A Primer on Real Options

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Abstract

Practitioners frequently experience problems with conventional methods for evaluating capital investment projects in situations where managerial flexibility subsequent to the project's commencement provides the firm with a real option. This paper provides a brief and intuitive introduction to methods of project evaluation that explicitly recognise managerial flexibility and its associated real options.

1 Introduction

How should a manager decide whether an investment project is financially viable? Virtually all standard textbooks in finance, accounting, or economics advocate the use of the net-present-value (NPV) rule. That is, forecast the project's future cashflows, discount each component of expected cashflow at the risk-adjusted cost of capital, sum all these discounted components, and from this subtract the initial investment cost. If the resulting number (the project's NPV) is positive, then the project is financially viable; otherwise it is not. General acceptance of the NPV rule is not confined to the world of textbooks: surveys of international corporate practice (see, for example, Kester et al, 1999) indicate that over 90% of firms use the NPV rule and that, for most firms, this is the primary method of project evaluation.

And yet despite this success in convincing practitioners of its worth, the NPV paradigm is increasingly being called into question. Although perfectly suitable (indeed optimal) for projects that, once launched, require no further decisions or actions by the firm investing in them, the NPV rule provides little guidance for projects that offer managerial flexibility in dealing with future contingencies. Fortunately, recent developments in a branch of finance theory known as "real-options analysis" (ROA) enable us to say something sensible (and hopefully valuable) in response. Motivated by the realisation that investment managers are becoming increasingly interested in ROA as an alternative to NPV, but mindful of the fact that the mathematics of ROA can appear daunting, we aim in this article is to provide a simple and brief introduction to ROA methodology for readers who wish to obtain an intuitive understanding of this complex topic.[1]

When a firm has a real option, it has the opportunity, but not the obligation, to undertake an action in the future. For example, an investment may take place in stages with abandonment possible at each stage if conditions warrant (an abandonment option). Similarly, if the project goes well, it can be expanded (an expansion option). By ignoring the ability of managers to respond to new information, standard NPV analysis assumes away future managerial flexibility and thus underestimates a project's true value.

In contrast, ROA correctly values managerial flexibility and real options by explicitly considering the appropriate managerial action at future dates on which additional information about a project's profitability is revealed. In this respect, ROA is similar to decision-tree analysis (DTA) which takes specific account of managerial flexibility in a simple and intuitive manner. However, there is one important difference between DTA and ROA. Whereas implementation of DTA typically assumes that the risks of a project's real options are the same as the risk of the project itself, ROA makes no such assumption. Instead, it uses techniques originally developed to price financial options to correctly account for the risk of the real option.

It is important to stress that ROA represents an extension, not an overthrow, of NPV. ROA accepts the essential NPV insight - that value equals the sum of discounted future payoffs - but argues that the standard NPV framework is unable to correctly make this calculation when projects offer future managerial flexibility. Although DTA also makes this argument, and thus provides a better method than NPV for estimating future payoffs, it offers no explicit guidance as to the correct discount rate. ROA builds on the DTA extension by specifying exactly, albeit indirectly, how this discount rate should be calculated.

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[1] For alternative sources and more advanced treatements, see *Suggestions for Further Reading* appearing at the end of this paper.

In the next section, we argue that real options are pervasive in investment opportunities, illustrating this with examples from different sectors. In Section 3, we provide a general discussion of the differences between NPV, DTA, and ROA. Sections 4 and 5 contain numerical examples of these general principles while Section 6 offers some concluding remarks.

2 Types of Real Options

2.1 Options to Expand, Diversify, and Defer

One of the most important real options is the option to expand, an option that may be profitable to exercise if, for example, future demand turns out to be higher than expected. Airports with surplus land, for example, can build extra runways if demand increases sufficiently. Even space-constrained airports may still build extra terminals or retail space.

Closely related to expansion options are options to diversify into new lines of business. Toll-road companies may, for example, have an option, but no obligation, to develop land adjacent to the roads they own. Utility companies that own rights of way may have options to diversify into other network businesses at a low cost. Gas companies may have options to run fibre-optic cable through gas ducts and thus diversify into telecommunications. If the value of bandwidth rises enough, they can increase their profits by exercising the options; if the value of bandwidth stays too low, they simply leave their options unexercised. And, to take a prominent example outside the infrastructure and utility sectors, Internet companies such as Amazon.com may have been highly valued in part because they had options to add new lines of business if their "business model" proved successful. Scalability was the buzzword (although whether the options were *correctly* valued before the tech crash is, of course, questionable).

Options to expand or diversify increase project value. Thus, a project that NPV analysis indicates is not financially viable (because it has negative NPV) can become attractive when the value of its flexibility options is incorporated. We illustrate this point in Sections 4 and 5.

Options to *defer* investment (timing options) have similar characteristics. A company that has not yet invested has an option to invest that it exercises (i.e., gives up) when it commits itself to investing. A mobile-telecommunications company considering when to roll out a new "3G" network, for example, is considering whether it is more valuable to immediately undertake this investment or to wait in order to obtain further information about its prospects (i.e., keep the timing option alive). Clearly, a project that can be delayed is more valuable than one that is otherwise equivalent, but must be launched now or not at all. Thus, as with options to expand or diversify, a timing option increases project value. However, unlike those options, a timing option discourages investment. When a firm invests, it *acquires* options to expand or diversify, thereby lowering the NPV needed to justify investment, i.e., investment in a project with negative NPV can be justified by the acquisition of valuable options that make up the shortfall. By contrast, choosing to invest requires the firm to *sacrifice* its timing option (since the project cannot usually be undertaken again sometime in the future), thereby *raising* the NPV needed to justify investment, i.e., investment in a project with positive NPV is not optimal unless the NPV is sufficiently high to offset the loss of the timing option.[2]

2.2 Options to Contract or Abandon

Investors may also own options to contract or abandon a project if it turns sour. An oil-exploration-and-production company has the option to abandon drilling if initial results are not promising. That option makes the company more valuable than a (hypothetical) company that had to commit itself to drilling, come what may.

Real-options analysis also offers a new perspective on the old problem of sunk costs: the projects are worth less than they would be in the absence of sunk costs because the exercise prices of the abandonment and contraction options tend to be low (i.e., the salvage value of the assets is low). With low exercise prices, the options and the business that owns them are worth less than they would otherwise be. For example, the sunk nature of many infrastructure investments means their abandonment options are less valuable than

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2 See Dixit and Pindyck (1994, Chapter 2) for some simple examples of this principle.

they might be. Once built, a toll road cannot be moved to another location or profitably used for some other purpose if traffic volumes are disappointing. Big reductions in demand for the road thus leave the investor "stranded". Even in the absence of changes in demand, customers may also pressure governments to prevent the owners of sunk assets from recovering the cost of their investment - knowing that the asset will remain in service so long as revenue covers variable costs, which are often very low.

Interestingly, governments sometimes *grant* infrastructure investors real options that partially compensate them for the sunk nature of their investments. In emerging markets, toll-road companies sometimes receive "revenue guarantees" which ensure that their revenues cannot fall below a certain level, no matter how low traffic volumes fall. Such guarantees are simply another form of real option, and can be valued using real-options analysis.

2.3 Complex Options

Some businesses are best analysed as *portfolios* of real options. R&D-based businesses typically have complex compound options: when a drug company undertakes the first stage of research into a new pharmaceutical, for instance, it creates an option to go to the next stage if the first stage is promising, which in turn creates further options. Such companies have options on options on options.

The value of peak-load power generators can be analysed as a series of options to generate electricity. The exercise price of the options is the cost of the fuel used to generate power (e.g., coal or gas), and electricity is the commodity on which the option is written. It is profitable to exercise the option when the price of electricity is high relative to the cost of the fuel.

3 An Intuitive Overview of Real-Options Analysis

Before attempting to get to grips with the mechanics of real-options analysis (ROA), it may be helpful to re-emphasise its intuitive foundations, particularly the ways in which it relates to, and differs from, net-present-value (NPV) and decision-tree analysis (DTA).

In doing so, a useful aid is the value-additivity principle. Although the mathematics underlying this result is quite complex (the interested reader can find a proof in LeRoy and Werner, 2001, Chapter 2), its meaning is straightforward: the value of an asset equals the sum of the values of its constituent parts. In the case of a project with managerial flexibility (i.e., a real option), this means we can write

Value of project	=	Value of project without flexibility	+	Value of flexibility
	=	NPV	+	Real-option value

What this means is that in attempting to value any project that contains a real option, we can either (i) incorporate the option payoffs into the project itself and value the project-with-flexibility directly or (ii) separately calculate the value of the option cashflows and add this to the project's NPV. The value-additivity principle tells us that the answer is the same. Although the choice of method is therefore immaterial to the ultimate decision, method (ii) is particularly helpful for illustrating the differences between NPV, DTA, and ROA.

Using method (ii), the differences between these three approaches can be summarised as follows:

Net Present Value

NPV **=** [sum of project's expected cashflows discounted at a rate reflecting the risk of these cashflows] minus initial cost of project

Decision Tree Analysis

DTA Value = NPV plus [sum of real option's expected net cashflows discounted at a rate reflecting the risk of the *project's* cashflows]

Real Options Analysis

ROA Value = NPV plus [sum of real option's expected net cashflows discounted at a rate reflecting the risk of the *option's* cashflows]

Standard NPV analysis takes account of uncertainty about the future by, first, estimating *expected* cashflows, taking into account the distribution of possible cashflows, and second, by discounting these expected cashflows at a rate that reflects the riskiness of the cashflow distribution. Its flaw is that it does not attempt to estimate the value management can create by responding to the resolution of uncertainty. DTA does attempt to estimate the value management can create in this way, but proceeds by assuming that the resulting cashflow distribution has the same risk as that of the project without flexibility, which is generally not the case. By contrast, ROA not only discounts project cashflows at an appropriate risk-adjusted rate, but also discounts the cashflows attributable to managerial flexibility at a rate appropriate to *their* risk.

To summarise, NPV ignores managerial flexibility; DTA takes managerial flexibility into account, but generally fails to value it correctly; only ROA takes managerial flexibility into account *and* values it correctly.

ROA values managerial flexibility not by explicitly calculating the risk-adjusted discount rate appropriate to the option component of the project's cashflows (which in most cases is very difficult), but rather by applying a no-arbitrage principle (the law of one price) which states that any two assets that yield the same cashflows in all future states of the world must have the same price. More specifically, ROA proceeds first by constructing a "replicating portfolio" of existing assets whose cashflows exactly mimic the real-option cashflows. Since the replicating portfolio consists of assets with known values, its value too is known. The second step then consists of appealing to the law of one price: the value of the real option must be equal to the value of the replicating portfolio.

4 Example I: The Option to Expand

To illustrate how real-options analysis (ROA) works and how it differs from net-present-value (NPV) and decision-tree analysis (DTA), we focus first on a very simple expansion option. Consider a project that costs $1100 to begin now and whose value in one year's time is dependent on the (currently unknown) demand for the project's product. If demand turns out to be high, the project will be worth $1800; if demand turns out to be low, it will be worth only $675.[3] These good and bad outcomes (which we shall refer to as high-demand and low-demand *states* respectively) are equally likely, i.e., each occurs with probability 1/2. Call this the base project.

The expansion option embedded in the project is an option to scale up production at a given cost in one year's time. Specifically, assume that the firm has the option to increase production by 25% (thereby increasing the project's gross value to $2250 in the high-demand state and to $844 in the low-demand state) in exchange for a payment of $320. Also let the riskless rate of interest be 4% and the appropriate risk-adjusted discount rate (cost of capital) for the base project be 13%.

Table 1 collects the project information.

Table 1: Parameters for Example I

Parameter	Value
Cost of base project	$1100
Value of base project in one year's time	
if demand is high	$1800
if demand is low	$675
Probability of demand being high	50%
Cost (in one year) of expansion	$320
Increase in base project value if expansion occurs	25%
Riskless rate of interest	4%
Risk-adjusted discount rate for base project	13%

Table 2 summarises the estimated value of this project according to the three methods of evaluation.

Table 2: Outcomes for Example I

Method of project evaluation	Estimated project value
NPV	-$5
DTA	+$53
ROA	+$47

In the remainder of this section, we outline the sources of these respective outcomes.

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[3] We do not specify the relationship between these values and the evolution of the project's cashflows since this is immaterial to our analysis. There are in fact an infinite number of possible projects that yield the values given. For example, a 5-year project that is expected to produce constant cashflows of $453/year (from the beginning of Year 1) if demand is high, $170/year if demand is low, and has a cost of capital of 13%.

4.1 Net present value (NPV)

The base project's present value is estimated by discounting *expected* cashflows at the 13% cost of capital. We illustrate this calculation in Figure 1 where (as in Figures 2-5 that follow) dark arrows represent the possible future values of the investment while light arrows explain the relationship of these future values to the investment's present value.

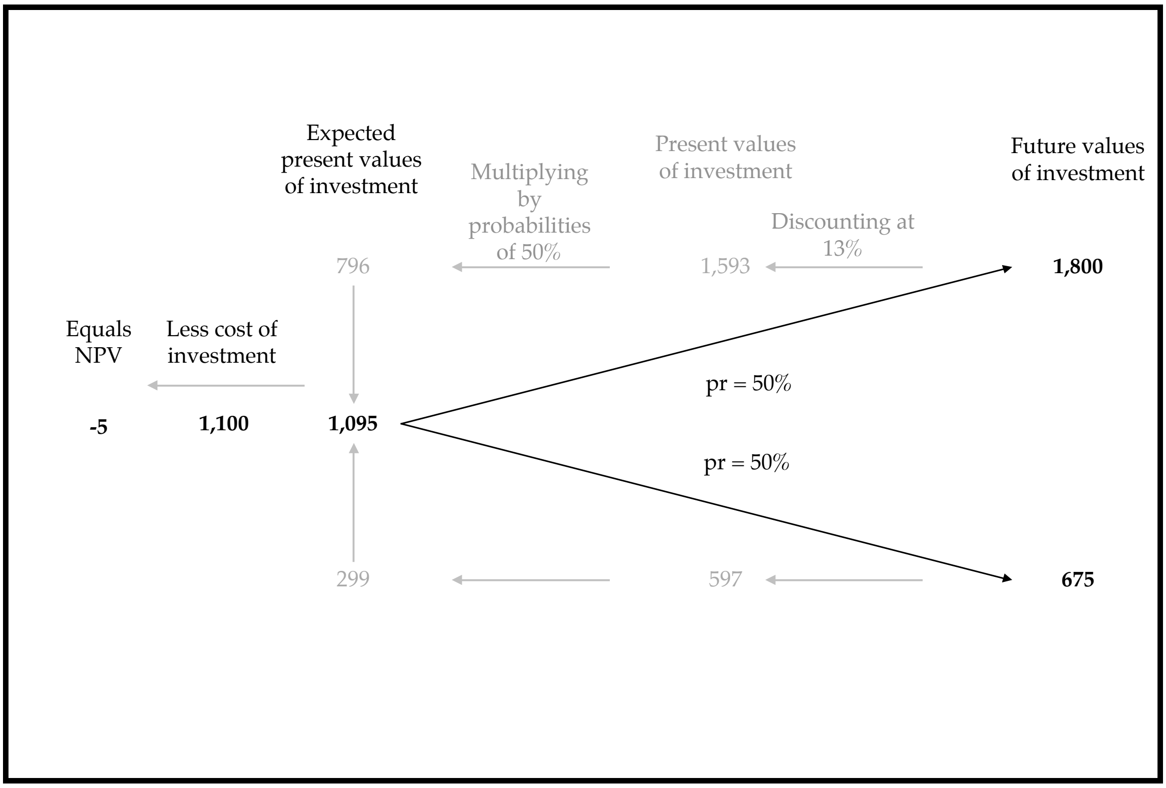

Figure 1: NPV

The \$1095 present value of the project cashflows is calculated as follows:

$$\text{Present Value} = 0.5^* \left(\frac{\$1800}{1.13}\right) + 0.5^* \left(\frac{\$675}{1.13}\right)$$
$$= 0.5^* (\$1593) + 0.5^* (\$597)$$
$$= \$796 + \$299$$
$$= \$1095$$

And the project's NPV is

$$NPV = \$1095 - \$1100 = -\$5$$

Thus, according to the NPV rule, the firm should not invest. Of course, this ignores the option to expand. Calculation of the NPV requires discounting expected future payoffs at the cost of capital, but the presence of an option means that the expected payoffs are dependent on future management decisions regarding whether or not to exercise the option. Standard NPV analysis deals with this by implicitly assuming that management commits to a particular course of action (expand or not expand) at the time the project is launched, regardless of the conditions that subsequently prevail. This severely underestimates the value of the expansion option since it ignores the ability of management to respond to new information.[4]

4.2 Decision tree analysis (DTA)

Standard NPV analysis is faulty because it assumes that the firm must decide now whether or not to expand next year. In practice, the decision to expand (or not) will only be made once the state of the world next year is known. The simplest way of extending NPV to cover such contingencies is to assume that management makes the optimal decision in all future states. In the example, this means that expansion will proceed only in those states where doing so increases the value of the firm. This is the essential insight of DTA.

In the example given, the payoffs to expansion are as follows:

High-demand state: Expansion costs \$320 and project value increases by \$450 (i.e., 0.25*\$1800), so there is a net gain of \$130.

Low-demand state: Expansion costs \$320 and project value increases by \$169 (i.e., 0.25*\$675), so there is a net loss of \$151.

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[4] In the example, the project NPV if the firm commits to not expanding is, as seen above, -\$5. If it commits to expansion, the NPV is -\$14. In this case, expansion helps if demand turns out to be high, but exacerbates the problem if demand is low, and the latter effect outweighs the former.

Clearly, the firm will choose to expand if and only if the high-demand state occurs, in which case the payoff is \$130 and the project is worth \$1930 (i.e., \$1800 + \$130). In the low-demand state, the firm maintains the status quo and the project is worth \$675.

The effect of expansion flexibility on the project's future payoffs is illustrated in Figure 2.

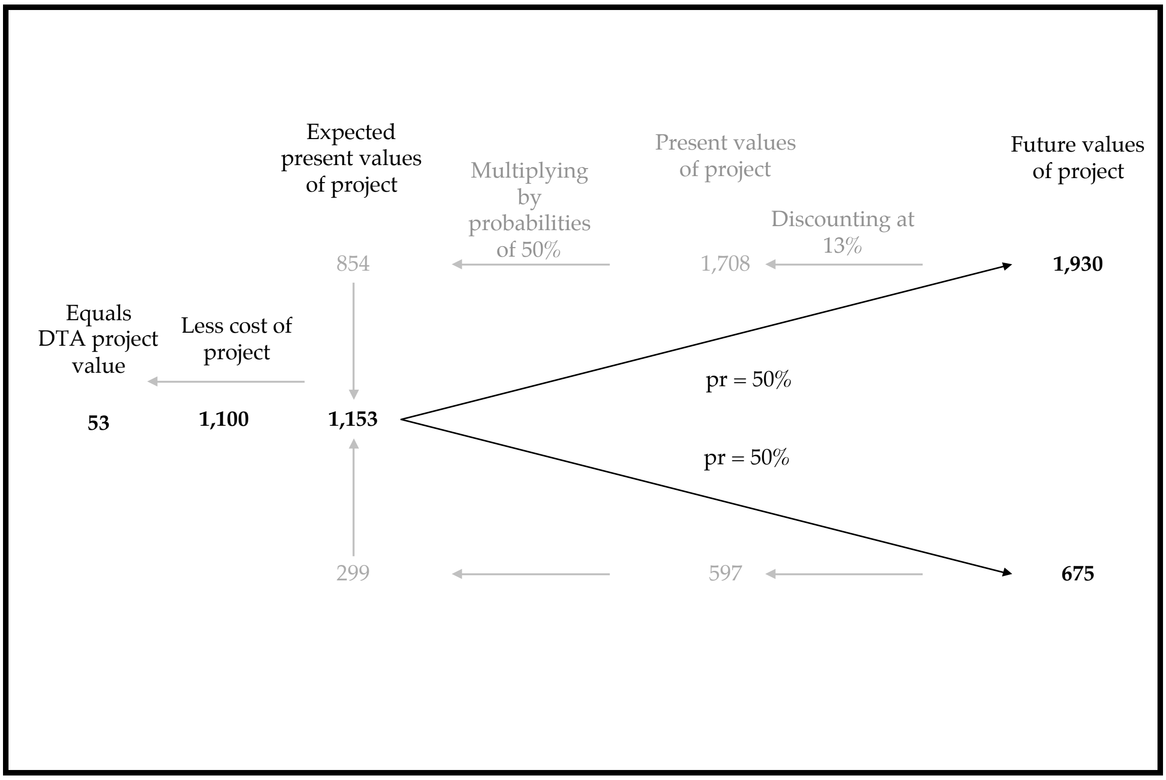

Figure 2: DTA (Project)

where

$$\text{DTA project value} = 0.5^* \left(\frac{\$2250 - \$320}{1.13}\right) + 0.5^* \left(\frac{\$675}{1.13}\right) - \$1100$$

$$= 0.5^* (\$1708) + 0.5^* (\$597) - \$1100$$

$$= \$854 + \$299 - \$1100$$

$$= \$53$$

Hence, once the flexibility offered by the expansion option is recognised, the project increases firm value by $53 ($1153 - $1100), so investment should proceed. Thus, the DTA rule yields a different recommendation to the NPV rule.

Recalling the value-additivity principle, the same conclusion is obtained by calculating the value of the expansion option directly. In the high-demand state, this option is exercised and the payoff is $130; in the low-demand state, expansion is unprofitable, so the option is not exercised and the payoff is $0. This is illustrated in Figure 3

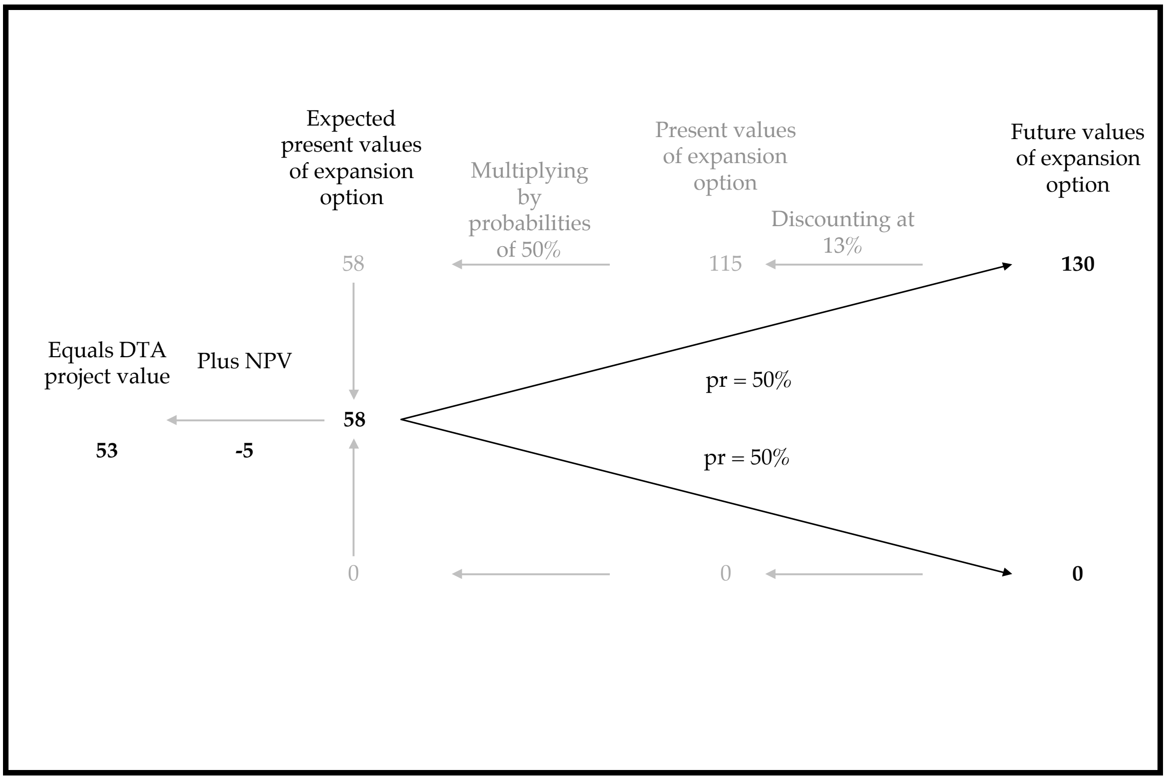

Figure 3: DTA (Expansion Option)

Thus

$$\text{DTA option value} \quad = 0.5^*\left(\frac{\$130}{1.13}\right) + 0.5^*\left(\frac{\$0}{1.13}\right)$$

$$= 0.5^*\,(\$115) + 0.5^*\,(\$0)$$

$$= \$58$$

which, when added to the base project NPV of -$5, yields the DTA project value of $53.

By recognising the value of managerial flexibility, DTA is superior to NPV. It also has the advantage of being relatively simple to implement. Yet it has a flaw. It assumes that the discount rate appropriate for valuing the expansion option is equal to the discount rate appropriate for valuing the base project - in this case 13%. By assumption, we know that 13% is the appropriate discount rate for an asset with the same risk as the base project (i.e., one that offers equally likely payoffs of $1800 and $675), but there is no reason to suppose that the risk of the expansion option (which offers equally likely payoffs of $130 and $0) meets this criterion.[5] So we have no sound basis for choosing 13% - and no obvious way of choosing any other rate. Even if our other inputs are correct, we can have no faith that the true value of the project to the firm is $53.

4.3 Real options analysis (ROA)

ROA overcomes the above problem by appealing to the no-arbitrage principle that *assets providing identical payoffs in the future must have the same present value*. The value of the expansion option is obtained by first constructing a "replicating portfolio" that has the same payoffs as the option and then applying the no-arbitrage principle: the value of the expansion option is equal to the value of the portfolio. By using the techniques originally developed to value financial options, ROA properly accounts for the risk of the expansion option and thereby provides more accurate valuations.

As before, the expansion option is known to be worth $130 in the high-demand state (because it is exercised in that state) and $0 in the low-demand state (because it is not exercised in that state). In arriving at a value for the expansion option, the ROA approach first identifies a portfolio that exactly mimics the option's state-contingent payoffs. That is, it constructs a portfolio of *existing* assets (for which current values are known) that provides a payoff of $130 in the high-demand state and $0 in the low-demand state.

One way of constructing such a portfolio is to find the proportion of the base project that, when combined with borrowing at the riskless rate of interest, yields the same

[5] For example, the expansion option payoffs have higher variance than those of the base project.

payoffs as the expansion option in each state of the world.[6] For example, suppose an investment in half of the base project partly financed by \$150 of borrowing yielded, after paying off the creditors, a payoff of \$130 in the high-demand state and \$0 in the low-demand state.[7] Then a portfolio consisting of (i) a long position (i.e., owning) of half the base project and (ii) a short position (i.e., borrowing) of \$150 in a riskless bond, would constitute the ROA "replicating portfolio".

To identify the replicating portfolio in the expansion option example, let m denote the proportion of the base project in the portfolio and b denote the portfolio's dollar holdings of the riskless bond (which will be negative in the case of borrowing). Recalling that the riskless interest rate is 4% (see Table 1), such a portfolio has payoff

$$m*1800 \; + \; b*1.04$$

in the high-demand state and has payoff

$$m*675 \; + \; b*1.04$$

in the low-demand state. If this portfolio is to replicate the option, m and b must satisfy the two equations

$$m*\$1800 + b*(1.04) = \$130$$
$$m*\$675 + b*(1.04) = \$0$$

The first equation says that the portfolio has the same payoff as the expansion option in the high-demand state; the second equation says that it has the same payoff as the expansion

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6 In principle, any two assets with imperfectly correlated payoffs could be used to construct the replicating portfolio, but notational and mathematical complexity are minimised by using the base project and the riskless bond. Copeland and Antikarov (2001) argue that this is also frequently the most practical way of constructing the replicating portfolio.

7 Of course, it isn't usually possible to buy half the project, so the ROA assumption that such divisibility is possible is an approximation to reality. However, this assumption is also contained in the NPV and DTA calculations, so its counter-factual nature is not unique to ROA. Specifically, the risk-adjusted discount rate used in NPV and DTA is, either implicitly or explicitly, typically set equal to the expected return on an asset with the same risk characteristics, regardless of any differences in size.

option in the low-demand state. The mathematically inclined reader will recognise this as a system of two linear equations with two unknown variables (m and b). Solving this system in the usual manner yields the solutions for m and b:

$$m = 0.1156$$

$$b = -\$75$$

That is, owning a portfolio of 0.1156 units of the base project and $75 of riskless borrowing is equivalent to holding the expansion option.[8]

The second part of the ROA approach states that, since they have the same future payoffs, the present value of the expansion option must be the same as the present value of the replicating portfolio. This yields

ROA option value	=	value of replicating portfolio
	=	value of 11.56% of base project minus $75
	=	(0.1156*$1095) - $75
	=	$52

which is $6 less than the DTA option value. Similarly, the ROA project value is $1147 (equal to the base project value of $1095 plus the ROA option value of $52), which is also $6 less than its DTA counterpart.

Figure 4 summarises the ROA valuation process.

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[8] Obviously, such a procedure is not likely to be very practical for most real options since the composition of the replicating portfolio will change whenever there is a change in the value of either the base project or the riskless interest rate. However, it turns out that the replicating-portfolio method of valuation is mathematically equivalent (see Cox, Ross and Rubenstein, 1979) to the so-called risk-neutral approach where the option's expected payoffs are calculated using the probability distribution that would apply in a risk-neutral world and then discounted at the riskless rate of interest. For practical purposes, the advantage of this alternative approach is that the risk-neutral probabilities are independent of the values of the base project and the riskless interest rate. However, the intuition is, at least initially, rather less transparent, so we focus on the replicating portfolio approach in this paper. For examples of the risk-neutral approach, see Trigeorgis (2000).

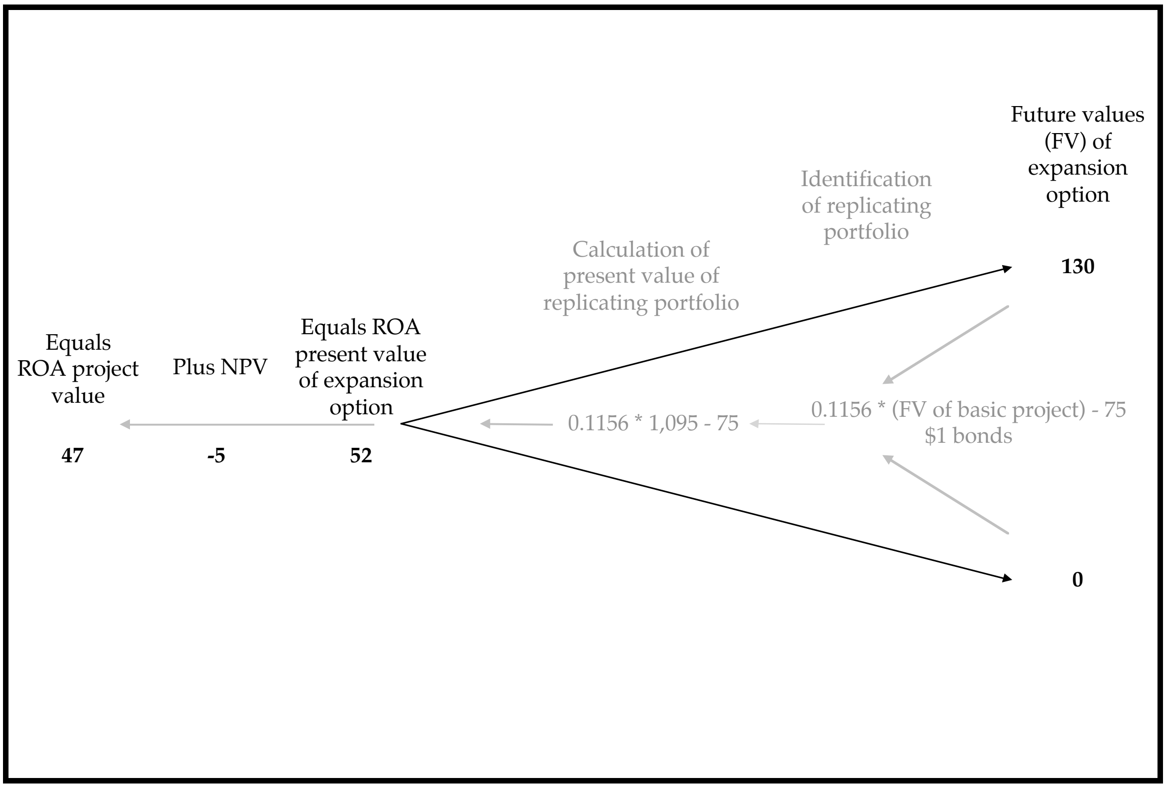

Figure 4: ROA (Expansion Option)

Alternatively, due to the value-additivity principle, the ROA project value could be calculated directly, as summarised by Figure 5

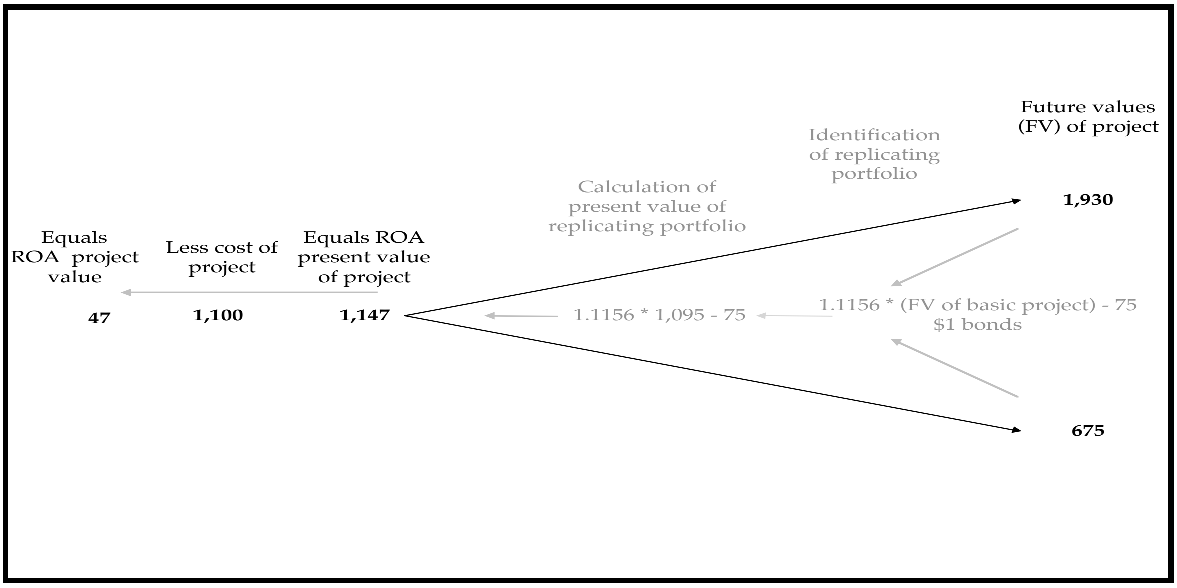

Figure 5: ROA (Project)

where the replicating portfolio consists of 1.1156 units of the base project partly financed by the sale of \$75 of riskless bonds, a combination that has present value of \$1147.[9]

Since the project cost is \$1100, investment increases firm value by \$47 and is therefore justified. Whereas the NPV rule said that the project should not go ahead, ROA, like DTA, says that it should. Here, DTA and ROA result in the same decision, so one might conclude that the additional complexity of ROA adds little to a manager's kit of project evaluation tools. Such a conclusion is premature. If, for example, the project cost were \$1150 (rather than \$1100), then investment in the project would increase firm value by \$3 according to DTA, but by -\$3 according to ROA. In that case, using DTA would lead to the wrong decision. Thus, although DTA is an improvement on NPV, it still provides inaccurate estimates of project value and can therefore lead to erroneous investment decisions. Only ROA provides accurate valuations and ensures correct decisions.

Further insight into the comparison of ROA with DTA can be obtained by considering the risk-adjusted discount rates each uses to calculate the value of the expansion option. Under DTA, this was assumed to be 13%, the base project's cost of capital. Although ROA does not explicitly use a discount rate, it is implicit in the calculations. Specifically, the ROA risk-adjusted discount rate k satisfies the following equation:

$$\$52 = 0.5 * \left(\frac{\$130}{1+k}\right) + 0.5 * \left(\frac{\$0}{1+k}\right)$$

$$k = 0.26$$

The implied discount rate for the option is 26%, twice that of the project, so the DTA approach of using the 13% base project rate leads to an overestimate of the option value and therefore an overestimate of the value of the project. It turns out that this is always the case

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[9] The replicating portfolio is formed by solving the following system of simultaneous equations:

$$m*\$1800 + b*(1.04) = \$1930$$
$$m*\$675 + b*(1.04) = \$675$$

for options to expand (although not, as we see in the next example, for options to contract or abandon the project). To see why, note that the expansion option payoff is perfectly positively correlated with the base project payoff: the former is high ($130) when the latter is high ($1800) and vice versa ($0 and $675 respectively). Thus, the expansion option effectively "magnifies" the risk of the base project payoffs. It follows that the expansion option is riskier than the base project and thus requires a higher discount rate, contrary to the DTA assumption.

5 Example II: The Option to Abandon

In this section, we briefly outline an abandonment option example. The base project is the same as before, but instead of an expansion option, the firm is assumed to have an option to abandon operations in one year's time in return for a salvage value of $775. The abandonment payoffs are therefore:

High-demand state: Abandonment yields $775, but sacrifices project value of $1800, so there is a net loss of $1025.

Low-demand state: Abandonment yields $775, but sacrifices project value of $675, so there is a net gain of $100.

Clearly, the firm will choose to abandon if and only if the low-demand state occurs, in which case the payoff is $100. In the high-demand state, the firm continues operations and the option payoff is $0.

The future payoffs generated by the abandonment option appear in Figure 6 (where, to simplify, we show only the values and not the calculations).

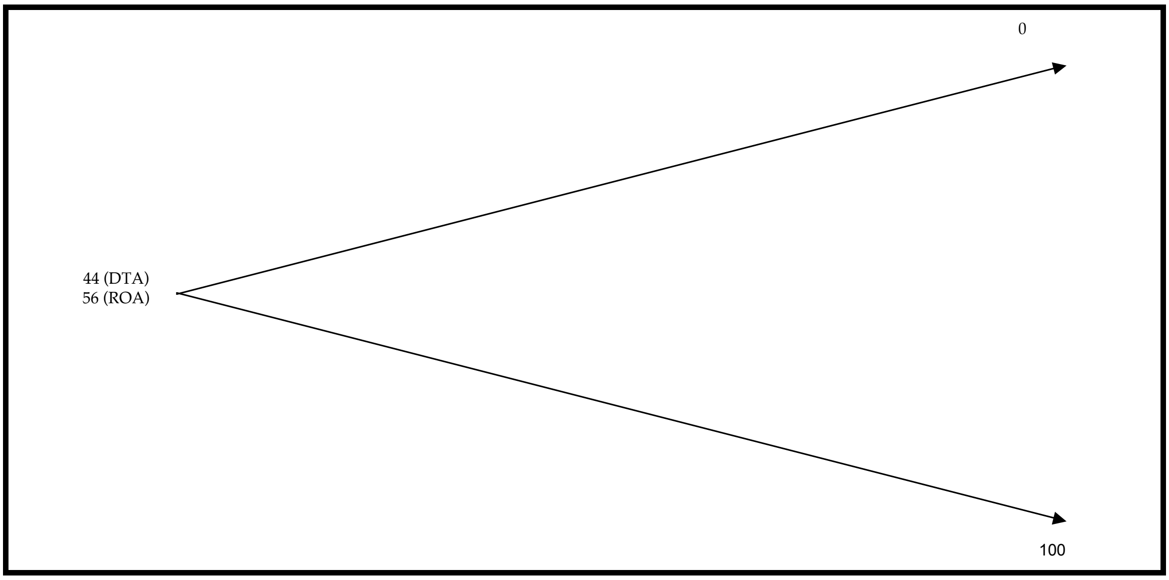

Figure 6: Abandonment Option

As the other parameters are the same as before, the base project NPV is still -\$5. Taking into account the flexibility offered by the abandonment option, DTA calculates net project value as

NPV + value of option to abandon

$$= -\$5 + 0.5^* \left(\frac{\$100}{1.13}\right) + 0.5^* \left(\frac{\$0}{1.13}\right)$$

$$= \$39$$

so the additional value due to the abandonment flexibility is \$44.

Forming the replicating portfolio as before, ROA calculates net project value as[10]

NPV + value of option to abandon

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[10] The replicating portfolio is formed by solving the following system of simultaneous equations:

$m^*\$1800 + b^*(1.04) = \$0$

$m^*\$675 + b^*(1.04) = \$100$

which yields $m = -0.089$ and $b = \$153.85$.

$$= -\$5 + \$154 - 0.09*\$1095$$

$$= \$51$$

so the additional value created by the abandonment option is $56.

In this case, DTA and ROA both indicate that the value of the abandonment option is more than sufficient to offset the negative NPV and thus justify investment. Once again though, such agreement will not always occur. If the project cost were $1145, then investment in the project would increase firm value by $6 according to ROA, but *decrease* it by $6 according to DTA. In that case, using DTA would lead to the wrong decision - the investment would be rejected when it should not be.

The outcome that abandonment option value is higher according to ROA than it is according to DTA is a general result. To see why, note that the abandonment option payoff is perfectly *negatively* correlated with the base project payoff: the former is high ($100) when the latter is low ($675) and vice versa ($0 and $1800 respectively). Thus, the abandonment option effectively "hedges" the risk of the base project payoffs. It follows that the abandonment option is less risky than the base project and thus requires a lower discount rate. The DTA approach of using the 13% base project rate therefore leads to an underestimate of both the abandonment option value and the value of the project.

6 Conclusion

Real options are pervasive and valuable, and standard NPV analysis fails to account for them, thereby forcing managers to rely heavily on qualitative “strategic” judgment when valuing investment opportunities. DTA recognises that real options have value because of the managerial flexibility they provide, but fails to value them accurately since it offers no way of estimating appropriate discount rates. Consequently, it simply assumes that the option's discount rate is the same as that of the base project. By contrast, ROA offers a simple and consistent way to value the flexibility inherent in real options.

The examples presented in this paper are highly simplified, indeed contrived. They assume there are only two points in time and only two possible future states, and ignore various real-world complications. Despite their artificial nature, these examples capture the essence of ROA since they can, relatively easily in many instances, be extended to much more complex and realistic situations. Although the analysis becomes more complicated, it relies on the same fundamental principle: *assets with the same future payoffs must have the same present value*. Applying this principle to more complex problems frequently requires some knowledge of stochastic calculus, a very effective entry barrier for many. But, as Copeland and Antikarov (2001) show, the resulting calculations can often be approximated in a simple lattice form on a spreadsheet. Even when neither of these methods is feasible, numerical techniques such as Monte Carlo simulation usually provide an accessible way of implementing ROA. Further discussion of these methods is beyond the scope of this paper, but the interested reader can find out more from the studies listed in our reference section.

To conclude, real-options analysis - with its reliance on no-arbitrage principles and its somewhat opaque terminology - takes some getting used to. But it is conceptually sound and, by now, relatively well understood in application. For many firms, the benefits of an investment in real-options technology are likely to exceed the costs.

Suggestions for Further Reading

Introductory

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