FORWARD LOOKING ESTIMATES OF THE MARKET RISK PREMIUM

ASSOCIATE PROFESSOR MARTIN LALLY

SCHOOL OF ECONOMICS & FINANCE
VICTORIA UNIVERSITY OF WELLINGTON
21 JUNE 2005

ISCR
NEW ZEALAND INSTITUTE FOR THE STUDY OF COMPETITION AND REGULATION INC.

Theory

The market value of equities is equal to the present value of future dividends to existing shareholders in these companies, discounted at $E(R_m)$:

$$P_m = \frac{DIV_m(1 + g_1)}{1 + E(R_m)} + \frac{DIV_m(1 + g_1)(1 + g_2)}{(1 + E(R_m))^2} + \ldots$$

i.e.,

$$1 = \frac{D_m(1 + g_1)}{1 + E(R_m)} + \frac{D_m(1 + g_1)(1 + g_2)}{(1 + E(R_m))^2} + \ldots$$

Estimation of $E(R_m)$ from this equation requires

- Market cash dividend yield D_m

- Short-run expected dividend growth rates g_1, g_2, …

- Long-run expected dividend growth rate (g)

The estimates of the MRP in two versions of the CAPM are then as follows

- $$MRP_S = E(R_m) - R_f$$

- $$MRP_T = E(R_m) - R_f(1 - .33)$$

Estimates

The upper bound on *g* is the long-run expected growth rate in aggregate dividends for existing coys, and …

the upper bound on this is the long-run expected growth rate in aggregate dividends for all companies (present and future), and….

this matches the long-run expected growth rate in GDP

$\Rightarrow g$ = long-run expected GDP growth
   – effect of share issues
   – effect of new coys

Parameter Estimates

- Cash dividend yield = 5%

- Short-run expected dividend growth rates:
  5%, 5%, 5%   5.1%, 5.8%   9.9%, 8.6%, 7.3%

- Long-run expected growth rate in GDP:
  CPI = 2.1%, Real GDP = 2.9% $\Rightarrow$ 5%

- Deduction from GDP growth rate = 1% $\Rightarrow$
  g = 4%

- Risk free rate = ten year government bonds
  = 5.9% (May 2005)

ISCR NEW ZEALAND INSTITUTE FOR THE STUDY
OF COMPETITION AND REGULATION INC.

Results

Suppose the highest set of short-run growth rates are invoked (9.9%, 8.6%, 7.3%) and

… suppose the long-run expected growth rate of dividends per share = 5%   and

… suppose convergence takes 10 years

$\Rightarrow$expected growth rates in dividends per share over the next 10 years are

.099, .086, .073, .070, .0664, .0631, .060, .0566, .0533, .050

Results cont

$\Rightarrow$ $E(R_m) = .111$

$\Rightarrow$ $MRP_S = .111 - .059 = .052$

      $MRP_T = .111 - .059(1-.33) = .071$

Table 1: Estimated Market Risk Premiums: High EPS Forecasts

	$N = 10$	$N = 20$	$N = 30$
$g = .030$	.056 (.037)	.063 (.044)	.067 (.048)
$g = .040$	.063 (.044)	.069 (.050)	.072 (.053)
$g = .050$	.071 (.052)	.075 (.056)	.077 (.058)

Table 2: Estimated Market Risk Premiums: Low EPS Forecasts

	$N = 10$	$N = 20$	$N = 30$
$g = .030$	.047 (.028)	.050 (.031)	.053 (.034)
$g = .040$	.055 (.036)	.056 (.037)	.057 (.038)
$g = .050$	.063 (.044)	.063 (.044)	.063 (.044)

Conclusions

- the central estimates for MRP_T are .056 and .069, averaging .062

- the range of estimates is .047 to .077

- the central estimate for MRP_S is .043 with a range from .028 to .058

- these central estimates are below those from historical averaging

- the central estimates are similar to those in Lally (2001)

Limitations

- the long-run expected growth rate in dividends per share for existing companies is unclear

- the short-run expected growth rates in EPS for existing companies are unclear

- the period over which short-run growth rates converge on the long-run growth rate is unclear

- the current market value of equities is assumed to be "rational"

- the market's pricing model is consistent with the model into which the MRP estimates are inserted

ISCR NEW ZEALAND INSTITUTE FOR THE STUDY
OF COMPETITION AND REGULATION INC.

THE OPTIMAL ESTIMATOR OF THE MARKET RISK PREMIUM

ASSOCIATE PROFESSOR MARTIN LALLY

SCHOOL OF ECONOMICS & FINANCE
VICTORIA UNIVERSITY OF WELLINGTON
21 JUNE 2005

ISCR

NEW ZEALAND INSTITUTE FOR THE STUDY OF COMPETITION AND REGULATION INC.

Theory:

Methods for estimating the market risk premium for a particular market

- **Historical averaging for that market (Ibbotson)**

- **Historical averaging of the reward-to-risk ratio coupled with a current estimate of risk (Merton)**

- **Forward-looking methods**

- **Historical averaging for other markets (Ibbotson)**

ISCR NEW ZEALAND INSTITUTE FOR THE STUDY
OF COMPETITION AND REGULATION INC.

Some estimators dominate others, but not when they draw upon different sources of information. Examples:

- **Historical averaging for that market (Ibbotson)**

- **Forward-looking methods involving dividends**

- **Historical averaging for other markets (Ibbotson)**

- **Forward-looking methods involving accounting numbers**

In this case, the best estimator will be some combination of the individual estimators

If estimators are unbiased, the best combination (weights w_1, w_2….w_n) minimises standard deviation, and these weights satisfy the following system of equations.

$$W_1 \sigma_{11} + W_2 \sigma_{12} + \ldots \ldots W_n \sigma_{1n} = \lambda$$

$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

$$W_n \sigma_{n1} + W_2 \sigma_{n2} + \ldots \ldots W_n \sigma_{nn} = \lambda$$

$$w_1 + w_2 + \ldots \ldots w_n = 1$$

If some estimators are biased (with biases B_1, B_2....B_n), the best set of weights minimises root mean squared error (RMSE), and these weights satisfy the following system of equations.

$$w_1(B_1 B_1 + \sigma_{11}) + ...w_n(B_1 B_n + \sigma_{1n}) = \lambda$$

$$...$$

$$w_n(B_n B_1 + \sigma_{n1}) + ...w_n(B_n B_n + \sigma_{nn}) = \lambda$$

$$w_1 + w_2 + ...w_n = 1$$

EXAMPLE: TWO ESTIMATORS

Suppose the estimators are as follows:

- **Ibbotson estimator for local market: standard deviation = .02**

- **Forward-looking estimator involving dividends: standard deviation = .02**

- **These estimators are uncorrelated**

If the two estimators are unbiased, the weights on the two estimators are chosen to minimise standard deviation

- Weights are 50% on each estimator

- Standard deviation of the combined estimator is .014

- Reduction in standard deviation compared to the best individual estimator is 30% of either individual estimator

If the Ibbotson estimator is biased by .02, then weights are chosen to minimise RMSE

- Weights are 33% on the Ibbotson estimator and 67% on the forward-looking estimator

- RMSE of the combined estimator is .0164

- Reduction in RMSE compared to the best individual estimator is 18%

Table 1: Optimal Estimator Weights With Two Estimators

σ	Ibbotson Bias		
	0	.02	.04
.01	.20, .80 (10%)	.11, .89 (6%)	.05, .95 (2%)
.02	.50, .50 (30%)	.33, .67 (18%)	.17, .83 (9%)
.03	.69, .31 (17%)	.53, .47 (27%)	.31, .69 (17%)

Table 1: Optimal Estimator Weights With Two Estimators cont

This table shows the optimal weights on the Ibbotson estimator and an unbiased forward-looking estimator respectively, with the bracketed figure being the reduction in RMSE from this optimal estimator relative to the best individual estimator. The optimal weights depend upon the bias in the Ibbotson estimator and the standard deviation of the forward-looking estimator (σ). The standard deviation on the Ibbotson estimator is assumed to be .02

EXAMPLE: THREE ESTIMATORS

Suppose the estimators are as follows:

- Ibbotson estimator for the local market: standard deviation = .02

- Forward-looking estimator involving dividends: standard deviation = .02

- Ibbotson estimator for the rest of the world: standard deviation = .014

- The forward-looking estimator is uncorrelated with the other two

- The correlation between the Ibbotson estimators = .60

If all three estimators are unbiased, the weights on the three estimators are chosen to minimise standard deviation:

- **Weights are 7%, 32% and 61%**

- **Standard deviation of the combined estimator is .0114**

- **Reduction in standard deviation compared to the best individual estimator is 19% of the best individual estimator**

If the Ibbotson estimator for the rest of the world is biased by .02, the weights are then chosen to minimise RMSE:

- **Weights are 36%, 45% and 19%**

- **RMSE of the combined estimator is .0134**

- **Reduction in RMSE compared to the best individual estimator is 33%**

Table 2: Optimal Estimator Weights With Three Estimators

σ	World Ibbotson Bias		
	0	.02	.04
.01	.03, .66, .31 (19%)	.16, .76, .08 (13%)	.18, .79, .03 (11%)
.02	.07, .32, .61 (19%)	.36, .45, .19 (33%)	.41, .53, .06 (30%)
.03	.08, .18, .74 (10%)	.48, .26, .26 (23%)	.62, .29, .09 (19%)

This table shows the optimal weights on the Ibbotson estimator for the local market, the forward-looking estimator, and the Ibbotson estimator for the world market respectively, with the bracketed figure being the reduction in RMSE from this optimal estimator relative to the best individual estimator. The optimal weights depend upon the bias in the Ibbotson estimator for the world market and the standard deviation of the forward-looking estimator (σ). The standard deviation on the Ibbotson estimator for the local market is assumed to be .02, that on the Ibbotson estimator for the world market is assumed to be .014, and the last two estimators are assumed to have a correlation coefficient of .60.

CONCLUSIONS

- With two estimators, the reduction in RMSE is up to 30% of the best individual estimator

- With three estimators, the reduction in RMSE is up to 35% of the best individual estimator

- With four estimators, the reduction in RMSE is up to 38% of the best individual estimator

- These maximum reductions in RMSE occur when the RMSE for the individual estimators are equal

- These diversification benefits are similar in principle to those arising in portfolio analysis