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August 11, 2004

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*For helpful comments, we are grateful to Scott Chaput, Timothy Crack, Vivien Pullar, Tom Smith, and seminar participants at Otago, Canterbury, and ANU.

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Abstract

Popular investment advice recommends that the stock/bond and stock/wealth ratios should rise with investor risk tolerance and investment horizon respectively, prescriptions that are difficult to reconcile with the simple mean-variance model. Canner et al. (1997) point out that the first piece of advice can potentially be explained by human capital considerations, but only by invalidating the second piece of advice. We show that extending the mean-variance model to include human capital, without any other modifications, *can* simultaneously justify both recommendations, so long as the correlation between human capital returns and stock market returns lies within a range determined by market and investor-specific parameters. Historical data from 11 countries generally satisfy this requirement, although the statistical precision of our estimates is fairly weak.

JEL classification: G11

Keywords: asset allocation, human capital, investment advice

I Introduction

One of the cornerstones of modern portfolio theory is the two-fund separation theorem. This seminal result, originally due to Tobin (1958), states that the composition of the optimal portfolio of risky assets depends solely on the stochastic structure of market returns and is independent of investor-specific characteristics such as risk tolerance. However, Canner et al. (1997) document investment advice recommending that less risk-tolerant investors hold a higher ratio of bonds to stocks, a phenomenon they refer to as the asset allocation puzzle. As they point out, this is perplexing not only because the advice differs from that implied by theory, but also because it is more complicated than theory.¹

For many investors, human capital is a significant part of their overall portfolio, but it plays no role in the one-period mean-variance model that gives rise to the two-fund separation theorem.² Extending the model to incorporate human capital can potentially resolve the asset allocation puzzle. To see this, suppose that returns to human capital are perfectly correlated with those on stocks. Then human capital and stocks are perfect substitutes, so the separation theorem implies that the ratio

$$\frac{\text{BONDS}}{\text{HUMAN CAPITAL} + \text{STOCKS}}$$

is independent of investor risk tolerance. An increase in risk tolerance increases both the numerator and denominator of this ratio in the same proportion. But at any point in time, the quantity of human capital is non-tradable and thus fixed, so the quantity of stocks must rise proportionately more than the quantity of bonds. That is, the ratio of stocks to bonds is greater for more risk-tolerant investors, just as popular advice recommends.

However, Canner et al. (1997) reject this explanation on two grounds. First, human capital and stocks are unlikely to be perfect substitutes for many investors. Second, if human capital returns are strongly correlated with stock returns, then it becomes difficult to reconcile theory with another popular piece of investment advice: that young investors with long investment horizons should hold more of their wealth in stocks than older investors.³ In general, young investors have more human capital than their older counterparts, so a high positive correlation

¹This advice also seems to be followed in practice. Degeorge et al. (2004) examine the participation decisions of France Telecom employees in that firm's 1997 privatization share offering and find that civil servant workers invest proportionately more in the most "bond-like" vehicle than do their (plausibly less risk averse) private sector counterparts. Other proxies for risk aversion yield similar patterns.

²Nevertheless, the potential importance of human capital for financial decisions has long been recognized. For example, Mayers (1972) shows that the presence of non-marketable assets such as human capital introduce an extra term into the CAPM risk premium. More recently, Bodie et al. (1992), Heaton and Lucas (1997), and Viceira (2001), among others, examine the role of labour and business income for various portfolio decisions.

³A simple rule of thumb recommends that the portfolio percentage devoted to stocks should equal 100 minus

between stocks and human capital implies that young investors optimally allocate less of their wealth to stocks, thereby contradicting the standard advice. By contrast, if human capital is a close substitute for bonds or cash, then younger investors should indeed hold a higher proportion of their wealth in stocks, just as popular advice dictates. But then the advice that more risk-tolerant investors hold a higher ratio of stocks to bonds cannot be explained.

Thus, human capital considerations seem unable to resolve the asset allocation puzzle without simultaneously creating another equally perplexing puzzle. On the one hand, the strong correlation between human capital returns and stock returns required to justify the asset allocation advice makes the investment horizon advice more puzzling. On the other hand, the weak correlation between human capital returns and stock returns required to justify the investment horizon advice exacerbates the asset allocation puzzle. More succinctly, it seems impossible for the human capital of any investor to simultaneously justify both pieces of advice.⁴

One explanation for this conundrum is that investment advisors are simply wrong.⁵ However, such a pessimistic conclusion warrants further scrutiny. Specifically, we ask two questions. First, despite the misgivings outlined above, is it theoretically possible for human capital considerations to reconcile the simple mean-variance model with popular advice on asset allocation and investment horizon? Second, if such a theoretical explanation does exist, is it empirically plausible? That is, given the ubiquitous nature of these investment recommendations, do the conditions required for human capital to offer an explanation in theory seem likely to also exist in practice?

For the first question, we use a simple extension of the Campbell and Viceira (2002) log-linear version of the mean-variance model, and show that human capital factors *can* justify popular advice about both asset allocation and investment horizon decisions so long as the correlation between stock and human capital returns falls within some range defined by market and investor-specific parameters. To address the second question, we use historical data on aggregate asset returns and labour income from 11 countries to estimate these parameters and find that, at least for the various data sets we employ, the stock-human capital correlation generally falls within the allowed range.

the investor's age; see also the Vanguard Group advice quoted in Ameriks and Zeldes (2001). This process is sometimes known as 'time diversification'; see Kritzman (1994) and Jagannathan and Kocherlakota (1996) for particularly lucid discussions.

⁴Of course, one could argue that the investment horizon advice is justified by other considerations. For example, long-term stock returns could be mean-reverting, so that stocks are less risky over a long investment horizon. However, in 75 years of data from 30 countries, Jorion (2003) finds little evidence of this. Moreover, Jagannathan and Kocherlakota (1996) argue that human capital considerations represent the only convincing explanation for the view that the stock/wealth ratio should rise with investment horizon.

⁵See Anonymous (1997) for an example of this interpretation.

Previous research has identified other possible solutions for the asset allocation puzzle. Elton and Gruber (2000) argue that theory and popular advice can be reconciled by introducing various constraints into the mean-variance model, while Shalit and Yitzhaki (2003) suggest that the advice is not necessarily inefficient for alternative investor preferences. In addition, Brennan and Xia (2000) and Campbell and Viceira (2001) show that time-varying expected returns can justify the advice for an infinitely-lived investor. None of these, however, considers the relevance of their analysis for investment horizon considerations. Other authors, such as Bodie et al. (1992), Jagannathan and Kocherlakota (1996), and Viceira (2001), show that human capital considerations can justify the popular investment horizon advice, but do not discuss the asset allocation puzzle. None of this work, therefore, considers whether or not recognition of non-tradable human capital can simultaneously justify *both* pieces of investment advice.

An interesting exception to this is Gomes and Michaelides (2002), who calibrate a multi-period model with non-mean-variance preferences and a fixed market entry cost and find optimal behaviour that is broadly consistent with both pieces of investment advice. Our work differs from theirs in two ways. First, their primary focus is on other matters, so they do not explore the source of this consistency in any detail. Second, and more importantly, their model is much more complex than ours. The primary contribution of our work is to show that inclusion of human capital in the simple mean-variance model, *without any further modifications*, can potentially resolve the investment advice puzzles associated with that model. Although the dynamic effects associated with multi-period decision-making, for example, are undoubtedly important for explaining other types of investment behaviour and practice, our analysis indicates that they are not essential for explaining the puzzles documented in this paper.

II Optimal asset allocation in the presence of non-tradable human capital

An investor has some initial endowment of financial wealth $\bar{W} > 0$ which is used to construct a portfolio that generates the random rate of return R_p . One period later, the investor consumes the portfolio's liquidation value $\bar{W}(1 + R_p)$ and labour income L earned over the period.

At the beginning of the period, the investor chooses the portfolio that maximizes the expected utility of terminal wealth $W = \bar{W}(1 + R_p) + L$. This decision is determined by the power utility function

$$U(W) = \frac{W^{1-\gamma}}{1-\gamma}, \quad (1)$$

where $\gamma > 0$ is the coefficient of relative risk aversion.

In Campbell and Viceira (2002), the investor's portfolio decision consists of choosing the optimal combination of two assets, one of which is riskless while the other is risky. To address the asset allocation puzzle of Canner et al. (1997), we require two risky assets, so we extend the Campbell and Viceira model to a three-asset setting. Asset f , which we call cash, is riskless and offers the rate of return R_f over the period. Asset s , which we call stocks, is risky with a random rate of return R_s . Asset b , which we call bonds, is also risky and has a random rate of return R_b . We assume that $1 + R_s$, $1 + R_b$, and labour income L are lognormal random variables.

The portfolio shares allocated to assets s and b are α_s and α_b respectively. Thus, the investor chooses these portfolio shares to maximize the expected value of (1) subject to the budget constraint

$$W = \overline{W}(1 + R_p) + L. \quad (2)$$

This problem is nonlinear, so we apply some linear approximations that effectively reduce it to a mean-variance setting. The details of this procedure are straightforward, but tedious, so we relegate them to an appendix. There we show that the optimal asset allocations are

$$\alpha_s = \frac{1}{\Delta} \left(\frac{1}{\rho\gamma} (\sigma_b^2 \mu_s - \sigma_{sb} \mu_b) + \left(1 - \frac{1}{\rho}\right) (\sigma_b^2 \sigma_{ls} - \sigma_{sb} \sigma_{lb}) \right), \quad (3)$$

$$\alpha_b = \frac{1}{\Delta} \left(\frac{1}{\rho\gamma} (\sigma_s^2 \mu_b - \sigma_{sb} \mu_s) + \left(1 - \frac{1}{\rho}\right) (\sigma_s^2 \sigma_{lb} - \sigma_{sb} \sigma_{ls}) \right), \quad (4)$$

where, for $i = s, b$, $l = \log L$, $r_i = \log(1 + R_i)$, $\sigma_i^2 = \text{Var}[r_i]$, $\sigma_{sb} = \text{Cov}[r_s, r_b]$, $\sigma_{li} = \text{Cov}[l, r_i]$, $\Delta = \sigma_s^2 \sigma_b^2 - \sigma_{sb}^2$ is the determinant of the variance-covariance matrix, $\mu_i = E[r_i] - r_f + \sigma_i^2/2$ is the logarithmic risk premium for asset i , and $\rho \in [0, 1)$ is a monotonic transformation of the expected ratio of financial wealth to human capital, which is defined in detail in the appendix.

Note that without labour income, the second terms in the large brackets in (3) and (4) both equal zero, so the ratio α_s/α_b is independent of investor risk attitudes γ , i.e., the two-fund separation theorem applies. With labour income, however, this independence disappears. In the next section, we determine whether this can change the model's implications in a way that is consistent with popular investment advice.

III Risk aversion, investment horizon, and optimal asset choice

We first determine the effect of risk aversion γ on the ratio $\alpha \equiv \alpha_s/\alpha_b$. According to the two-fund separation theorem, α and γ are independent, but popular advice, as documented in Canner et al. (1997), recommends that less risk-tolerant investors hold a lower ratio of stocks to bonds. That is, $\partial\alpha/\partial\gamma$ should be negative.

In our model, the sign of $\partial\alpha/\partial\gamma$ is determined by the sign of (see the appendix)

$$\mu_s \sigma_{lb} - \mu_b \sigma_{ls}. \quad (5)$$

Letting $c_{li} = \sigma_{li}/\sigma_l\sigma_i$ denote the linear correlation coefficient for human capital and asset i returns, (5) is negative if and only if

$$c_{ls} > \underline{H}, \tag{6}$$

where $\underline{H} = c_{lb}(\mu_s/\sigma_s)/(\mu_b/\sigma_b)$.⁶ Thus, more risk-tolerant investors should indeed hold a higher proportion of stocks, so long as their labour income is sufficiently strongly correlated with stock returns, the required extent of which is determined by the relative size of the stock and bond Sharpe ratios. This condition reflects the balancing of the two determinants of asset demand: the ability to hedge human capital returns and the risk-return trade-off as measured by the Sharpe ratio. If (6) is satisfied, the hedging capabilities of the bond are sufficient to offset the risk-return properties of the stock, so investors who wish to reduce their risk exposure hold less of both stocks and bonds, but reduce stock holdings by more since they must continue to hold their non-tradable human capital.

This result is a simple extension of the Canner et al. (1997) argument that human capital considerations can justify popular asset allocation advice if stocks and human capital are perfect substitutes. It shows that human capital need only be relatively more “stock-like” than “bond-like”, thereby negating Canner et al.’s concern that perfect substitutability is unlikely to be the case for most investors. What remains unresolved is whether this weaker condition can also overcome Canner et al.’s other, more important, objection: that relatively “stock-like” human capital is inconsistent with popular advice on the relationship between investment horizon and optimal stock holdings. This advice is neatly summarized by Malkiel (1996):

“... the longer the time period over which you can hold on to your investments, the greater should be the share of common stocks in your portfolio.”

However, if human capital is strongly correlated with the stock market, then the holding of “stock-like” assets automatically increases with the investment horizon, thereby implying that the share of traded stocks should be smaller the longer the time period over which the portfolio can be held. If human capital considerations are to be a plausible explanation for the asset allocation puzzle, then the required condition (6) should not rule out the recommended relationship between stock holdings and investment horizon, i.e., the required high correlation between stock market returns and labour income should not be so high as to imply that investors with a long time horizon should hold a lower share of common stocks in their portfolios.

To address this issue, we use the parameter $1/\rho$ as a proxy for the length of investment horizon. This can be justified by noting that ρ , as defined in the appendix, is decreasing in

⁶To avoid unnecessary complications associated with negative numbers, we anticipate our subsequent empirical findings and assume $\mu_b > 0$.

the ratio of human capital wealth to financial wealth. Young investors, with long investment horizons and long working lives, have high human capital but low financial wealth, so they have lower ρ than do older investors with shorter investment horizons.⁷

Differentiating (3) with respect to $1/\rho$ yields

$$\frac{\partial \alpha_s}{\partial (1/\rho)} = \frac{1}{\Delta} \left(\frac{1}{\gamma} (\sigma_b^2 \mu_s - \sigma_{sb} \mu_b) - (\sigma_b^2 \sigma_{ls} - \sigma_{sb} \sigma_{lb}) \right).$$

To justify popular advice, this expression must be positive. This occurs if and only if

$$c_{ls} < \bar{H}, \tag{7}$$

where $\bar{H} = (1/\gamma \sigma_l \sigma_s)(\mu_s - (c_{sb} \sigma_s / \sigma_b) \mu_b) + c_{sb} c_{lb}$. Thus, long-horizon investors should indeed allocate a greater proportion of their wealth to stocks so long as the correlation between stock returns and labour income is not too high.⁸ The condition appearing in (7) gives concrete expression to what is meant by “not too high”. If (7) is satisfied, then stocks are a good hedge for non-tradable human capital, so a young investor with a long investment horizon puts more into stocks than an older investor with a shorter horizon, just as popular advice recommends.

Of course, what we are primarily interested in is whether the joint distribution of labour income and stock and bond returns can justify popular advice in relation to both asset allocation and investment horizon; that is, whether (6) and (7) can hold simultaneously. This occurs if and only if

$$\underline{H} < c_{ls} < \bar{H}. \tag{8}$$

Thus, as long as the correlation between stock returns and labour income lies between two bounds, investors should allocate less of their wealth to stocks as their investment horizon shortens and they should adjust their bond/stock ratios downwards in response to any increase in tolerance for risk.

Various parameter combinations would automatically disqualify this requirement. For example, if the stochastic structure of asset returns and labour income were such that $\underline{H} \geq \bar{H}$, or $\bar{H} \leq -1$, or $\underline{H} \geq 1$, then (8) cannot hold. However, inspection of \underline{H} and \bar{H} reveals that there are combinations of parameters for which (8) is satisfied, so it is theoretically possible for human capital considerations to reconcile the simple mean-variance model with popular advice on asset allocation in a way that does not conflict with other popular advice on the relationship between stock holdings and the investment horizon. What remains unclear is whether such a possibility is empirically plausible; that is, whether *actual* parameter values satisfy (8).

⁷Young investors also have greater future liabilities (consumption) than their older counterparts, but this important difference cannot be captured in a static model.

⁸See Bodie et al. (1992) and Jagannathan and Kocherlatoka (1996) for a similar conclusion.

Table 1: U.S. estimates of c_{ls} , \underline{H} , and \overline{H}

Popular investment advice on (i) stock/bond allocation and risk tolerance and (ii) stock/wealth allocation and investment horizon can both be justified if and only if $\underline{H} < c_{ls} < \overline{H}$, where c_{ls} is the linear correlation between labour income and stock returns, and \underline{H} and \overline{H} are constants that depend on market and investor-specific parameters. For the U.S., for each year in the period 1930–1999, market return data are obtained from Ibbotson Associates and labour income data from the Department of Commerce’s Bureau of Economic Analysis. γ is the coefficient of relative risk aversion. Based on a Wald test, * indicates that $c_{ls} - \underline{H}$ (or $\overline{H} - c_{ls}$) is positive at the 10% significance level, ** that it is positive at the 5% level, and *** that it is positive at the 1% level.

Bond type	Sample		c_{ls}	\underline{H}	\overline{H}			γ for which $c_{ls} < \overline{H}$
	Begin	End			$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	
Long-term corporate bonds	1930	1999	0.11	-0.02	4.34***	1.74***	0.87***	$0 < \gamma < 75.4$
Long-term government bonds	1930	1999	0.11	-0.12	4.66***	1.86***	0.92***	$0 < \gamma < 75.1$
Intermediate government bonds	1930	1999	0.11	0.03	5.05***	2.02***	1.01***	$0 < \gamma < 91.5$

IV Empirical estimates of c_{ls} , \underline{H} , and \overline{H}

To determine whether our model’s justification of popular advice is plausible, we use historical data to estimate c_{ls} , \underline{H} , and \overline{H} . If these estimates satisfy (8), then this is consistent with the view that popular investment advice implicitly incorporates human capital considerations; failure to satisfy (8) suggests that popular investment advice cannot be justified by human capital considerations, at least not in the way envisaged by our model.

To obtain estimates of the terms in (8), we first examine a long time series of U.S. returns and average labour income data. Specifically, we use annual U.S. market returns data from Ibbotson Associates and per-capita income data from the U.S. Department of Commerce’s Bureau of Economic Analysis (BEA) for the period 1930–99.⁹ Ibbotson Associates report real returns for long-term government bonds, intermediate-term government bonds, and corporate bonds, so we calculate \underline{H} and \overline{H} for each bond type; real stock returns are calculated from the large company index.

With these data, we calculate estimates of the various means, standard deviations, and correlations that appear in equations (3) and (4). We then substitute these estimates into the terms appearing in (8), a process that yields the results in Table 1. Focusing first on the difference between \underline{H} and c_{ls} , the labour-stock correlation is 0.11, but the estimates of \underline{H}

⁹The BEA data are available from <http://www.bea.doc.gov/>. Ideally, we would use actual human capital data, but we are unable to locate reliable sources of this variable for most of the countries we subsequently examine. Since labour income is equal to human capital in a one-period world, we use the former as a proxy for the latter.

range from -0.12 to 0.03 . Thus, regardless of the class of bond, the first inequality in (8) is satisfied. Turning to the second inequality, we report estimates of \overline{H} for low, medium, and high values of γ ($\gamma = 2, 5, 10$ respectively).¹⁰ Most of these estimates are greater than unity, thereby automatically exceeding c_{ls} ; even the smallest estimate of \overline{H} is almost eight times as large as the labour-stock correlation. In the final column of Table 1, we express this result in a different way by reporting the range of γ values for which $\overline{H} - c_{ls}$ is positive; for this not to occur, γ must attain at least the implausibly-high value of 75.¹¹

While these results are consistent with our model, an important caveat applies. The terms in (8) depend in part on means and correlations, parameters that are notoriously difficult to estimate with any precision. Thus, the point estimates appearing in Table 1 are likely to be subject to considerable error. To address this issue, we use a Wald test of both inequalities in (8).¹² For the first inequality, we define

$$h_1 = \mu_b \sigma_{ls} - \mu_s \sigma_{lb}$$

and test the null hypothesis that $h_1 \leq 0$ against the alternative that $h_1 > 0$. For the second inequality, we define

$$h_2 = \sigma_b^2 \mu_s - \sigma_{sb} \mu_b - \gamma(\sigma_b^2 \sigma_{ls} - \sigma_{sb} \sigma_{lb})$$

and test the null hypothesis that $h_2 \leq 0$ against the alternative that $h_2 > 0$. In either case, rejecting the null supports the corresponding inequality in (8).

The results of this test procedure also appear in Table 1 and provide both good and bad news for our story. The good news is that h_2 is positive at the 1% significance level in all cases, even at the highest level of risk aversion. The bad news is that h_1 is insignificantly different from zero in all cases. This difference reflects the fact that the estimated standard errors for \overline{H} and c_{ls} are quite small, whereas that for \underline{H} is large.¹³ Thus, our U.S. data strongly support the notion that human capital considerations can explain the investment horizon advice, but are rather more reticent about the asset allocation advice.

To examine this issue further, we estimate the terms in (8) with data from other countries. For asset returns, we use the data series generated by Dimson et al. (2002); these contain annual

¹⁰Because $\sigma_b^2 \mu_s - \sigma_{sb} \mu_b > 0$ in our data, \overline{H} is monotonically decreasing in γ .

¹¹We also calculate c_{ls} , \underline{H} , and \overline{H} assuming that asset returns are lagged one year, reflecting possible lags in labour income (see Campbell et al., 2001). Although this results in different estimates of c_{ls} and \underline{H} individually, it has virtually no effect on the difference between them. Similarly, the difference between \overline{H} and c_{ls} remains large and positive.

¹²See, for example, Greene (1993, pp. 131–133).

¹³Indeed, in order for the point estimates in Table 1 to be able to reject the null that $h_1 \leq 0$ at the 5% significance level, we would need approximately 350 years of data in the case of long-term government bonds, and even more for the other two bond categories.

real returns on equities, bonds and bills from 1901 to 2002 for 15 countries (not including the U.S.).¹⁴ For labour income, we use International Financial Statistics data published by the IMF, deflating these nominal series by their corresponding CPI values. The income series are of shorter duration than the Dimson et al. returns series and, moreover, are not available for all 15 countries. In all, we are able to calibrate our model with data from 10 additional countries over post-WWII periods of varying length. For completeness, we also include the U.S. to check that our earlier results are not sensitive to the source of labour income data.

The results from using these data appear in Table 2. Several features are apparent. First, the post-WWII results for the U.S. are very similar to those for the longer time period reported in Table 1. Second, for eight of the other 10 countries, the bounds specified by (8) are satisfied, consistent with the view that popular investment is motivated by human capital considerations. Moreover, in most of these eight countries, c_{ls} differs from its two bounds by fairly large margins. In the two countries (Canada and Italy) where the first bound is violated, the difference is small. Third, c_{ls} is less than \bar{H} at conventional significance levels in all countries for all reported values of γ . Moreover, $c_{ls} \geq \bar{H}$ only for implausibly-high risk aversion. Fourth, c_{ls} is greater than \underline{H} at conventional significance levels in three countries (France, Japan, and the U.K.).

Overall, our results from international data paint much the same picture as those generated earlier by U.S. data: they provide strong support for the link envisaged by our model between human capital and investment horizon advice, but statistically weaker support for the human capital link with asset allocation advice.

V Concluding remarks

Can human capital considerations resolve the asset allocation puzzle of Canner et al. (1997)? Those authors are doubtful, primarily because the strong correlation between stock returns and labour income gains that would be required also implies that investors with a long investment horizon should allocate less of their financial wealth to stocks, exactly the opposite of popular investment advice. However, once non-tradable human capital is explicitly modelled, the optimal stock-bond ratio depends not only on the correlations of these assets with labour income, but also on the simple risk-return trade-offs offered by these assets. As a result, the correlation between stock returns and labour income gains required to resolve the asset allocation puzzle does not, after all, have to be all that high, leaving open the possibility that it can be sufficiently low to also justify the investment horizon advice.

The principal contributions of this paper have been, first, to confirm the theoretical validity

¹⁴These data are available from Ibbotson Associates.

Table 2: International estimates of c_{ls} , \underline{H} , and \overline{H}

This table repeats the calculations of Table 1, but for alternative data sets. Market returns are calculated from the real bond, equity, and bill indices in Dimson et al. (2002). Labour income for each country is generated using IMF labour income data, deflated by the respective CPI series. As in Table 1, *, **, and *** indicate that the corresponding bound on c_{ls} in (8) is statistically significant (using a Wald test) at the 10%, 5%, and 1% significance levels respectively.

Country	Sample		c_{ls}	\underline{H}	\overline{H}			γ for which $c_{ls} < \overline{H}$
	Begin	End			$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	
Australia	1963	2002	-0.29	-0.68	7.82**	3.11**	1.53**	$\gamma > 0$
Canada	1949	2002	-0.00	0.09	10.15**	4.06**	2.03**	$\gamma > 0$
France	1950	2002	0.14	-0.07*	4.85*	1.92*	0.94*	$0 < \gamma < 57.3$
Ireland	1949	2002	0.38	0.08	7.81***	3.13***	1.57**	$0 < \gamma < 42.0$
Italy	1960	2002	-0.25	-0.21	3.16*	1.24*	0.60*	$\gamma > 0$
Japan	1949	2002	0.34	-0.18*	4.46***	1.78***	0.89**	$0 < \gamma < 26.0$
Netherlands	1950	2002	-0.02	-0.12	9.38***	3.75***	1.88***	$\gamma > 0$
Spain	1961	2002	-0.13	-0.52	3.77**	1.43**	0.65**	$0 < \gamma < 826.8$
Sweden	1961	2002	-0.03	-0.08	6.21**	2.48**	1.24**	$\gamma > 0$
UK	1957	2002	-0.16	-1.03**	8.58**	3.29**	1.53**	$0 < \gamma < 241.0$
US	1949	2002	0.14	-0.03	14.60***	5.84***	2.92***	$0 < \gamma < 207.4$

of the above logic, and second, to assess its empirical validity using historical data from a number of countries. The results of the latter exercise are somewhat ambiguous. On the one hand, the critical inequalities identified by our model are almost always evident in the data. On the other hand, the imprecision of our parameter estimates means that a number of these inequalities are statistically insignificant. In particular, the correlation between stock returns and labour income is a little too close to its lower bound, thereby making it difficult to conclude that this bound is truly satisfied in most cases. Thus, while the data generally support our story that ubiquitous investment advice can be justified by human capital considerations, they cannot definitively reject Canner et al.'s (1997) view that these two phenomena are independent. Analysis of disaggregated labour income data using longer time series than are currently available may ultimately help to shed further light on this issue.

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Appendix

Proof of (3) and (4)

Maximizing the expected value of (1) subject to (2) yields the first-order conditions

$$E[(R_i - R_f)W^{-\gamma}] = 0, \quad i = s, b,$$

which can be re-written as

$$\log E[(1 + R_i)W^{-\gamma}] = (1 + R_f) \log E[W^{-\gamma}], \quad i = s, b. \quad (\text{A-1})$$

To make this problem analytically tractable, we use two loglinear approximations developed by Campbell and Viceira (2001, 2002). First, a Taylor expansion of the logarithmic form of (2) gives

$$w \approx k + \rho(\bar{w} + r_p) + (1 - \rho)l, \quad (\text{A-2})$$

where $r_p = \log(1 + R_p)$, $l = \log L$, $\bar{w} = \log \bar{W}$,

$$\rho = \frac{\exp(\bar{w} + E[r_p - l])}{1 + \exp(\bar{w} + E[r_p - l])} < 1,$$

and

$$k = \log(1 + \exp(\bar{w} + E[r_p - l])) - \rho(\bar{w} + E[r_p - l]).$$

Second, a Taylor expansion of $\log(1 + R_p)$ yields

$$r_p \approx r_f + \alpha_s(r_s - r_f) + \alpha_b(r_b - r_f) + \frac{1}{2} (\alpha_s(1 - \alpha_s)\sigma_s^2 - 2\alpha_s\alpha_b\sigma_{sb} + \alpha_b(1 - \alpha_b)\sigma_b^2), \quad (\text{A-3})$$

where $r_i = \log(1 + R_i)$, $\sigma_i^2 = \text{Var}[r_i]$, and $\sigma_{sb} = \text{Cov}[r_s, r_b]$.

As r_s and r_b are jointly normal, (A-3) implies that r_p also has a normal distribution. Then, since l is also normal, (A-2) implies that w is normal as well. Thus, both terms inside the expectations operator in (A-1) are lognormally distributed. Using the standard properties of a lognormal random variable, we obtain

$$\begin{aligned} \log E[(1 + R_i)W^{-\gamma}] &= E[r_i - \gamma w] + \frac{1}{2}\text{Var}[r_i - \gamma w], \quad i = s, b, \\ (1 + R_f)\log E[W^{-\gamma}] &= r_f - \gamma E[w] + \frac{1}{2}\text{Var}[-\gamma w]. \end{aligned}$$

Substituting these back into (A-1) yields

$$\begin{aligned} E[r_i] - r_f + \frac{\sigma_i^2}{2} &= \gamma \text{Cov}[r_i, w] \\ &= \gamma \text{Cov}[r_i, \rho r_p + (1 - \rho)l] \\ &= \gamma \rho \text{Cov}[r_i, \alpha_s r_s + \alpha_b r_b] + \gamma(1 - \rho)\text{Cov}[r_i, l], \end{aligned}$$

where we used (A-2) and (A-3). This is a system of two linear equations in the two unknowns α_s and α_b . Solving this system produces (3) and (4).

Proof of (5)

From (3) and (4), we can write

$$\alpha = \frac{g_s/\gamma + h_s}{g_b/\gamma + h_b},$$

where

$$\begin{aligned} g_s &= \frac{\sigma_b^2 \mu_s - \sigma_{sb} \mu_b}{\rho}, \\ g_b &= \frac{\sigma_s^2 \mu_b - \sigma_{sb} \mu_s}{\rho}, \\ h_s &= \left(1 - \frac{1}{\rho}\right) (\sigma_b^2 \sigma_{ls} - \sigma_{sb} \sigma_{lb}), \\ h_b &= \left(1 - \frac{1}{\rho}\right) (\sigma_s^2 \sigma_{lb} - \sigma_{sb} \sigma_{ls}). \end{aligned}$$

Then

$$\frac{\partial \alpha}{\partial \gamma} = \frac{g_b h_s - g_s h_b}{(g_b + \gamma h_b)^2},$$

where

$$g_b h_s - g_s h_b = \frac{1}{\rho} \left(1 - \frac{1}{\rho}\right) \Delta (\mu_b \sigma_{ls} - \mu_s \sigma_{lb}).$$

Since $1 - 1/\rho < 0$ and $\Delta > 0$, $g_b h_s - g_s h_b$ has the sign of $\mu_s \sigma_{lb} - \mu_b \sigma_{ls}$.