differ between hedgers and non-hedgers, and that hedging propensities do not depend on standard measures of growth opportunities.

JEL classification: G31, G32

Keywords: hedging, investment timing flexibility

*For helpful comments on earlier drafts, we are grateful to Glenn Kentwell, Stephen Gray, Vivien Pullar, Steven Li, and, especially, Peter McKay, and to seminar participants at Auckland, Otago, and the 2003 AFAANZ and ESAM conferences. Any remaining errors and ambiguities are our responsibility.

[†]Corresponding author. School of Economics and Finance, Victoria University of Wellington, PO Box 600, Wellington, New Zealand. Email: graeme.guthrie@vuw.ac.nz Ph.: (64)(4)463-5763 Fax: (64)(4)463-5566

Hedging the Value of Waiting

I Introduction

Firms enhance shareholder wealth by making productive investments. Building on this simple observation, Bessembinder (1991) and Froot, Scharfstein and Stein (1993) show that hedging allows firms to alleviate the under-investment problem associated with imperfect capital markets. When external financing is costly, a shortfall in internal funds can lead to a sub-optimal reduction in investment. By ensuring that internal funds are sufficient to allow profitable investment projects to proceed, hedging adds to firm value.

The models of Bessembinder (1991) and Froot et al. (1993) assume that projects are available at a single future date and thus do not allow for managerial flexibility in the timing of investment. As Dixit and Pindyck (1994) point out, most investment decisions have some degree of timing flexibility, although this may be constrained by factors such as competition and financing uncertainty.¹ In this paper, we examine the role of hedging when the firm's investment opportunities are available for some finite period of time, a simple extension to a dynamic world that has significant implications for the optimal hedging policy. When the timing of investment is flexible, a firm subject to external financing restrictions faces the risk that its ability to finance investment may disappear in the future, perhaps exactly at the time it would wish to invest. Consequently, the firm may decide to invest in states where it has sufficient funds to do so, even when the optimal unconstrained policy would entail delay of investment. By reducing the risk of future funding difficulties, hedging allows the firm to improve the timing of investment; without hedging, the firm might have to rush into investment and sacrifice some of the project's value. Or, to put it another way, hedging adds value not only because it allows investment to occur, as in the "now-or-never" environment of Froot et al., but also because it allows investment to be delayed. Thus, hedging not only permits more investment, but also paradoxically encourages less investment.

The need for hedging in a dynamic world with investment timing flexibility differs from its static counterpart in two contrasting ways. First, by extending the period of time in which investment can occur, timing flexibility increases the probability of the firm accumulating sufficient funds for financing that investment and thus lowers the need for hedging. Second, by creating valuable options that also require protection, timing flexibility increases the need for hedging. These two effects change the sensitivity of hedging demand to the firm's stochastic environment. For example, in the absence of any flexibility, a 1% increase in firm cash flow volatility has approximately the same effect on the optimal hedge as does a 1% fall in the correlation between firm cash flow and project value. With flexibility, however, hedging demand becomes significantly more sensitive to the former.

As well as Bessembinder (1991) and Froot et al. (1993), our work is also related to several

¹On these issues, see, for example, Pindyck (1993) and Boyle and Guthrie (2003) respectively.

other studies. Grossman and Vila (1992) examine the optimal trading strategy of an investor subject to a financing constraint, but do not explicitly consider the roles of hedging and investment timing flexibility. Mello et al. (1995) consider the situation of a multinational firm that can exit production in one or more countries and, like Froot et al., conclude that hedging allows continued investment in circumstances where exit would otherwise have occurred. Mello and Parsons (2000) analyze the role of hedging in protecting the value of a project's abandonment option for a firm subject to capital market frictions, and compare the performance of alternative hedging rules, but do not address the initial decision to invest. McDonald and Siegel (1986) provide the seminal treatment of investment timing flexibility, but assume that markets are frictionless and thus allow no role for hedging. Boyle and Guthrie (2003) extend the McDonald and Siegel model by introducing external financing restrictions, but do not allow the firm to hedge its cash flow risk and thus do not consider the optimal hedging policy or its interaction with investment.

In the next section, we outline a simple example that intuitively captures the role of investment timing flexibility in determining the optimal hedge position. We then develop a more general model of investment and hedging in Section III. In Sections IV to VI, we show that the optimal hedging policy protects not only the firm's ability to undertake investment, but also its ability to delay investment, and that this additional role may have implications for our understanding of empirical work on hedging. Finally, we summarize our findings and offer some concluding remarks in Section VII.

II A simple example

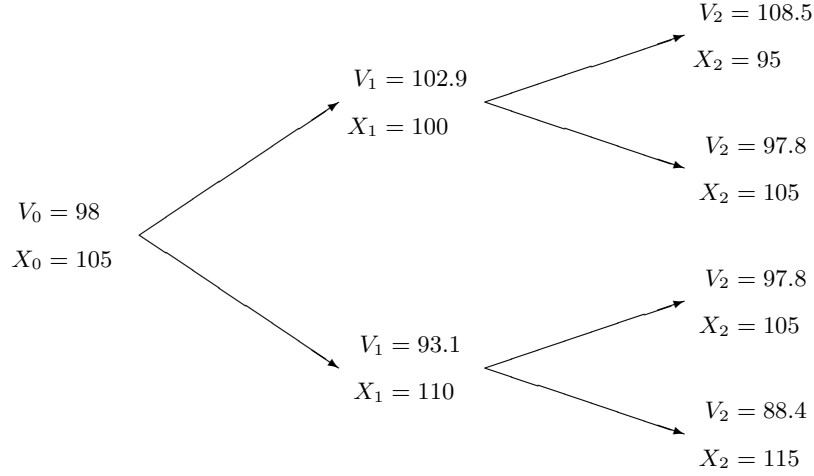
We begin with a numerical example that compares the hedging decision of a firm with flexibility in the timing of investment with that of a firm without such flexibility. Although this example is highly stylized, it illustrates the basic insight of our story in a simple and easily-understood manner.

A firm has the rights to a \$100 project and it must finance this from internal funds. There are three dates; we ignore discounting. Currently (at date 0), the project value V is \$98 and the firm's cash stock X is \$105. Subsequently, these variables follow the negatively correlated binomial processes shown in Figure 1: in one state, V rises by 5% and X falls by \$5; in the other state V falls by 5% and X rises by \$5.² All outcomes occur with equal probability.

At date 0, the firm can hedge date 1 cash flow by holding h units of an instrument that offers payoffs which are perfectly negatively correlated with cash stock innovations. As a result,

²The perfect negative correlation between X and V helps simplify the example, but as later sections illustrate, is unnecessary in general. The distinction between multiplicative shocks to V and additive shocks to X anticipates our more general model of Section III. Unlike project market value, the absolute magnitude of shocks to the firm's cash stock should not depend on its initial value.

Figure 1: Evolution of project value V and cash stock X without hedging



Notes. At date 0, the project is worth \$98 and is 5% higher or lower at subsequent dates. The firm's cash stock X is \$105 at date 0 and is \$5 higher or lower at subsequent dates. All changes occur with equal probability.

the date 1 states are either

$$\text{or} \quad \begin{aligned} V_1 &= 102.9, & X_1 &= 105 - 5(1 - h), \\ V_1 &= 93.1, & X_1 &= 105 + 5(1 - h). \end{aligned} \quad (1)$$

For these state-contingent payoff and cash flow realizations, we consider two investment scenarios and calculate the date 0 hedging decision for each.

In the first scenario, the investment option expires at date 1, so the firm cannot invest at date 2. This corresponds to the standard static investment decision where the firm must invest at a particular date (date 1) or not at all. In this case, the firm wishes to invest in the high- V state ($V_1 = 102.9$) at date 1, but not in the low- V state ($V_1 = 93.1$). Moreover, internal funds are sufficient in the former state to allow the desired investment to occur, so no hedging is required at date 0.

In the second scenario, the investment option expires at date 2, so the firm can invest at either date 1 or date 2. This represents a simplification of the standard dynamic investment decision where the firm has flexibility in the timing of an investment whose future payoffs are uncertain. The optimal investment policy is to exercise the investment option at a particular date if and only if the payoff from doing so exceeds the value of waiting and retaining the option.

To facilitate comparison with the first scenario, we analyze this more complex problem in three stages. First, we outline a benchmark case where the firm is unconstrained (i.e., we ignore the uncertainty about X) and the optimal first-best policy is to invest at date 2. Second, we extend the benchmark case to allow for uncertain cash flow and find that, in the absence of hedging, the risk that X may be insufficient to finance investment at date 2 induces a second-

best policy where investment occurs at date 1. Third, we show that hedging allows the firm to retrieve the first-best policy, i.e., investment occurs at date 2 rather than date 1. As a result, hedging has the effect of *reducing* investment at date 1.³

Turning to the specifics of the first stage where uncertainty about X is ignored, the firm considers investment at date 1 if $V_1 = 102.90$, but not otherwise (in the latter case, not only does the firm not invest at date 1, but the option itself is worthless since investment can never be profitable at date 2 either). If it invests, it obtains a payoff equal to $102.90 - 100 = 2.90$. However, if the firm chooses to wait until date 2, it then has a 50% probability of obtaining a payoff equal to $108.05 - 100 = 8.05$, and a 50% probability of obtaining a zero payoff (when $V_2 = 97.8 < 100$). From the perspective of date 1, this payoff has an expected present value of $0.5(8.05) = 4.025$. Since $4.025 > 2.90$, the value of waiting and retaining the investment option exceeds the date 1 payoff from exercising, so the optimal investment policy is to wait until date 2.

However, incorporating uncertainty about X yields a different outcome. From Figure 1, we see that the date 2 payoff of 8.05 is not achievable, since $X_2 = 95$ in that state, i.e., the firm has insufficient funds to pay the investment cost of 100. Thus, choosing to wait until date 2 guarantees a payoff of zero, either because investment is unprofitable ($V < 100$) or because it is impossible ($X < 100$). Consequently, the date 1 value of retaining the option is zero. Since $2.90 > 0$, the firm chooses to invest at date 1.

Although this decision is optimal given the financing constraint, firm value is lower than in the unconstrained case. From the perspective of date 0, the value of the investment option is $0.5(2.90) = 1.45$ when X is uncertain, but is $(0.5)(0.5)(8.05) = 2.0125$ if there is no risk of being unable to finance the project at date 2. Thus, the financing constraint reduces firm value by 0.5625. That is, relative to the first-best unconstrained outcome, the firm adopts a sub-optimal early investment policy.

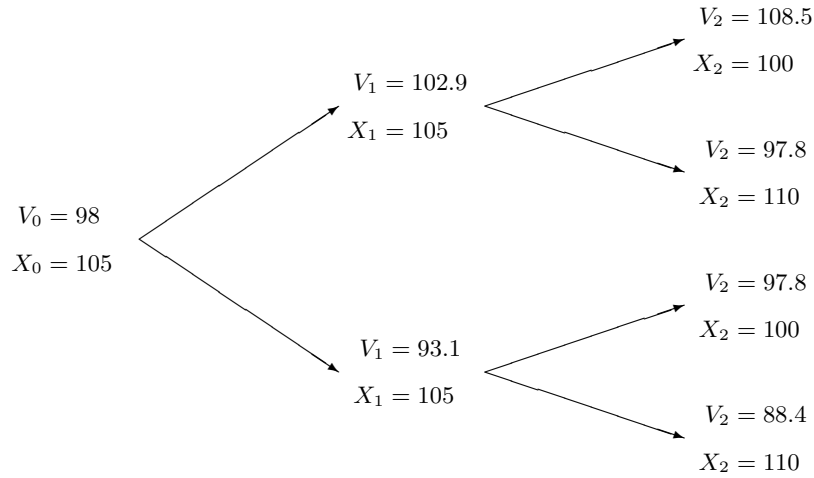
In this situation, hedging at date 0 is valuable because it saves the firm from having to invest prematurely.⁴ Suppose we set $h = 1$ in equation (1).⁵ Then the joint evolution of V and X is shown in Figure 2. Now the firm has sufficient cash to invest in all date 2 states; in particular, it has sufficient cash in the state where $V_2 = 108.5$. With hedging, the risk of losing the ability to finance the project at date 2 disappears, so the firm can delay investment at date 1 confident in the knowledge that sufficient funding will be available at date 2 should it wish to invest at

³Moreover, it does so in the context of maximizing firm value. High levels of hedging might also be associated with low investment if, for example, the firm were run by an undiversified and/or highly risk averse manager, but then firm value would not be maximized.

⁴Note that the firm is not allowed to hedge at date 1, as this would simply repeat the first scenario in which the firm knows that it faces a “now-or-never” investment decision, thereby assuming away the problem we wish to analyze. In practice, a firm with investment timing flexibility is always uncertain about whether it will begin or further delay investment at the next date, so this set-up captures this uncertainty within a 3-date world.

⁵ $h = 1$ is the minimum hedge necessary to fully restore the investment option. If hedging is costless, then the firm is indifferent between all $h \geq 1$.

Figure 2: Evolution of project value V and cash stock X when the hedge ratio $h = 1$



Notes. At date 0, the project is worth \$98 and is 5% higher or lower at subsequent dates. The firm's cash stock X is \$105 at date 0 and is \$5 higher or lower at subsequent dates. All changes occur with equal probability. The firm can hedge date 1 cash flow by holding h units of an instrument that offers payoffs which are perfectly negatively correlated with cash stock innovations. In this diagram, the firm sets $h = 1$ and thus guarantees sufficient funds for investment at date 2.

that date. Hedging thus restores the value of the investment option by eliminating the need for premature investment.

The principal lesson of this example is straightforward. When the firm has flexibility in investment timing, hedging allows it, should it so choose, to retain its investment option. Without hedging, the firm may have to accelerate investment in a sub-optimal fashion and thus give up its option; hedging allows it to retain flexibility.

The dynamic nature of the investment timing decision thus introduces an additional element into the hedging decision. At each date where it does not invest, the firm with timing flexibility knows that the optimal decision at the next date may be to invest, or it may be to delay further. Optimal hedging therefore requires that both alternatives be protected, i.e., hedging not only protects a firm's investment projects, but also the options on those projects. By contrast, optimal hedging in a static world of "now-or-never" investment decisions is concerned only with the former.

Although this example provides helpful intuition, it has obvious limitations. Not only does it allow the firm to perfectly hedge cash flow risk, but it also permits investment at only two dates. Moreover, project value and firm cash flow are perfectly negatively correlated. In the next section, we develop a more general model of the joint determination of investment and hedging.

III The model

The firm owns the rights to a project that first becomes available at date T_1 . These rights give it an option to invest in the project at any date t in the interval $[T_1, T_2]$; higher values of T_2 correspond to greater timing flexibility, reflecting, for example, the firm's competitive position. During this time, the project cannot be undertaken by any other firm, so at each date in $[T_1, T_2)$ the firm can either exercise the option and invest, or delay investment and retain the option. If the firm invests, it pays a fixed amount I and receives a project worth V . The risk-neutral process for V is

$$dV = (\mu - \kappa)Vdt + \sigma Vd\eta, \quad (2)$$

where μ , σ , and κ are constants, and η is a Wiener process. The parameter μ is the expected rate of growth in V , σ is the standard deviation, and κ is the market price of risk associated with V , i.e., $\mu - \kappa$ is the "certainty-equivalent" expected growth in V .⁶

In contrast to standard models of the investment timing decision (see, for example, McDonald and Siegel (1986) and Dixit and Pindyck (1994)), where the firm has unlimited costless access to capital markets, we assume that the firm is restricted to financing the project with internal funds.⁷ Specifically, at the date the firm wishes to exercise its investment option, it can do so if and only if its cash balance is greater than or equal to I .⁸ Prior to that date, however, it can enter into hedging contracts that alter the future distribution of its internal funds.

To model this constraint, we assume that the firm begins with an initial cash stock X which, over time, is augmented in three ways. First, if the firm does not launch the project, X is invested in riskless securities which earn interest at the rate r . Second, the firm's existing activities generate uncertain operating cash flow with risk-neutral dynamics

$$rGdt + \phi d\zeta,$$

where G is the present value of a perpetual claim to this cash flow, $\phi > 0$ is a constant and ζ is a Wiener process with $d\eta d\zeta = \rho dt$.⁹ Third, the firm hedges its operating cash flow. For this purpose, we suppose there is a futures contract for which the futures price P follows the process

$$dP = \psi P d\epsilon,$$

⁶If V is a traded asset, then κ is its market-determined risk premium. See, for example, Hull (2003, ch. 21).

⁷It is straightforward to allow the firm access to external funding (see Boyle and Guthrie, 2003), but as this has no qualitative effect on our results, we maintain the simpler structure here. Numerous articles in the financial press refer to the external funding difficulties faced by many firms in recent years. See, for example, Alsop (2001), Anonymous (2002), Chung (2002), and Zellner et al. (2003).

⁸The source of this constraint is immaterial for our purposes, but it could be due to severe information or agency problems, or because the firm does not wish to reveal information to competitors about the project at the investment stage.

⁹The values of G and ϕ can be interpreted as reflecting the size, nature and productivity of the firm's assets-in-place, as well as the financial commitments created by its capital structure policy.

where $\psi > 0$ is a constant and ϵ is a Wiener process satisfying $d\epsilon d\zeta = \lambda dt$ and $d\epsilon d\eta = \lambda \rho dt$ for some constant $\lambda \in (-1, 1)$.¹⁰ Thus, the change in the futures price is imperfectly correlated with shocks to the firm's cash flow, so the firm cannot eliminate all cash flow volatility.

A hedging policy consists of a choice of h futures positions; if $h > 0$, h is the number of long positions; if $h < 0$, $-h$ is the number of short positions. Thus, over the time interval dt , the firm's beginning cash stock yields interest $rXd t$, the firm's existing assets generate cash flow $rGdt + \phi d\zeta$, while marking-to-market the futures contracts earns $hdP = \psi hP d\epsilon$. Combining these sources of cash, the dynamics of the change in the firm's total cash stock are given by

$$dX = r(X + G)dt + \phi d\zeta + \psi hP d\epsilon.$$

Note that hedging affects only future values of the cash stock via the marking-to-market process. Thus, the firm cannot bypass the financing constraint by using the futures contract to raise funds directly.

It is convenient to rewrite the process for X in an equivalent form which involves only one Wiener process; that is,

$$dX = r(X + G)dt + (\phi^2 + 2\lambda\phi\psi H + \psi^2 H^2)^{1/2} d\xi, \quad (3)$$

where $H = hP$ and ξ is a Wiener process satisfying

$$d\xi d\eta = \frac{\rho(\phi + \lambda\psi H)}{(\phi^2 + 2\lambda\phi\psi H + \psi^2 H^2)^{1/2}} dt.$$

Given the firm's choice of hedging policy, h , its cash stock evolves according to (3). Together, equations (2) and (3) represent a system with state variables (X, V) and the control H . Consequently, the firm's hedging policy can be described by the function $H(X, V, t)$. Given this function, the corresponding hedge position comprises $h = H(X, V, t)/P$ long positions.

The firm can invest if and only if $T_1 \leq t \leq T_2$ and $X \geq I$, so the value $F(X, V, t)$ of the investment option depends on X and t as well as V . If the firm invests at date t , $F(X, V, t) = V - I$. Otherwise, it satisfies the partial differential equation (see the appendix for details; subscripts on F denote partial derivatives)

$$rF = \sup_H \left\{ F_t + \frac{1}{2} (\phi^2 + 2\lambda\phi\psi H + \psi^2 H^2) F_{XX} + \rho\sigma V(\phi + \lambda\psi H) F_{XV} + \frac{1}{2} \sigma^2 V^2 F_{VV} + r(X + G)F_X + (\mu - \kappa)V F_V \right\}. \quad (4)$$

Intuitively, the left-side of this equation is the expected return required by the firm in order to hold the project rights. The right-side is the expected return obtainable from holding the rights, given the optimal hedging strategy.

¹⁰This can be motivated by assuming that $d\epsilon = \lambda d\zeta + \sqrt{1 - \lambda^2} d\theta$ for some Wiener process θ satisfying $d\theta d\zeta = d\theta d\eta = 0$.

The value function $F(X, V, t)$ and the optimal hedge function $H^*(X, V, t)$ are determined simultaneously by solving the nonlinear system comprising, from (4),

$$H^*(X, V, t) = \frac{-\lambda\phi}{\psi} - \frac{\rho\lambda\sigma VF_{XV}}{\psi F_{XX}} \quad (5)$$

and

$$0 = F_t + \frac{1}{2} (\phi^2 + 2\lambda\phi\psi H^* + \psi^2 H^{*2}) F_{XX} + \rho\sigma V (\phi + \lambda\psi H^*) F_{XV} + \frac{1}{2} \sigma^2 V^2 F_{VV} + r(X + G)F_X + (\mu - \kappa)VF_V - rF. \quad (6)$$

The functions H^* and F are thus inextricably linked. We focus on this link between optimal investment and hedging policies in the next section and then turn to a more detailed examination of the determinants of hedging in Section V.

IV Hedging and the optimal investment policy

A The effect of hedging on the optimal investment policy

For projects with timing flexibility, the optimal rule is to invest if project value V exceeds some minimum threshold, but otherwise wait. Because investment is allowed if and only if $X \geq I$, the value of X places restrictions on the future states in which the investment option can be exercised, so the threshold is a function of X . Moreover, because stochastic fluctuations in future X can be altered by hedging, the threshold also depends on H . Thus, investment is justified if and only if

$$V \geq \hat{V}(X, t; H),$$

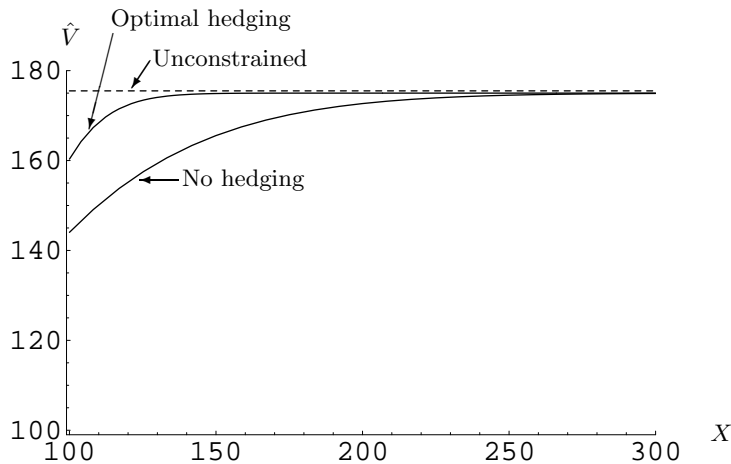
where \hat{V} is the investment threshold function.

Unfortunately, the complexity of the system (5) and (6) prevents us finding an analytical solution for \hat{V} . In order to shed some light on the relationship between hedging and the optimal investment policy, we obtain a numerical solution for $\hat{V}(X, t; H)$ and calculate this for different hedging policies. For the purposes of this exercise, we assume $I = 100$, $\sigma = 0.2$, $\mu - \kappa = 0$, $r = 0.03$, $\rho = 0.5$, $\phi = 60$, $G = 100$, $\psi = 0.2$, $\lambda = 0.9$, $T_1 = 1$, and $T_2 = 5$, and evaluate the threshold when $t = 1$.¹¹ Using these parameter values, we write the partial differential equation describing $F(X, V, t)$ as a partial difference equation and solve this using the explicit finite difference method. $\hat{V}(X, 1; H)$ is set equal to the smallest value of V that satisfies $F(X, V, 1) = V - I$. Further details of this procedure appear in the appendix.

Figure 3 illustrates the effect of hedging on the optimal investment policy. The bottom curve depicts the situation where the firm does not hedge. At low levels of X , the firm adopts a low threshold, reflecting the risk of future funding shortfalls. As this risk recedes with higher levels

¹¹This choice of parameter values is based on those used by other authors in obtaining numerical solutions to similar models, e.g., Mauer and Triantis (1994), Milne and Whalley (2000), and Boyle and Guthrie (2003). Different values have little qualitative effect on our results.

Figure 3: The investment threshold function: hedged and unhedged.



Notes. The top curve depicts the value of the firm’s investment threshold function when the optimal hedging policy is followed; the bottom curve corresponds to the case where there is no hedging; the dashed line shows the unconstrained threshold. Hedging decreases the risk that the firm will have insufficient cash to finance the project in the future, thereby increasing the value of waiting and raising the investment threshold. We use the parameter values $I = 100$, $\sigma = 0.2$, $\mu - \kappa = 0$, $r = 0.03$, $\rho = 0.5$, $\phi = 60$, $G = 100$, $\psi = 0.2$, $\lambda = 0.9$, $T_1 = 1$, and $T_2 = 5$, and evaluate the threshold when $t = 1$.

of X , the threshold increases. When the firm follows the optimal hedging policy (top curve), the relationship between \hat{V} and X is similar, but the threshold now lies above its unhedged counterpart for low and medium values of X ; only when X is approximately 2.5 times the investment cost I do the thresholds converge.¹² Thus, hedging leads to a higher investment threshold, reflecting its role in reducing the risk of future funding shortfalls. By hedging, the firm can confidently delay investment in circumstances where it would otherwise have had to invest prematurely. In other words, hedging allows the firm to improve the timing of its investment, as well as the quantity.

In confirming the findings of our Section II example, this outcome highlights a new motivation for hedging. When projects are of the “now-or-never” variety, cash has value because it allows the firm to invest, so insufficient hedging may cause the firm to forgo investment. Thus, hedging adds value because it allows investment to occur. By contrast, when there is flexibility in the timing of projects, cash also has value because it allows the firm to retain the option to invest, so insufficient hedging may cause the firm to invest prematurely. Thus, hedging adds value because it allows investment to be delayed.

¹²At that point, hedging has no effect on the value of the project rights and the investment threshold is the same as that for a financially-unconstrained firm. At lower levels of X , the threshold for the optimally-hedging firm is less than the unconstrained threshold because the imperfect nature of the hedging instrument leaves the firm with some unhedged funding risk.

B Empirical implications

The notion that hedging can result in less investment may seem a somewhat counter-intuitive and unlikely proposition, but it is consistent with a puzzling feature of data on the investment rates of hedging and non-hedging firms. To the extent that hedging mitigates the under-investment problem, one might expect hedging firms to invest more than non-hedging firms, all else being equal. However, evidence obtained by empirical researchers provides little support for this view. For example, after adjusting for size differences, Graham and Rogers (1999, Table 4) and Allayannis and Mozumdar (2000, Table 1) report little difference in the level of investment between hedgers and non-hedgers, while Géczy, Minton and Schrand (1997, Table III) find that non-hedgers invest more than hedgers.

Of course, there are some obvious explanations for these findings. Non-hedgers in the above researchers' samples may (i) have better investment opportunities than hedgers, or (ii) have more internal funds than hedgers, or (iii) face lower costs of external funding than hedgers. However, none of these is an unambiguous feature of the data: in the samples of Géczy et al. (1997), Graham and Rogers (1999), and Allayannis and Mozumdar (2000), hedgers (i) have similar (or in some cases higher) research and development expenditure, Tobin's Q, and/or market-to-book ratios, than non-hedgers, and (ii) are larger, have less or similar leverage, higher dividend yields, and higher operating cash flow. Thus, hedgers in these samples appear to have similar investment opportunities and face weaker financial constraints than non-hedgers, so neither of these factors can be unequivocal explanations for the independence of investment and hedging.

Our model suggests an alternative explanation: that hedging has an ambiguous effect on the incentive to invest. On the one hand, hedging allows firms to undertake more investment by reducing the number of states in which there is a funding shortfall. On the other hand, it also reduces the risk of future funding shortfalls and thus makes waiting more attractive. Since these two effects work in opposite directions, investment can be unrelated to hedging over any finite period of time; although hedging allows the firm to undertake more investment, it also allows it to delay more investment.

V Determining the optimal hedge

A The effect of investment timing flexibility on hedging

Having seen that investment timing flexibility creates an additional role for hedging, we now turn our attention to the implications of this for the optimal hedging policy. Although not a closed-form solution, equation (5) is helpful for addressing this issue, insofar as it identifies the fundamental components of dynamic hedging. To explore the implications of (5), we first provide a general intuitive discussion of its properties, then illustrate these with a numerical solution of the type described in Section IV.

In general, hedging reduces cash flow volatility by shifting cash from high-cash states to

low-cash states. In equation (5), the first term represents the ‘volatility-minimizing’ (henceforth VM) hedge position, i.e., the hedge that minimizes the variance of fluctuations in the firm’s cash flow. However, shifting cash from a high-cash state to a low-cash state is counter-productive if the marginal value of cash is high in the former state, but low in the latter state. Consequently, the optimal hedging policy does not shift cash from all high-cash states to all low-cash states, but only from high-cash states in which the marginal value of cash is low to low-cash states in which the marginal value of cash is high. The second term in (5) reflects these considerations. When the firm has investment timing flexibility, the marginal value of cash is given by F_X and the optimal hedging policy moves cash from states where F_X is low to states where F_X is high.¹³

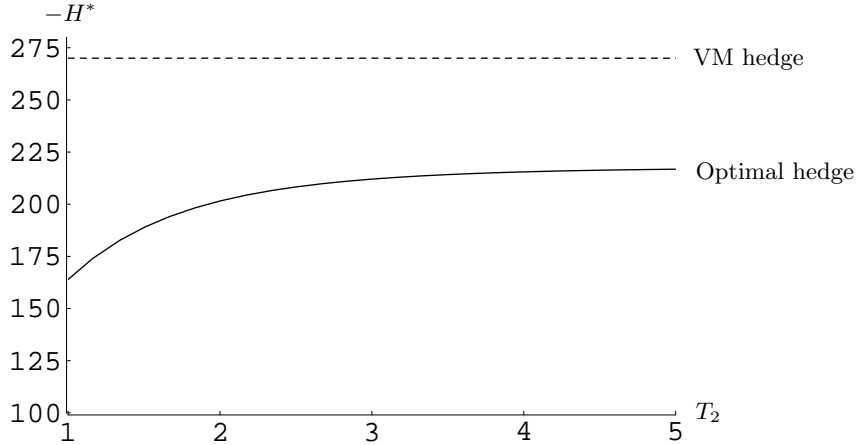
The crucial aspect of equation (5) is the dependence of H^* on the value function F . Since F reflects in part the value of delaying investment, this implies that the optimal hedging policy of a firm with investment timing flexibility will generally differ from that of a firm without this flexibility. When a project disappears if not taken at a particular date, the firm hedges to protect its ability to realize the project’s value at the specified date. By contrast, if the timing of the project is flexible, then the firm wishes to protect not only the immediate payoff from investment, but also the value of retaining the option to invest at a later date. The optimal hedge therefore has to consider both sources of value, not just the former.

Investment timing flexibility thus has two effects on the optimal quantity of hedging. On the one hand, allowing the firm to choose the timing of its investment reduces its need for hedging since it does not lose the project if funding is not available on a given date. On the other hand, the need to protect the value of the investment option increases the quantity of required hedging. For example, suppose X and V are positively correlated such that, at the next date, X exceeds I whenever V exceeds I . Then the optimal hedge is zero if the firm can invest only at that date. But for firms with an ongoing option to invest, the value of this option is positive even when $V < I$ and, moreover, this value is enhanced by additional X , since this increases the probability of eventual exercise. Consequently, firm value would be increased by a hedge that moved cash from states where X is more than sufficient to finance investment to states where X is low. This way, the firm not only has sufficient funds to invest if it wishes to do so, but it also maximizes the value of its investment option should it choose to retain it.

We provide a concrete illustration of these points by explicitly calculating the optimal hedge position when the firm has no discretion in the timing of investment, and then determining how this choice is affected by allowing for varying degrees of flexibility. For the purposes of this exercise, we use the same parameter values as in Section IV, but allow T_2 to vary. For the case with no timing flexibility, we set $T_2 = T_1$, solve the partial differential equation describing $F(X, V, t)$ using the explicit finite difference method, and then use (5) to obtain the optimal hedge position when $X = V = I$. We then repeat this procedure for successively higher values

¹³It is straightforward to show that the hedging policy given by (5) minimizes the variance of F_X . Mello and Parsons (2000) also emphasize this characteristic of optimal hedging policies.

Figure 4: The optimal hedging policy.



Notes. The solid curve depicts $-H^*$ (the optimal short position) as a function of the firm's investment timing flexibility (T_2). Greater flexibility increases the need for hedging in order to protect the valuable timing options created by this flexibility, but this effect levels off because greater flexibility also increases the probability that internal funds will be sufficient to finance investment. The dashed line indicates the volatility-minimizing hedge, given by the first term on the right-side of equation (5). We use the parameter values $I = 100$, $\sigma = 0.2$, $\mu - \kappa = 0$, $r = 0.03$, $\rho = 0.5$, $\phi = 60$, $G = 100$, $\psi = 0.2$, $\lambda = 0.90$, and $T_1 = 1$, and evaluate H^* at $X = V = I$ and $t = 0$.

of T_2 , each corresponding to a greater degree of flexibility.¹⁴

The results from this exercise are depicted in Figure 4.¹⁵ The VM hedge position is a constant, but the optimal hedge monotonically increases in the degree of timing flexibility, reflecting the need to protect the valuable options created by this flexibility. However, this effect becomes smaller as T_2 increases, reflecting the lower funding risk afforded by a longer potential investment period. For example, in the case depicted in Figure 4, increasing T_2 from 1 to 2 raises optimal hedging by 30%, but further increasing T_2 to 3 raises $-H^*$ by only another 5%.¹⁶

We also consider the sensitivity of hedging to the firm's underlying stochastic cash flow structure, and the dependence of this sensitivity on the degree of timing flexibility. Specifically, for various values of T_2 we calculate the percentage change in optimal hedging $-H^*$ in response

¹⁴We also experiment with different combinations of high, medium and low values of V and X , but these affect only the position, and not the general appearance, of the curves in Figure 4. Similar comments apply to the parameters T_1 , λ , ϕ , ρ , σ , and ψ .

¹⁵Setting $\lambda = 0.9$ means that hedging instrument returns are positively correlated with operating cash flow. As a result, the optimal hedge position is a short one, so we report its value as $-H^*$.

¹⁶By contrast, Mello et al. (1995) find that, for a given financial structure (given by X in our model), the size of the optimal hedge is a *decreasing* function of the multinational firm's flexibility in switching production between countries, a result that is also implicit in MacKay (2003). This difference arises because of the differing forms of flexibility. In Mello et al., greater flexibility reduces the probability that the firm will need to exit the industry, so hedging against this possibility is less necessary. In our model, greater flexibility increases the probability that the firm will wish to delay investment, so more hedging is needed to ensure that this is possible.

Table 1: The optimal hedging policy.

T_2	1	2	3	4	5
$-H^*$	153.89	200.18	209.80	215.60	219.88
Elasticities					
λ	4.088	2.054	1.795	1.619	1.520
ϕ	0.664	1.085	1.098	1.104	1.104
ρ	-0.593	-0.405	-0.322	-0.271	-0.233

Notes. This table illustrates the sensitivity of the optimal hedge to changes in parameters describing the stochastic structure of firm cash flow. When $T_2 = 1$, the firm can invest only at date 1; otherwise investment can occur at any time during the interval $[1, T_2]$. The first row thus gives the optimal hedge position for varying degrees of timing flexibility. Each cell in the following three rows shows the percentage change in the optimal hedge resulting from a 1% change in the corresponding parameter. The elasticity calculations use the same parameter values as in Figure 4.

to a 1% increase in each of firm cash flow volatility ϕ , the correlation between firm cash flow and project value ρ , and the correlation between firm cash flow and the hedging instrument price λ . Intuitively, these parameters represent, respectively, the risk of being unable to fund the project, the extent to which the project offers natural protection against this risk, and the effectiveness of the hedging instrument in protecting against this risk. As Table 1 shows, hedging demand is most sensitive to the effectiveness of the hedging instrument; without timing flexibility, a 1% increase in λ raises optimal hedging by 4.1%. However, this elasticity declines with T_2 , reflecting the fact that even an imperfect hedging instrument is able to ensure sufficient funds are available to allow investment at least once during a long window. Unsurprisingly, higher cash flow volatility also raises the demand for hedging, and this effect increases with the degree of timing flexibility, reflecting the greater potential for adverse cash flow shocks over a long period. Finally, greater correlation between V and X reduces the need for hedging (as suggested by Froot et al., 1993), but the absolute value of this effect declines with timing flexibility. Overall, it seems that timing flexibility can have a significant effect on both the optimal quantity of hedging and on the relative importance of the determinants of this quantity.

The role of the correlation ρ in determining the optimal quantity of hedging merits further attention. For a firm without timing flexibility, Froot et al. (1993) emphasize the importance of this parameter for the natural hedge properties of the project. If ρ is positive, then cash tends to be high precisely when it is most needed (i.e., when V is high). This pattern of volatility is more desirable than one in which cash is independent of its potential uses, so the optimal hedge is less than the VM hedge. On the other hand, if ρ is negative then cash tends to be low in states where it is most valuable, so the optimal quantity of hedging is greater than that needed for the VM hedge. This is precisely the pattern observed in Table 1: higher ρ leads to a lower $-H^*$.

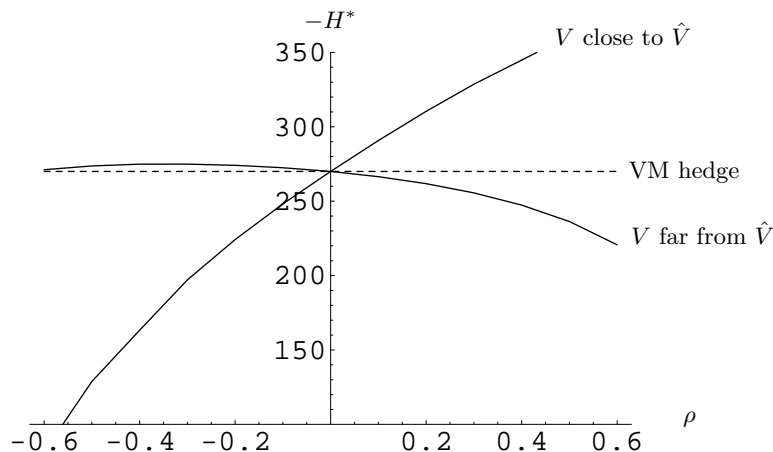
However, equation (5) shows that in a dynamic world, the direction in which the optimal hedge deviates from the VM hedge actually depends not on the sign of ρ , but rather on the sign of ρF_{XV} .¹⁷ That is, positive correlation between X and V reduces the need for hedging if and only if $F_{XV} > 0$. This latter condition is automatically satisfied when investment cannot be delayed, since additional cash allows a financially-constrained firm to move from a payoff of zero to a payoff of $V - I$, the value of which is clearly increasing in V . Consequently, the sign of ρ is sufficient to determine the direction of the optimal deviation from the VM hedge. However, this need not be the case when investment can be delayed. Suppose, for example, that the firm would prefer not to invest immediately (even though it has sufficient funds to do so), but faces a high risk of funding shortfalls in the future. Faced with this risk, the firm may choose to invest now. Extra cash in these circumstances reduces the risk of future funding difficulties, thereby allowing the firm to delay investment. Without such a cash injection, the firm invests immediately, so $F = V - I$ and a \$1 dollar increase in project value V increases F by an amount equal to the change in project value, i.e., by \$1. With a cash injection, the firm can afford to delay investment and retain the option, so F satisfies (4) and a \$1 increase in project value V increases F by an amount less than the change in project value, i.e., by less than \$1. As a result, F_V is smaller with the cash injection than without it, i.e., $F_{XV} < 0$.

If $F_{XV} < 0$, then the marginal value of cash F_X is high when project value V is low, and vice versa. In that case, a positive correlation between X and V tends to deliver high (low) cash when the value of that cash is low (high), i.e., the correlation does a poor job of protecting the investment option F . Consequently, higher ρ can *increase* the need for hedging in a dynamic world, in contrast to its effect in a static world.

To verify this point, we continue our numerical simulations and consider the relationship between $-H^*$ and ρ in two cases. In one case, the project is currently in a breakeven situation ($V = 100$); in the other case the project payoff is significantly positive ($V = 150$). As Figure 5 shows, the optimal quantity of hedging is decreasing in ρ in the first case, but increasing in ρ in the second, a difference that reflects the considerations discussed above. In the first case, the project value is a long way from the investment threshold, so the firm chooses to delay investment and both X and V contribute positively to the value of the investment option that the firm continues to hold. Consequently, a positive correlation between X and V increases the value of this option and reduces the need for hedging. In the second case, however, the project value is near the investment threshold and the firm's calculation is different. Because near-term investment is now a possibility, it wants the project to have a higher value in those future states where it chooses to invest (i.e., those states where waiting is risky because cash is low) and a lower value in the states where it can afford to delay (i.e., those that produce high cash). To achieve this pattern, it needs to offset any positive correlation between X and V , so the optimal quantity of hedging increases with the size of this correlation, as depicted by the upward-sloping

¹⁷The second order condition for the maximization problem in (4) ensures that $F_{XX} < 0$.

Figure 5: The optimal hedging policy and correlation ρ .



Notes. The graph depicts $-H^*$ (the optimal short hedge position) as a function of ρ (the correlation between the value of the project and shocks to the firm’s cash stock). Although the relationship between $-H^*$ and ρ is generally negative (shown in the figure for the case where $V = 100$ and $X = 150$), it can be positive when project value approaches the investment threshold (as shown in the figure for the case where $V = 150$ and $X = 150$). We use the following parameter values: $I = 100$, $\sigma = 0.2$, $\mu - \kappa = 0$, $r = 0.03$, $\phi = 60$, $G = 100$, $\psi = 0.2$, $\lambda = 0.90$, $T_1 = 0$, and $T_2 = 2$. The dashed line indicates the VM hedge.

curve in Figure 5.

B Empirical implications

The observation that optimal hedging protects dynamic investment opportunities has interesting implications for our interpretation of empirical work on hedging. Empirical studies of the determinants of corporate hedging generally proceed by regressing some measure of hedging activity on a range of variables designed to capture factors that theoretical research has shown to be important. However, several studies report little support for the hypothesis that more valuable investment opportunities enhance the propensity to hedge; see for example, Nance, Smith and Smithson (1993), Berkman and Bradbury (1996), Mian (1996), Howton and Perfect (1998), and Graham and Rogers (1999). Although Géczy et al. (1997) and Gay and Nam (1998) obtain different results, it is clear that the empirical evidence for this hypothesis is, at best, mixed.

A potential explanation for this finding is simply that many firms are not hedging optimally. However, our model raises the possibility that hedging rates can be unrelated to standard measures of investment opportunities even if firms *are* optimally hedging these opportunities. As we have emphasized, the value generated by hedging a given set of investment projects depends in part on the timing flexibility offered by those projects. When timing is flexible, the optimal hedging strategy protects not only the projects themselves, but also the option to delay those projects until conditions are more favorable. With “now-or-never” projects, only the former

consideration applies. Thus, firms with similar investment opportunities may adopt very different hedging policies, reflecting their different timing flexibilities. Consequently, in the absence of reliable controls for differences in timing flexibility, there is no reason to expect a robust relationship between hedging propensity and standard measures of investment opportunities.

Of course, rectifying this problem represents a considerable challenge for empirical research as there is no obvious way of distinguishing firms that have significant timing flexibility from those that do not. One possibility is to identify firms with considerable market power, as these face little competitive pressure to rush investment and thus have greater scope for delay. Another possibility is to identify firms that tend to make large and infrequent investments, as these have a strong incentive to time such investments in an optimal fashion. Differentiating these two types of firms from others may help to improve the explanatory power of empirical models of hedging and investment.

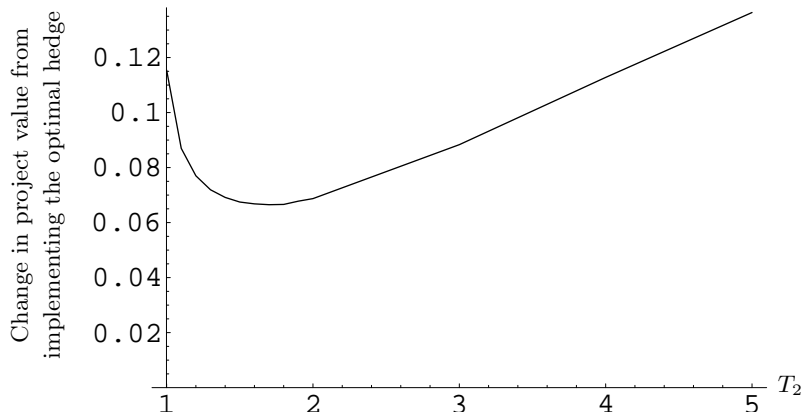
VI The value of hedging

Having described the interaction between the optimal investment and hedging policies in the previous two sections, the remaining issue of interest is the implications of this interaction for project value. More prosaically, what effect does hedging have on the value of a firm with investment timing flexibility, and how does this differ from a firm without such flexibility?

To address this issue, we adopt the same parameter values as used in Section V and numerically calculate the value added by hedging. That is, for varying degrees of flexibility (different values of T_2), we calculate the project value $F(X, V, t)$ when the firm follows the optimal investment and hedging policies, and compare this with the corresponding value under a no-hedging policy. As can be seen in Figure 6, the value of hedging is related to timing flexibility in a non-monotonic fashion. For $T_2 = T_1$ (i.e., no timing flexibility), the optimal hedge increases project value by 12%, but higher values of T_2 are initially associated with smaller hedging effects. As T_2 continues to increase, however, the optimal hedge has a proportionately bigger effect on project value. Thus, small amounts of timing flexibility reduce the value of hedging, but significant flexibility has the opposite effect.

This non-monotonicity is due to the competing effects discussed in Section V. When the project is “now-or-never”, insufficient funds for investment mean that it is lost forever. In these circumstances, hedging is relatively valuable because it reduces the probability of permanent abandonment. Allowing for timing flexibility alters this relationship in two ways. First, by eliminating the need to invest at just one date, the probability of abandonment falls and so hedging is less valuable. Second, flexibility in the choice of investment date increases the options available to the firm, thereby making hedging more valuable. When timing flexibility is low, the proportion of project value that is due to timing options is relatively small, so the first effect dominates and flexibility reduces the value of hedging. However, as flexibility increases, more and more of the project’s value emanates from timing options, so the second effect dominates

Figure 6: The value of hedging.



Notes. The curve depicts the proportional increase in $F(X, V, t)$ when the firm hedges optimally over its value when the firm does not hedge, as a function of the firm's investment timing flexibility (T_2). Greater flexibility initially reduces the value of hedging, but as flexibility increases, more and more of the project's value originates from timing options and hedging's ability to protect these options increases its value to the firm. We use the parameter values $I = 100$, $\sigma = 0.2$, $\mu - \kappa = 0$, $r = 0.03$, $\rho = 0.5$, $\phi = 60$, $G = 100$, $\psi = 0.2$, $\lambda = 0.90$, and $T_1 = 1$, and evaluate $F(X, V, t)$ at $X = V = I$ and $t = 0$.

and flexibility increases the value of hedging. Consequently, the relationship between timing flexibility and the value of hedging has the U-shape depicted in Figure 6.

VII Concluding remarks

The fundamental goal of hedging for a firm with valuable investment opportunities that are available only at single future dates is to protect the ability to fund these projects as they become available. For projects that offer a choice of investment dates, however, hedging has an additional goal: to protect the ability of the firm to utilize this timing flexibility. Without hedging, the firm may have to rush into investment because of the risk that waiting exposes it to the risk of future funding shortfalls. By reducing this risk, hedging is valuable. Not only does it allow investment to occur, but it also allows investment to be delayed.

This simple observation implies that the value of hedging depends not only on a firm's investment opportunities and financing constraints, but also on its investment timing flexibility, a conclusion that has two corollaries. First, it reinforces the view that incorporating dynamic features in capital investment models can yield predictions significantly different to those generated by the static, one-period, framework. Second, it has implications for our interpretation of empirical work on hedging. In particular, it suggests that the failure of empirical work to find significant relationships between hedging rates, investment rates and investment opportunities is not, after all, particularly surprising.

References

- Allayannis, G., and A. Mozumdar, 2000. Cash flow, investment, and hedging. University of Virginia working paper.
- Alsop, S., 2001. Oh, these are the days of glory. *Fortune*, 19 March 2001.
- Anonymous, 2002. Doors now closing; raising finance in America. *The Economist*, 26 October, 65–66.
- Berkman, H., and M. Bradbury, 1996. Empirical evidence on the corporate use of derivatives. *Financial Management* 25, 5–13.
- Bessembinder, H., 1991. Forward contracts and firm value: investment incentive and contracting effects. *Journal of Financial and Quantitative Analysis* 26, 519–532.
- Boyle, G., and G. Guthrie, 2003. Investment, uncertainty, and liquidity. *Journal of Finance* 58, 2143–2165.
- Chung, M., 2002. Signs of movement in IPO market. *Financial Times*, September 11, 29.
- Dixit, A., and R. Pindyck, 1994. *Investment Under Uncertainty*. Princeton: Princeton University Press.
- Froot, K., D. Scharfstein, and J. Stein, 1993. Risk management: Coordinating corporate investment and financing policies. *Journal of Finance* 48, 1629–1658.
- Géczy, C., B. Minton, and C. Schrand, 1997. Why firms use currency derivatives. *Journal of Finance* 52, 1323–1355.
- Gay, G., and J. Nam, 1998. The underinvestment problem and corporate derivatives use. *Financial Management* 27, 53–69.
- Graham, J., and D. Rogers, 1999. Is corporate hedging consistent with value maximization? An empirical analysis. Duke University working paper.
- Grossman, S., and J. Vila, 1992. Optimal dynamic trading with leverage constraints. *Journal of Financial and Quantitative Analysis* 27, 151–167.
- Howton, S., and S. Perfect, 1998. Currency and interest-rate derivatives in US firms. *Financial Management* 27, 111–121.
- Hull, John C., 2003. *Options, Futures, and Other Derivatives*. New Jersey: Prentice-Hall
- MacKay, P., 2003. Real flexibility and financial structure: An empirical analysis. *Review of Financial Studies* 2003, 1131–1165.

- Mauer, David C., and Alexander J. Triantis, 1994. Interactions of corporate financing and investment decisions: A dynamic framework. *Journal of Finance* 49, 1253–1277.
- McDonald, R., and D. Siegel, 1986. The value of waiting to invest. *Quarterly Journal of Economics* 101, 707–727.
- Mello, A., and J. Parsons, 2000. Hedging and liquidity. *Review of Financial Studies* 13, 127–153.
- Mello, A., J. Parsons, and A. Triantis, 1995. An integrated model of multinational flexibility and financial hedging. *Journal of International Economics* 39, 27–51.
- Mian, S., 1996. Evidence on corporate hedging policy. *Journal of Financial and Quantitative Analysis* 31, 419–439.
- Milne, Alistair, and A. Elizabeth Whalley, 2000. ‘Time to build, option value and investment decisions’: a comment. *Journal of Financial Economics* 56, 325–332.
- Nance, D., C. Smith, and C. Smithson, 1993. On the determinants of corporate hedging. *Journal of Finance* 48, 267–284.
- Pindyck, R., 1993. A note on competitive investment under uncertainty. *American Economic Review* 83, 273–277.
- Zellner, W., A. Palmer, W. Symonds, and S. Holmes, 2003. The main squeeze in the credit crunch. *Business Week*, 3 February, 42–43.

Appendix

Derivation of the partial differential equation for $F(X, V, t)$

In the region where the firm delays investment, it holds a portfolio consisting of physical assets worth G , the project rights worth $F(X, V, t)$, a cash stock of X , and h futures positions. Since G is a constant and the instantaneous value of the futures position is zero due to marking-to-market, Itô’s Lemma implies that the change in the value of this portfolio over the time interval dt (with respect to the risk-neutral probability measure) is

$$\begin{aligned}
 dR &= dF + dX \\
 &= \left(F_t + \frac{1}{2} (\phi^2 + 2\lambda\phi\psi H + \psi^2 H^2) F_{XX} + \rho\sigma V(\phi + \lambda\psi H) F_{XV} \right. \\
 &\quad \left. + \frac{1}{2} \sigma^2 V^2 F_{VV} + r(X + G) F_X + (\mu - \kappa) V F_V + r(X + G) \right) dt \\
 &\quad + (\phi^2 + 2\lambda\phi\psi H + \psi^2 H^2)^{1/2} (1 + F_X) d\xi + \sigma V F_V d\eta,
 \end{aligned}$$

where $H = hP$. The firm’s expected rate of return with respect to the risk-neutral probability measure equals the risk free interest rate, so

$$E[dR] = r(F + G + X).$$

Since H is a control variable, this implies the following partial differential equation for F :

$$rF = \sup_H \left\{ F_t + \frac{1}{2} (\phi^2 + 2\lambda\phi\psi H + \psi^2 H^2) F_{XX} + \rho\sigma V(\phi + \lambda\psi H) F_{XV} + \frac{1}{2} \sigma^2 V^2 F_{VV} + r(X + G)F_X + (\mu - \kappa)V F_V \right\}.$$

Numerical solution of equation (4)

Equation (4) is solved on a grid with nodes $\{(X_k, V_j, t_n) : j = 1, \dots, J, k = 1, \dots, K, n = 0, \dots, N\}$, where $X_k = k dX$, $V_j = j dV$, and $t_n = n dt$, and N is chosen so that $t_N = T_2$. We use $F_{j,k}^n$ to denote $F(X_k, V_j, t_n)$.

We start by using the terminal condition to solve for $F_{j,k}^N$ for all j and k . In particular, $F_{j,k}^N = 0$ if $X_k < I$ or $V_j < I$, and $F_{j,k}^N = V_j - I$ at all other nodes. We then work backwards through time in a series of steps, solving for $\{F_{j,k}^n : j = 1, \dots, J, k = 1, \dots, K\}$ for $n = N - 1$, $n = N - 2$, and so on. Each time step is broken into two stages.

1. In the first stage, we calculate $H_{j,k}^{n+1} \equiv H(X_k, V_j, t_{n+1})$ for all j and k . For interior points, if the second order condition $\partial^2 F / \partial X^2 < 0$ holds at (X_k, V_j, t_{n+1}) , we set $H_{j,k}^{n+1}$ equal to the finite difference approximation of equation (5), otherwise we set $H_{j,k}^{n+1} = 0$. For all other points we set $H_{j,k}^{n+1} = 0$.
2. In the second stage, we use the (explicit) finite difference approximation of equation (6), together with the solution $\{(F_{j,k}^{n+1}, H_{j,k}^{n+1}) : j = 1, \dots, J, k = 1, \dots, K\}$ at the previous step, to obtain the approximate solution to (6), which we denote $\hat{F}_{j,k}^n$. If $T_1 \leq t_n \leq T_2$ and $X_k \geq I$, so that investment is feasible, then $F_{j,k}^n = \max\{\hat{F}_{j,k}^n, V_j - I\}$, otherwise $F_{j,k}^n = \hat{F}_{j,k}^n$.