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The Welfare Gain from A New Good: An Introduction

John Creedy*

Abstract

This note provides an elementary introduction to the measurement of welfare gains from the introduction of a new good, based on the concept of the 'virtual price' and standard expressions for welfare changes arising from price changes.

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1 Introduction

Any attempt to measure changes in living standards using an income or consumption based welfare metric faces a limitation arising from the fact that it cannot deal with the introduction of new commodities. The role of new commodities over, say, a ten year period can be substantial. Furthermore, significant quality improvements can be regarded as involving an essentially new commodity. An individual with an unchanged total expenditure over a period may be better off as a result of innovations resulting in new goods.

This paper provides an elementary introduction to a method of allowing for new goods which makes use of standard measures of welfare change that are usually used in the context of price changes (and, in particular, price changes arising from indirect taxes).[1] There is now a substantial and often technical literature so the aim here is merely to clarify, and illustrate with a simple example, one approach to the analysis of the welfare gains arising from new goods.

The situation before a new good is introduced may be imagined to be one whe re the good exists, but with a 'virtual price' that is so high that the individual chooses not to consume any of the good.[2] The advent of the new good can then be regarded as being equivalent to involving a reduction in price, from the high virtual price to one which makes it worthwhile for the individual to consume the good. The welfare change can be expressed in money terms as the difference between expenditures, at new and old prices, involved in moving along a given indifference curve.

Section 2 introduces the concept of the virtual price and shows, with a simple diagram, how it can apply to a new good. This section also shows that the same ideas apply to the case where a good is rationed, except that the concept of a virtual income is also required. Hence, the only difference

[1] On welfare changes, see Creedy (1998)

[2] The basic idea of a virtual price and its role in rationing and the introduction of news goods goes back to Hicks (1940) and Rothbarth (1941). See Neary and Roberts (1980) and Hausman (1996) for later seminal and more technical contributions to, respectively, the analysis of rationing and new goods. For an application to historical data and further references see Hersh and Voth (2002). See also Nevo (2003) and, for a non-parametric approach, see Blow and Crawford (2003).

is that in the rationed case, the welfare change resulting from the end of rationing involves both a change in relative prices and a change in virtual income. It involves no basic additional principles.

Section 3 shows how the welfare change from the introduction of a new good can be calculated, by illustrating the steps involved for a simple form of direct utility function, a modification of the Cobb-Doublas form.

2 The Concept of Virtual Price

If only one good is available, the maximum amount that can be consumed, with a budget of m and a price of p_1 is simply m/p_1. This is indicated by point A in Figure 1. The individual is able to reach the indifference curve corresponding to utility, U^0. However, the consumption of only one good is consistent with a standard tangency solution at point A, given the budget line AB. This is a virtual budget line, given by the combination of m and p_1 with the virtual price of, say, p_2^*: the slope of AB is thus p_2^*/p_1.

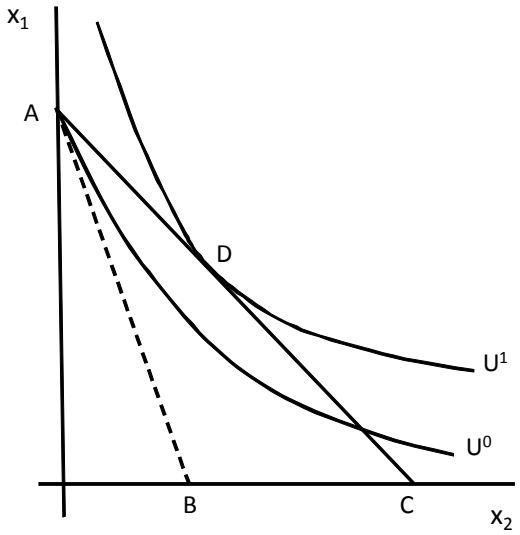

Figure 1: The Introduction of a New Good

The introduction of a new good can therefore be examined in terms of a reduction in the price of good 2. When good 2 is available at price, p_2,

the budget constraint becomes AC: with no change in the price of good 1, the constraint pivots around point A. The new tangency solution is at point D, representing a tangency position for a higher utility, U^1. Of course the gain from the new good cannot be represented by $U^1 - U^0$, given an ordinal concept of utility, but standard methods of obtaining welfare changes arising from price changes can be used (giving results that are invariant with respect to monotonic transformations of utility). The following section illustrates the calculation of the equivalent variation, which measures the difference between the expenditure levels required to reach the new indifference curve at the new and old prices. The compensating variation, which measures associated expenditure differences while moving along the old indifference curve, involves no basic differences and can be obtained following the same approach.

A similar approach can be applied to the situation where good 2 is initially rationed, rather than not being available at all. If x_2 is rationed at the level, C, in Figure 2, the initial budget constraint is the line ABC, and the highest indifference curve corresponds to U^0. The optimal position is a corner solution at B. However, this can be converted into a standard tangency solution using the virtual budget line – the dashed line in Figure 2 – which is effectively the tangent to U^0 at B. This virtual constraint is associated with both a virtual relative price of the two goods and a virtual budget, since the intersections on the axes correspond neither to A nor D.

When good 2 is no longler subject to rationing, the budget constraint becomes the straight line AD, and the tangency position, E, can be reached corresponding to a higher level of utility, U^1. The main difference between the case of a new good and rationing is that in the latter case the welfare change arises from the combination of a change in relative prices and the budget: here the virtual price ratio is higher than the actual price ratio (the slope is steeper than AB), and the budget (expressed using good 1 as numeraire) is lower than the actual budget, m. This can easily be handled using standard methods of obtaining welfare changes. The illustration presented in the following section therefore concentrates on the introduction of a new good, involving only a price change.

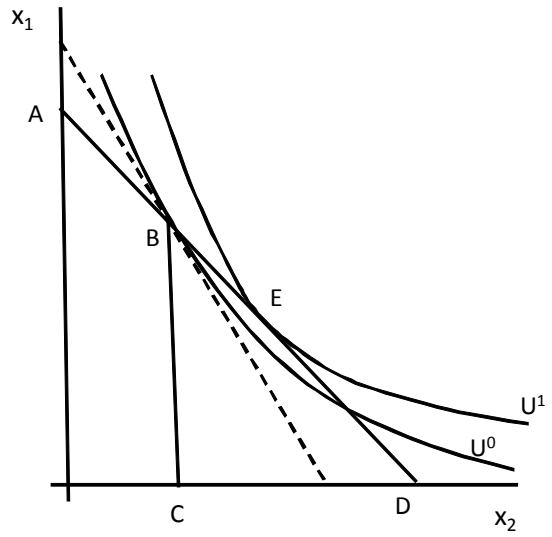

Figure 2: A Rationed Good

3 A Simple Model

This section illustrates the steps needed to calculate the welfare change from a new good, using a very simple and tractable form of direct utility function based on the Cobb-Douglas form.[3]

3.1 The Direct Utility Function

Suppose that initially the individual can consume only good 1, and has a budget of m. With a price of p_1, the amount consumed is $x_1 = m/p_1$. Then a second good is introduced, utility from consumption of two goods, x_1 and x_2, is:

$$U = x_1^{\alpha} (x_2 + 1)^{\beta} \tag{1}$$

where $\alpha + \beta = 1$. Hence in the initial situation, when $x_2 = 0$, utility is $U = x_1^{\alpha}$. This case corresponds to one in which the two goods are assumed to exist but there is a virtual price of the second good, p_2^*, for which optimal

[3]In the standard Cobb-Douglas form, it is required to consume positive amounts of both goods to obtain utility: indifference curves do not intersect the axes. Hence a slight modification is needed.

consumption of the second good is zero. This corresponds to the point A in Figure 1, where the highest indifference curve is tangential to the virtual budget line. The welfare gain from the invention or introduction of the second good can thus be represented as the gain arising from a reduction in the price to $p_2 < p_2^*$. To obtain an expression for this gain, it is first necessary to derive the expenditure function, expressing the minimum expenditure required to attain a given utility level for a specified set of prices.

First solve for optimal values. Form the Lagrangean:

$$L = U + \lambda \left(m - p_1 x_1 - p_2 x_2 \right) \tag{2}$$

The first-order conditions for a maximum are:

$$\frac{\partial L}{\partial x_1} = \frac{\alpha U}{x_1} - \lambda p_1 = 0 \tag{3}$$

and:

$$\frac{\partial L}{\partial x_2} = \frac{\alpha U}{x_2 + 1} - \lambda p_2 = 0 \tag{4}$$

Adding (3) and (4) and solving for λ (remembering that $\alpha + \beta = 1$)gives:

$$\lambda = \frac{U}{m + p_2} \tag{5}$$

and subsituting back into the first-order conditions in turn gives optimal solutions of:

$$x_1 = \frac{\alpha \left(m + p_2 \right)}{p_1} \tag{6}$$

and:

$$x_2 = \frac{\beta m}{p_2} - \alpha \tag{7}$$

Hence the virtual price in the initial situation is given by:

$$p_2^* = \frac{\beta m}{\alpha} \tag{8}$$

for which $x_1 = m/p_1$ and $x_2 = 0$.

3.2 The Expenditure Function

The indirect utility function, V, is obtained by substituting (6) and (7) into (1) to give:

$$V = (m + p_2) \left(\frac{\alpha}{p_1}\right)^{\alpha} \left(\frac{\beta}{p_2}\right)^{\beta} \tag{9}$$

Hence the expenditure function is given by:

$$E(p, U) = U \left(\frac{p_1}{\alpha}\right)^{\alpha} \left(\frac{p_2}{\beta}\right)^{\beta} - p_2 \tag{10}$$

This can be written as:

$$E(p, U) = UB - p_2 \tag{11}$$

where B is a weighted geometric mean of prices.

3.3 The Equivalent Variation

Using superscripts 0 and 1 to indicate initial values and those after the introduction of the new good, the equivalent variation is given by:

$$EV = E\left(p^1, U^1\right) - E\left(p^0, U^1\right) \tag{12}$$

This measures the change in expenditure along the new indifference curve (attained after the introduction of good 2) as a result of the price change.[4] The definition in (12) follows that generally used in the public finance literature, so that an increase in a price leads to a reduction in welfare, measured as a positive value of EV. Hence it is important to remember that the present context involves a fall in a price and hence a gain in utility, reflected in a negative value of the equivalent variation.[5]

Substitution using (11) gives:

$$EV = m - U^1 B^0 + p_2^* \tag{13}$$

[4] In cases where total expenditure (including virtual total expenditure) changes, it is only necessary to add $m^0 - m^1$ to the above expression for EV.

[5] The negative of EV is in fact the compensating variation, CV, for a price change in the opposite direction. Hence $|EV|$ in this context is the compensation needed, in the new situation, to return to the virtual price, p_2^*.

Using $U^1 = (m + p_2^1)/B^1$ this becomes:

$$EV = (m + p_2^*) - \left(m + p_2^1\right)\left(\frac{B^0}{B^1}\right) \tag{14}$$

with:

$$\frac{B^0}{B^1} = \left(\frac{p_1^0}{p_1^1}\right)^{\alpha}\left(\frac{p_2^*}{p_2^1}\right)^{\beta} \tag{15}$$

which is a weighted geometric mean of price relatives. This has the advantage that absolute prices are not required.

3.4 Money Metric Utility

Money metric utility, m_E, is defined as the total expenditure, at some 'reference set of prices', which gives the same utility as the actual total expenditure. Clearly, if the initial prices are chosen as the reference prices (p_1^0 and $p_2^0 = p_2^*$), the initial money metric utility is simply $m_E^0 = m$, and after the introduction of the second good:

$$m_E^1 = m - EV \tag{16}$$

The proportional change in money metric utility is thus conveniently given simply by EV/m. Writing $p_2^1 = p_2^*(1 + \dot{p}_2)$, the equivalent variation in (14) can be written as:

$$EV = (m + p_2^*)\left(1 - \frac{B^0}{B^1}\right) - p_2^*\dot{p}_2\left(\frac{B^0}{B^1}\right) \tag{17}$$

But from (8) the virtual price is $p_2^* = (\beta/\alpha)\, m$, and substitution gives:

$$\frac{EV}{m} = 1 - \frac{B^0}{B^1} + \frac{\beta}{\alpha}\left\{1 - \left(\frac{B^0}{B^1}\right)(1 + \dot{p}_2)\right\} \tag{18}$$

In the present context, only the price of the new good changes, so that $\dot{p}_1 = 0$ and it can be seen that:

$$\frac{B^0}{B^1} = \left(\frac{1}{1 + \dot{p}_2}\right)^{\beta} \tag{19}$$

Hence:

$$\frac{EV}{m} = \frac{1}{\alpha} - \left(\frac{1}{1 + \dot{p}_2}\right)^{\beta}\left\{1 + \frac{\beta}{\alpha}(1 + \dot{p}_2)\right\} \tag{20}$$

In this context, the price of good 2 falls from its virtual price of p_2^*, so that $\dot{p} < 0$, and consequently $EV < 0$, representing a welfare gain. The welfare gain depends on the three terms α, β and $\dot{p}_2$. Clearly, when $\dot{p}_2 = 0$, this reduces to $EV/m = 0$.

It is also convenient to normalise the price of good 1, so that $p_1^0 = p_1^1 = 1$. Initially all expenditure is on this good, so that $x_1^0 = m$, and it can be seen that after the introduction of the new good, the proportion of expenditure on that good becomes:

$$\frac{p_2^1 x_2^1}{m} = -\beta \dot{p}_2 \tag{21}$$

For example, suppose $\alpha = 0.7$ and $\beta = 0.3$. Suppose $\dot{p}_2 = -0.2$, so that the price effectively falls from its virtual price by 20 per cent, the value of money metric utility increases by 0.73 per cent, and 6 per cent of total expenditure is devoted to good 2. If the price reduction is instead $\dot{p}_2 = -0.4$, 12 per cent of expenditure is spent on good 2 and money metric utility increases by 3.68 per cent.

4 Conclusions

This paper has given a simple illustration of the calculation of a change in money metric utility arising from the introduction of a new good. The crucial ingredient of the approach is the recognition that a new good can be modelled as the equivalent of a price change, using the concept of a virtual price and virtual budget constraint.

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