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# On trend robustness and end-point issues for New Zealand's stylised business cycle facts

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## Abstract

We present new empirical evidence on trend robustness and end-point issues, utilising the macroeconomic data set investigated in McKelvie and Hall (2012). We consider the relative merits of non-robust Hodrick-Prescott (HP) and robust loess (LOCAL regrESSion) trend filtering methods, and assess the sensitivity of HP1600 stylised facts to (i) the considerable “supply shock” deviations from trend associated with New Zealand's 1992 power crisis, and (ii) an alternative HP100 specification and the loess approach. On end-point issues, we assess value-added from the use of seven-point triangular moving average and HP1600 filters, relative to insights from a 21-quarter uniform moving average filter.

**JEL Classification:** E32, C54

**Keywords:** trend robustness; end-point issues; growth cycle analysis; stylised business cycle facts; New Zealand

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# On trend robustness and end-point issues for New Zealand's stylised business cycle facts

## 1 Introduction

In assessing stylised facts for New Zealand's business cycles, it has been traditional (i) to use well-established univariate and bivariate growth cycle methodology to quantify volatility, persistence and co-movements of aggregate output with the economy's key macroeconomic variables, and (ii) to illustrate the variability of these measures over time using techniques such as 21-quarter moving average filters. On the former, see McKelvie and Hall (McKH) (2012), McCaw (2007), Hall, Kim and Buckle (HKB) (1998) and Kim, Buckle and Hall (KBH) (1994); and on the latter, see Magrini, Gerolimetto and Duran (2013) for U.S. states, and McKH, HKB and KBH for New Zealand. However, it is also well-known that empirical results from these methods can be sensitive to alternative trend filtering methods (e.g. Canova (1994, 1998)), and that end-point problems associated with 21-quarter and other types of moving average can limit the usefulness of movements-over-time measures for forecasting and policy purposes.

In this paper, we present new empirical evidence on both trend robustness and end-point issues, utilising the data set of New Zealand macroeconomic variables investigated in McKH for the period 1987q2 to 2010q4.

We consider the relative merits of non-robust Hodrick-Prescott (HP) (1997) and robust loess (LOcal regrESSion) trend filtering methods<sup>1</sup>, and assess the sensitivity of HP1600 stylised facts to (i) the considerable "supply shock" deviations from trend associated with New Zealand's 1992 power crisis, and (ii) an alternative HP100 specification and the loess approach.

On end-point issues<sup>2</sup>, we assess value-added from the use of seven-point triangular moving average and HP1600 filters, relative to insights from a 21-quarter uniform moving average filter.

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<sup>1</sup> Alternative trend filtering/detrending methods were considered but not progressed in this work. In some cases this was because the resulting series have not led to materially different results for New Zealand or Australian stylised business cycle facts or turning points: e.g. Harvey and Jaeger (1993) structural time-series-based classical turning points for New Zealand in Kim, Buckle and Hall (1995) and stylised business cycle facts for Australia in Tawadras (2011); also Baxter and King (1999) band-pass New Zealand regional and Australasian regional growth cycle cross-correlations in Hall and McDermott (2007, 2011). A common trends approach (e.g. Kozicki (1999)), while potentially suitable for small numbers of macroeconomic time series, would not seem sufficiently suitable for the very much larger number of variables in our data set. Ng and Wright (2013) have recently updated U.S. business cycle facts in the context of the *Great Recession* and recessions with financial origins using factor analysis, but this approach was considered beyond the scope of this paper.

<sup>2</sup> End-point issues could be addressed further by adding forecast observations to the sample, prior to applying trend filters (i.e. so-called "forecast extension"). This aspect has not been addressed here, but could be the subject of further work.

The methodology for our trend filters, and for our analysis of trend deviations and associated cross-correlations is described in Section 2. Our empirical results on trend filter robustness are reported in Section 3. Empirical results for end-point issues are then assessed in Section 4. Section 5 concludes.

## 2 Methodology

To maintain consistency with previous studies for New Zealand’s stylised business cycle facts, we have adopted growth cycle rather than classical cycle methodology. Our trend filtering methodology is set out in section 2.1, and our methodology for analysis of trend deviations and their cross correlations is in section 2.2.

### 2.1 Trend filters

Here we are concerned with the additive decomposition of a non-seasonal quarterly macroeconomic time series  $x_t$  into an unobserved or hidden trend  $g_t$  and its deviation from the trend  $d_t$  so that

$$x_t = g_t + d_t.$$

The decomposition and its conceptual components are identified by assuming that  $g_t$  is smooth, yet follows the secular general movement of the time series concerned, whereas  $d_t$  reflects shorter-term fluctuations and cyclical behaviour not accounted for by the trend. Typically  $g_t$  is estimated by a linear trend filter of the form

$$\hat{g}_t = \sum_s w_t(s) x_{t-s}$$

where the  $w_t(s)$  are given filter weights which can be time-varying or time invariant. In this study we have restricted attention to the Hodrick-Prescott filter (Hodrick and Prescott, 1997), the *loess* filter (Cleveland et al., 1992) and simple moving average trend filters which are all linear filters of the general form given above.

The **Hodrick-Prescott filter** is widely used as a general-purpose empirical trend filter for quarterly non-seasonal macroeconomic time series. Its original purpose was to decompose such time series into a growth component (trend) and cyclical component (trend deviation), but its general utility and applicability has seen it widely used in many different contexts. Here  $\hat{g}_t$  minimises the criterion

$$F + \lambda S = \sum_t (x_t - \hat{g}_t)^2 + \lambda \sum_t (\Delta^2 \hat{g}_t)^2$$

where  $\Delta$  is the first difference operator  $\Delta x_t = x_t - x_{t-1}$  and  $\lambda$  is a trade-off parameter balancing the fidelity  $F$  of  $\hat{g}_t$  to the data  $x_t$  with the smoothness  $S$  of  $\hat{g}_t$ . The smaller  $F$  the

closer  $\hat{g}_t$  follows the data, and the smaller  $S$  the closer  $\Delta^2 \hat{g}_t$  is to zero and the closer  $\hat{g}_t$  is to a simple linear trend. In most quarterly applications the choice of  $\lambda$  is  $\lambda = 1600$  (the **standard** Hodrick-Prescott filter), but other choices are possible, depending on the balance of smoothness and fidelity required. In this study we consider the standard Hodrick-Prescott filter as well as one with  $\lambda = 100$  which has higher fidelity, but is not as smooth as the standard Hodrick-Prescott filter.

The Hodrick-Prescott filter has its origins in the graduation of life tables and the pioneering work of Whittaker (1923). More recently it has been set in the context of structural time series models (see Harvey, 1989, and Durbin and Koopman, 2001, for example) where it can be regarded as the optimal predictor of  $g_t$  for the model where  $\Delta^2 g_t$  and  $d_t$  follow independent Gaussian white noise processes with  $\lambda$  given by the ratio of the two variances. In such a context  $\lambda$  can be estimated by techniques such as maximum likelihood (see Harvey and Jaeger, 1993, in particular). An advantage of such an approach is that optimal predictors of future values or missing values of  $g_t$  are also available. In this study we have chosen to regard the Hodrick-Prescott filter as an empirical trend filter which can, if necessary, be tuned by varying the value of  $\lambda$ .

The **loess filter** uses moving time windows within each of which a linear (or polynomial) time trend is fitted by weighted least squares and the fitted value in the centre of the window taken as the corresponding estimate of  $g_t$ . This approach has its origins in the seminal work of Macaulay (1931). In the linear case used here and for windows of length  $2n+1$  time points, this amounts to estimating  $g_t$  by  $\hat{g}_t = \hat{\alpha}$  where  $\hat{\alpha}, \hat{\beta}$  are the values of  $\alpha, \beta$  that minimise the weighted least squares criterion

$$\sum_{s=-n}^n W(s/n)(x_{t+s} - \alpha - \beta s)^2$$

where the weight function  $W(s)$  is the tri-cube function  $(1 - |s|)^3$ . The shape of  $W(s)$  as  $|s|$  approaches 1 makes it appropriate for smoothing, but the tri-cube function has other advantages (see Cleveland and Devlin, 1988, for example). For the first and last windows, loess uses the locally fitted trend to provide estimates of  $g_t$  over the half windows at either end of the series. It also takes advantage of some robust fitting procedures. Although loess has the advantage of providing trend estimates over the entire time range, there can sometimes be issues with its trend estimates at the ends of the time series (see Gray and Thomson, 1990). A variety of choices of window length and local trend polynomial are available. In this study we have selected a loess filter which fits local linear trends and uses moving windows of length 11 ( $n=5$ )<sup>3</sup>.

The centred symmetric **moving average filter** given by

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<sup>3</sup> This particular loess filter was chosen as a reasonable match to the HP100 filter, although not quite as smooth.

$$\hat{g}_t = \sum_{s=-n}^n w(s)x_{t-s} \quad (w(s) > 0, w(s) = w(-s))$$

provides a simple, readily understood, estimate of  $g_t$ . As with other trend filters, the weights  $w(s)$  can be selected to give reasonable fidelity and smoothness properties. An example is the centred triangular moving average of length  $2n+1$  where  $w(s) = c(n+1-|s|)$  and the constant  $c$  is chosen so that the  $w(s)$  sum to 1. This particular filter can be built up from sequential uniform moving averages of length  $n+1$  which can be useful for eliminating any seasonal cycles of length  $n+1$ . Here we use the 7 point ( $n=3$ ) triangular and 21 point ( $n=10$ ) uniform moving average filters. The 7 point filter has higher fidelity than a 21 point filter, but is not nearly as smooth. On the other hand, the 7 point filter provides more information near the ends of the series and eliminates any residual seasonality left in the seasonally adjusted quarterly data used. Both are readily understood and provide unambiguous trend estimates in the body of the series.

In sections 3 and 4, we consider the relative merits of the HP1600, HP100 and loess trend filtering methods, in the context of their fidelity versus smoothness. We also assess in those sections, in the context of end-point issues, the relative value added from our 7 point triangular and 21 point uniform moving averages, and from an HP1600 filter.

## 2.2 Analysis of trend deviations and their cross-correlations

The estimated trend deviations

$$\hat{d}_t = x_t - \hat{g}_t$$

are assumed to be approximately stationary and vary about a long-term mean of zero. To provide a graphical check for time-varying volatility, a smoothed estimate of the squared deviations  $\hat{d}_t^2$  can be obtained by using a suitable trend filter such as one of those given in the previous section. Taking square roots yields

$$s_t = \sqrt{\sum_s w_t(s)(x_t - \hat{g}_t)^2}$$

which is a time-varying estimate of the standard deviation or volatility. Here the weights  $w_t(s)$  relate to the trend filter chosen to smooth the squared deviations  $\hat{d}_t^2$  which can be different from the trend filter used to estimate  $g_t$ . Any marked departure of this quantity from the overall standard deviation of the  $\hat{d}_t$  is evidence of time-varying volatility.

If the trend deviations  $\hat{d}_t$  have constant standard deviation  $\sigma$  in addition to being zero-mean stationary, then a simple robust estimate of  $\sigma$  is given by the mean absolute deviation (*mad*)

$$s_{mad} = \frac{c}{T} \sum_{t=1}^T |\hat{d}_t| \quad (c = \sqrt{\pi/2})$$

where the constant  $c$  has been chosen to make  $s_{mad}$  an unbiased estimator of  $\sigma$  in the case where the  $\hat{d}_t$  are Gaussian. This estimator is less sensitive to the influence of large deviations than the more conventional root-mean-square (*rms*) estimator which estimates  $\sigma$  as the square root of the average squared deviation. Furthermore,  $s_{mad}$  is directly proportional to a simple average of the autocorrelated  $|\hat{d}_t|$  and, as a consequence, the standard error of  $s_{mad}$  is readily determined by standard time series limit theorems. In the cases considered here it was found sufficient to approximate the autocorrelation structure of the  $|\hat{d}_t|$  by a first-order autoregressive process.

Now suppose that another non-seasonal macroeconomic time series  $y_t$  is available with trend  $h_t$ , trend deviation  $e_t$  and decomposition

$$y_t = h_t + e_t.$$

As before, the trend  $h_t$  can be estimated using a suitable trend filter  $\hat{h}_t$  and the trend deviations

$$\hat{e}_t = y_t - \hat{h}_t$$

determined. In addition to examining the time-varying volatility of the trend deviations  $\hat{e}_t$  it is of interest to construct estimates of time-varying contemporaneous correlation between the  $\hat{d}_t$ , the estimated trend deviation for  $x_t$ , and  $\hat{e}_t$ , the estimated trend deviation for  $y_t$ . These time-varying estimates can also be checked for any departure from the overall (time-invariant) correlation with significant departures being of particular interest.

To construct a suitable estimate of time-varying contemporaneous correlation we first form the smoothed cross-products of the trend deviations given by

$$c_t^{dd} = \sum_s w_t(s) \hat{d}_t^2, \quad c_t^{de} = \sum_s w_t(s) \hat{d}_t \hat{e}_t, \quad c_t^{ee} = \sum_s w_t(s) \hat{e}_t^2$$

where the trend filter used is the *same for all three smoothed cross-products* with common weights  $w_t(s)$ . This common filter can be different from either of the filters used for estimating  $g_t$  and  $h_t$ . Given these quantities, a suitable estimate of time-varying correlation between the two sets of trend deviations is now given by

$$r_t = \frac{c_t^{de}}{\sqrt{c_t^{dd} c_t^{ee}}}.$$

When the filter weights  $w_t(s)$  are non-negative, a simple application of the Cauchy-Schwarz inequality guarantees that  $r_t$  will always be bounded between -1 and 1 as expected. For the most part this condition is met, especially in the body of the series and always for the centred



symmetric moving average filter. However, near the ends of the series, the Hodrick-Prescott and loess filters can have negative weights which may occasionally yield values for  $r_t$  with absolute value greater than 1. In such cases one can infer only that the magnitude of the correlation is large and, perhaps, focus more on the values of  $r_t$  determined using the triangular 7 point moving average or equivalent. In practice, such anomalies will be relatively rare and  $r_t$  should provide a useful graphical estimate of time varying correlation between the two sets of trend deviations.

If the volatility of the trend deviations  $\hat{d}_t$  and  $\hat{e}_t$  can be assumed to be constant then a simple (and conventional) estimate of the overall contemporaneous correlation is given by

$$\hat{\rho} = \frac{1}{T} \sum_{t=1}^T \frac{\hat{d}_t}{s_d} \frac{\hat{e}_t}{s_e}$$

where  $s_d$ ,  $s_e$  are the usual estimates of the standard deviations of  $\hat{d}_t$ ,  $\hat{e}_t$  respectively and  $T$  denotes the number of cross-products. While formulae are available for the standard error of  $\hat{\rho}$ , especially in the case where  $\hat{d}_t$ ,  $\hat{e}_t$  are jointly Gaussian or fourth-order stationary, these formulae are relatively complicated and involve a more comprehensive analysis of the joint properties of  $\hat{d}_t$  and  $\hat{e}_t$ . Here we adopt a simpler, consistent, but less efficient, strategy and assume only that the product  $\hat{d}_t \hat{e}_t$  can be regarded as a weakly stationary time series with second-order properties that are well-approximated by a finite moving average process. Then, the standard error of  $\hat{\rho}$  can be determined using conventional techniques such as the Newey-West estimator (Newey and West, 1987). For this study there was little serial correlation in the time series of cross-products  $\hat{d}_t \hat{e}_t$  and we chose to estimate the standard error of  $\hat{\rho}$  by fitting a moving average process of order one (in effect, a Newey-West estimator with lag 1). Once obtained,  $\hat{\rho}$  and its confidence bounds can be superimposed on the plot of the estimate of the time-varying correlation to provide an informal test of whether departures of the time-varying correlation from the constant correlation are significant.

The above correlation analysis is not restricted to contemporaneous correlation (lag 0), but can also be applied to estimating cross-correlations at any other lag of interest. This is because the contemporaneous correlation will not always be the most informative. Here, as was established in McKH (2012), it is sufficient to compute co-movement of each series with real GDP as far as fifth-order leads and lags. Specifically, the cyclical component of the candidate variable at time  $t + k$  (represented in Tables 1 and 3 by  $x_{t+k}$ ) is associated with the cyclical component of real GDP at time  $t$ , for  $-5 \leq k \leq 5$ . Under this approach, a maximum correlation at, for example,  $k = 3$  indicates that the cyclical component of the candidate variable tends to lag the aggregate business cycle by three quarters. The contemporaneous correlation coefficients may be described as either procyclical,

countercyclical or acyclical.<sup>4</sup>

### 3 Empirical results: on trend filter robustness

Our raw data series have been sourced from Statistics New Zealand (SNZ), the Reserve Bank of New Zealand (RBNZ), and the New Zealand Treasury, as documented in McKH (2012, Appendix C). Additional to series from that data set, our CPI tradables and non-tradables series for the sample period 1988(1) to 2010(4) are from the RBNZ.

Series were seasonally adjusted as required and then log transformed, with the exception of those containing negative observations (e.g. net exports) or those already expressed as a percentage (e.g. interest rates).

All computations and graphical analysis were carried out in the R statistical environment (R Development Core Team, 2004). The HP filter implementation used is `hpfiler()` given in the R package `mFilter` (Balcilar, 2007).

#### 3.1 Real gdpe trends

Figure 1 shows movements in the logarithm of New Zealand's real expenditure-based Gross Domestic Product (gdpe), for the period 1987q2 to 2010q4, together with the HP1600, HP100 and loess trend filter series. Over this period, gdpe has grown considerably from its original level, and fluctuations in the series over time can be captured by a wide range of measures for New Zealand's classical business cycle recessions and recoveries<sup>5</sup>, and for New Zealand's growth cycle stylised facts.

The most sizable fluctuations can be associated with macroeconomic events of historical significance, and for our sample period such events include: the short two-quarter recession of 1991q1 and 1991q2 experienced simultaneously with the U.S. and Australia; the relatively modest three-quarter slowdown from 1997q3 to 1998q1 associated with the Asian financial crisis and New Zealand's successive summers of drought; and most recently, the more substantial six-quarter recession from 2008q1 through to 2009q2 which followed the onset of the global financial crisis.

One particularly striking feature from Figure 1 is the prolonged ten-quarter period of real gdpe activity below HP1600 trend, during the period 1991q1 through to 1993q2. This was associated first with the above-mentioned two-quarter 1991q1 and 1991q2 classical recession, and subsequently with a "supply shock"-related interruption to New Zealand's recovery path.

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<sup>4</sup> A variable is procyclical when its deviations from trend are *contemporaneously* correlated with those of output in a positive fashion; countercyclical when its deviations from trend are *contemporaneously* correlated with those of output in a negative fashion; and acyclical when its deviations from trend exhibit a *contemporaneous* correlation with output that is close to zero.

<sup>5</sup> Measures for New Zealand's classical business cycle recessions and recoveries have recently been documented in Hall and McDermott (2014). Descriptive accounts of New Zealand's business cycles over the period 1998 to 2011 can be found in Chetwin (2012), and of past recessions are available in Reddell and Sleeman (2008). Summary statistics for New Zealand's post-war business cycles can be found in Hall and McDermott (2009).

Accordingly, before assessing the alternative trend paths associated with our HP1600, HP100 and loess filters, we consider the extent to which key HP1600 real business cycle facts may have been sensitive to this supply shock episode associated with what has been termed the 1992 power crisis.

### **3.2 Direct adjustment of “supply shock” observations**

New Zealand experienced a period of drought during the early part of 1992, with the result that hydro-power generation of electricity was severely affected. This (along with other influences) led to real gdpe observations for 1992q3 through to 1993q1 being significantly lower than would otherwise have been the case.

We assessed the sensitivity of key HP1600 stylised facts to this supply shock, by directly adjusting real gdpe, real private consumption (cons), and real non-durables consumption (consnd) for assumed direct and indirect effects of the shock. For these three variables, the direct adjustment was designed to achieve a smooth path back to trend over the six quarters 1992q3 to 1993q4.

Results presented in Table 1 for the supply shock versus no supply shock series are confined to those real sector variables most likely to have been directly materially affected. Not surprisingly, the absolute volatilities for gdpe, private consumption and non-durables consumption were reduced, though only gdpe’s reduction from 1.39% to 1.24% can be considered material. Persistence was not noticeably different for any of the three variables. Contemporaneous cross correlations with gdpe, for each of private consumption, non-durables consumption, durables consumption (consd), non-residential investment (invnonres), and employment (emp), remained statistically significant and not materially lower in magnitude. The cross correlation with residential investment (invres) was unchanged.

These results, investigating the impact on key real variable stylised facts from a significant supply shock over a short six-quarter period, are not sufficiently different to call into question the property of HP1600-based “real variable regularity”.

### **3.3 Alternative trend filters**

So, how sensitive are HP1600 stylised business cycle facts to our two other trend filtering methods? The results we present and comment on below are for a subset of variables only.<sup>6</sup>

#### **3.3.1 Trends for variables other than real gdpe**

As was the case for the real gdpe paths (Figure 1), it is clear from Figures 2a-2i for real private consumption, real residential investment, real government consumption (gconfa<sup>7</sup>),

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<sup>6</sup> Results for the remaining variables are available on request from the corresponding author.

<sup>7</sup> The suffix ‘fa’ for the government consumption, net exports share and imports of goods and services variables denotes that the value of frigates purchases has been subtracted for the observations recorded for 1997q2 and 1999q4.

real net exports share (nxshrfa), real imports of goods and services (mtotfa), real 90-day interest rates (ninetyr), gross government debt to gdp (gdebtgdp), the unemployment percentage (unemp) and labour productivity (labpy) that HP1600 trends show least fidelity and loess trends show greatest fidelity to the defined data series. It follows that HP1600 trends are the smoothest (relative to a linear trend) and loess the least smooth.

It is also clear from the trend movements for *gdpe* and these other variables that the HP100 and loess filtered variables follow each other much more closely than those from an HP1600 filter. This illustrates that the issue of fidelity versus smoothness is an empirically important one<sup>8</sup>.

### **3.3.2 Differences for volatility**

Given the above evidence on fidelity versus smoothness, it is not surprising that the sample volatilities for all variables are greatest for HP1600 filtered variables and least for those which are loess filtered. For example, *gdpe* volatility is 1.39% for HP1600, and 0.65% for loess (Table 2).

There is no consistent pattern across filtering methods for volatilities relative to *gdpe*.

The additionally important issue of robustness of movement over time in the time-varying estimates for these variables is considered below in section 4.1.

### **3.3.3 Differences for persistence**

Again not surprisingly, persistence is consistently higher for all HP1600 filtered variables, relative to persistence for the corresponding HP100 and loess filtered variables (Table 2)<sup>9</sup>. This reflects the fact that as trends become smoother, deviations become more persistent.

More surprising, though, is the extent to which there is considerably less persistence from loess filtered variables than from their HP100 filtered counterparts. This is associated with the somewhat more pronounced smoothness of HP100 trends relative to those from loess filtered variables.

### **3.3.4 Differences for cross correlations**

The robustness of our cross-correlation associations is assessed by considering the greatest contemporaneous or lagged cross correlation between real *gdpe* and each of the other variables (Table 3).

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<sup>8</sup> The balance of smoothness relative to fidelity in the context of business cycle properties and stylised facts remains an important topic for further research.

<sup>9</sup> Our HP1600 volatilities and our persistence values being greater than those for our HP100 filtered variables is consistent with the Hodrick and Prescott results (1997, Tables 1 and Appendix Table A1) which show that for U.S. GNP, as  $\lambda$  increases from 400 to infinity standard deviations increase and persistence is greater.

The evidence is clearest for those variables whose greatest cross correlation is **contemporaneous**. Here, for real private consumption and investment expenditure variables and their major components, for gross fixed capital formation, for labour productivity and for real unit labour costs (rulc), cross correlations remain statistically significant and of consistent sign across all three filtering methods, albeit with decreasing absolute magnitudes.<sup>10</sup> For example, the procyclical cross correlations for residential investment decline from 0.73 for HP1600 to 0.47 for loess, and countercyclical real unit labour cost correlations also fall in absolute terms (from -0.62 to -0.38). The procyclical labour productivity correlations provide a counterexample to the declining pattern, taking successive values of 0.54, 0.64 and 0.57.

Overall then, for these contemporaneously correlated variables, it seems reasonable to conclude that “real variable regularity” has been maintained across the three trend filters.

However, the degree of robustness of results is not as clear cut for variables whose greatest cross correlation is **non-contemporaneous**.

For four of these variables, signs, statistical significance and lag length have been maintained, and absolute magnitudes have declined across filter method. The four variables are: real government consumption expenditure, significantly lagging gdpe by 5 quarters; net exports share, significantly negatively associated with gdpe and (through movements in its real imports component) lagging by two quarters; real ninety-day interest rates, significantly positively associated and lagging by three quarters; and real labour costs (rlc), significantly negatively associated and lagging by five quarters.

For nine further variables, real total government expenditure, real government investment expenditure, net government expenditure to GDP, gross government debt to GDP, imports of goods and services, terms of trade, employment, unemployment and real average hourly earnings, sign and lag length remained consistent but magnitudes declined to the point where statistical significance could not be maintained for loess and/or HP100 filtered variables.

Then, for a third small group of variables, real TWI, CPI and its tradable and non-tradable components, and real M3, the signs, lag lengths and statistical significance for HP100 and loess filtered variables are such that the cross correlations from these two filters are not sufficiently robust.

Movements over time for the cross correlations considered sufficiently robust are considered below in section 4.2.

#### **4 Empirical results: movements over time and end-point issues**

Pros and cons for use of 21 quarter uniform moving average, seven point triangular moving average and HP1600 filters were established above in section 2. We now assess their

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<sup>10</sup> Note, however, that the cross correlations for non-residential investment are not statistically significant at the 5% level for HP100 and loess filtered variables (Table 3).

relative empirical usefulness, in the context of movements over time in volatilities and cross correlations, and the relative strengths and weaknesses of the moving average filters in an end-point context.

#### 4.1 On the variability over time of volatilities

Three of our variables are singled out for illustration (Figures 3a-3c<sup>11</sup>): *gdpe*, with HP1600 volatility of 1.39%; residential investment, with a high HP1600 volatility of 8.67%; and real imports of goods and services, also with a relatively high HP1600 volatility of 4.90%.

Recall too, that the absolute values of these volatilities are highest for the HP1600 filter and lowest for the loess filter, e.g. for *gdpe*, 1.39% for HP1600, 0.86% for HP100 and 0.65% for loess.

For variables filtered by all three methods, the movements over time shown by the HP1600 and 21 quarter uniform filters are very similar. For example, for these two moving average filters, the volatility of *gdpe* is shown in all three panels of Figure 3a to have risen to a peak around 1993, declined to a low around 2004, lifted again to a still below average peak around 2007, and subsequently trended lower.

But by way of contrast, and consistent with what the seven point moving average filter is designed to do, this moving average filter produces movements which show the greatest variability over time. Also, towards both ends of the sample period, this filter provides guidance for a greater number of quarters than does the 21 quarter filter, and lesser but more pronounced guidance than the HP1600 filter. Seven point triangular movements for HP100 and loess filtered data provide very similar information, but display peak and trough volatilities which at times are somewhat different from those associated with HP1600 filtered data. For example, for residential investment, the peak volatility for HP1600 seven point triangular is reached around 2001, whereas the peak for HP100 and loess filtered data arrives around 2000.

Overall, though, movements of these volatilities over time around their sample averages provide empirically valuable information. For *gdpe* for example, HP1600 smoothed standard deviations not only display trend increases and decreases over time which are generally within plus and minus two standard deviation limits when assuming constant volatility, but perhaps more importantly also highlight those periods for which the two-standard-deviation bounds are exceeded (Figure 3a).

#### 4.2 On the variability over time of cross correlations

We illustrate and provide comment for three contemporaneously correlated variables (private consumption, residential investment and labour productivity), and five variables for

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<sup>11</sup> In Figures 3a-3c, the black line depicts the trend deviation, the black dashed line is our robustly estimated standard deviation (the mean absolute deviation, *mad*), the red dashed line is the traditionally reported time-invariant standard deviation (the root-mean-square deviation, *rms*), and the grey-shaded rectangular area covers two standard deviations around the robustly estimated standard deviation.

which their greatest cross correlation is non-contemporaneous (net exports share lagging gdpe by two quarters, ninety-day interest rates lagging three quarters, government consumption lagging five quarters, gross government debt to GDP lagging 3 quarters and unemployment lagging one quarter). (Figures 4a-4c and 5a-5e respectively<sup>12</sup>).

From the three panels for each variable, we first consider movements over time around trend filtered time-varying maximum cross correlations. For example, the sample period contemporaneous cross correlation for private consumption is 0.75 for HP1600 filtered data, 0.58 for the HP100 series, and 0.43 for the loess series (Table 3), and the movements over time for these and their underlying standardised trend-deviation cross-products are represented in Figure 4a for each of our three moving average methods.

For all three of our **contemporaneously correlated variables**, and as was the case for our moving volatilities, the HP1600 and the 21 quarter moving average method provide the smoothest paths and the seven point triangular moving average method produces a path closest to that of the volatile cross-product observations.

However, greater attention to detail is required when deriving messages on high and low correlations around the sample average<sup>13</sup>. For example, again for private consumption, the lowest cross correlation from HP1600 smoothing occurs around the year 1999 for the HP1600 filtered series, but around 1995 for the HP100 and loess filtered series. Similarly, while the lowest cross correlation from seven point moving averaging is around 1994 for HP100 and loess data, the lowest for HP1600 data is in the vicinity of 1998/99<sup>14</sup>.

Secondly, and more specifically in an end-point context, while adoption of seven point triangular moving averages leads to the loss of three observations at the ends of each series relative to observations produced from HP1600 smoothing, it provides better guidance on movements in cross correlations over shorter-term intervals of time. Moreover, this guidance could be further enhanced if this moving average method were used in conjunction with (say) two years of quarterly forecast observations.

Broadly similar conclusions can be reached when one examines movements over time for the other two contemporaneously cross correlated variables, residential investment and labour productivity (Figures 4b and 4c).

Specific conclusions for variables whose greatest cross correlations are **non-contemporaneous** will vary according to which variable is of interest (e.g. see Figures 5a-5e for nxshrfa lag(-2), ninetyr lag(-3), gconfa lag(-5), gdebtgdp lag(-3) and unemp lag(-1)).

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<sup>12</sup> In Figures 4a-4c and 5a-5e, the grey lines are the cross-products of the standardised trend deviations, the black dashed lines are the sample period greatest cross correlations, and the grey-shaded rectangular area covers two standard deviations around the greatest cross correlation.

<sup>13</sup> It might also be noted that, as foreshadowed in section 2.2 on methodology, use of the HP and loess filters has for some variables occasionally led to moving average cross correlation values with absolute value greater than one. This is because these filters can have negative filter weights near ends of the series.

<sup>14</sup> As was the case for the variability over time of volatilities, it is important to note here those periods around which the greatest and least cross correlations go beyond the plus and minus two standard deviation bounds, when assuming constant cross correlations.

However, the more general points made above for contemporaneous variables remain valid for variables which have credible non-contemporaneous greatest cross correlations. These points are that: the HP1600 filter and 21 quarter uniform moving average method produce similarly smooth paths, and therefore provide useful evidence for medium-term intervals of time; seven point triangular moving averaging reflects cross product movements more closely than do the other two averaging processes, and therefore contribute the more useful information for movements over shorter-term time horizons; the moving average paths for HP100 and loess filtered series are similar and can provide highs and lows for cross correlations which are somewhat different from those from HP1600 filtered series.

In summary on **end-point issues**, while the HP1600 filter provides the greatest number of movement-over-time observations, and the seven point triangular and 21 quarter uniform moving averages lose three and ten observations respectively at each end, the end of period HP1600 observations may not be sufficiently informative in some cases.

## 5 Conclusion

We present new empirical evidence on trend robustness and end-point issues, set against the HP1600 stylised business cycle facts for New Zealand reported in McKelvie and Hall (2012).

Our new evidence comes firstly from assessing results from HP1600 filtered series against those from HP100 and loess filtered series. Secondly, when evaluating volatility and cross correlation movements over time, our evidence comes from evaluating the relative merits of 21 quarter uniform and seven point triangular moving average methods, and from an HP1600 filter. End-point issues are also considered.

On **robustness**, we first evaluated the impact on key McKH real business cycle facts of the supply shock deviations from trend associated with New Zealand's 1992 power crisis. For key real sector variables, we directly adjusted relevant quarterly observations for deviations which were considered significant over a short six-quarter period. The results for volatility, persistence and cross correlations from the adjusted series were not sufficiently different from those for the unadjusted series to call into question the property of HP1600-based "real variable regularity".

Judgements on alternative trend filters can be set in the context of whether a researcher's primary purpose is better served by a trend filter which exhibits greater fidelity to the data or greater smoothness (relative to a linear time trend). For example, a trend filter with greater fidelity may be preferred for a forecasting exercise, but greater smoothness may be more helpful when one's aim is to establish robust stylised business cycle facts.

Adopting an often-used HP1600 trend filter for quarterly data will produce measures of volatility which are greater than those emanating from an HP100 or loess filter, and measures of persistence which are also greater. Statistical significance is likely to be preserved, even for the lower volatility values.



Greatest cross correlations established for HP1600 filtered series will also generally be higher than those produced from HP100 and loess filtered series. Robustness, in the sense of “real variable regularity” has been preserved for data from all three filtered series, where the real variables are *contemporaneously* correlated with real gdp. Specifically, the variables for which this real variable regularity is preserved are: total private consumption, non-durables consumption and durables consumption; gross fixed capital formation; total private investment, residential investment, and other investment; labour productivity and real unit labour costs.

However, for variables whose greatest cross correlation with real gdp is *non-contemporaneous*, and which were established as statistically significant for HP1600 cross correlations in McKH, not all relationships remained robust in sign, statistical significance and/or greatest cross correlation lag length, when HP100 and loess filtered cross correlations were computed. Within this category, robustness of sign, statistical significance and greatest cross correlation lag length is preserved for real government consumption expenditure, net exports share (through movements in its real imports component); real ninety-day interest rates, and real labour costs. The robustness of sign and greatest cross correlation lag length (but not statistical significance for loess and/or HP100 filtered variables) is preserved for nine further variables, including total government and government investment expenditure, gross government debt to GDP, imports of goods and services, employment, unemployment and real average hourly earnings.

For some variables, and especially so for their cross correlations, there is evidence of significant movement over time around sample averages. This evidence provides valuable insights as to the waxing and waning over time of volatilities and cross correlations, and also identifies periods when volatilities and cross correlations have breached two-standard-deviation-limits.

On **end-point issues**, evaluated in the context of movements over time in volatilities and greatest cross correlations, continued use of traditionally-used 21 quarter uniform (or similar) moving averages provides less valuable information than either an HP1600 filter or a seven point triangular moving average. The 21 quarter moving average provides some value for medium-term slices of time, but its loss of 10 observations at each end is a major disadvantage for nowcasting<sup>15</sup>, forecasting and policy purposes.

Using an HP1600 filter produces relatively smooth movements over time, similar to those provided from 21 quarter averaging. It has an advantage over 21 quarter averaging for broad judgements over medium-term time horizons in that it provides a greater number of credibly smooth observations, but in some cases it may also provide moving average observations at the very beginning and end of each sample which have unsatisfactory properties.

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<sup>15</sup> On nowcasting, see for example papers from the “Nowcasting with Model Combination Workshop”, 11-12 December 2008,

[http://www.rbnz.govt.nz/research\\_and\\_publications/seminars\\_and\\_workshops/december2008/3421588.html](http://www.rbnz.govt.nz/research_and_publications/seminars_and_workshops/december2008/3421588.html)

This means, especially if one's primary interest is in shorter-term moving average volatilities and cross correlations, that the use of seven point triangular moving averages could provide the best value. Only three observations are lost at each end of the sample, and this disadvantage could be lessened if this moving average method were used in conjunction with (say) up to two years of quarterly forecast observations for each variable of interest.

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**TABLE 1**  
*Cyclical Behaviour of Key Real Sector Variables*  
*Quarterly Deviations from HP1600 Trend: 1987(2) - 2010(4)*

Variable $x$	Including early 1990s "supply shock" to GDPE				No early 1990s "supply shock" to GDPE			
	Volatility SD%	Relative volatility	Persistence	Cross correlation	Volatility SD%	Relative volatility	Persistence	Cross correlation
GDP (expenditure)	1.39 (0.21)	1.00	0.76 (0.06)		1.24 (0.19)	1.00	0.74 (0.07)	
Consumption	1.50 (0.17)	1.08	0.81 (0.06)	0.75 (0.15)	1.47 (0.16)	1.19	0.81 (0.06)	0.72 (0.14)
Non-durables	1.39 (0.13)	1.00	0.65 (0.08)	0.60 (0.13)	1.36 (0.12)	1.10	0.66 (0.08)	0.55 (0.11)
Durables	3.09 (0.46)	2.22	0.76 (0.06)	0.82 (0.17)	-	2.49	-	0.76 (0.15)
Investment								
Residential	8.67 (1.02)	6.24	0.76 (0.07)	0.73 (0.13)	-	6.99	-	0.73 (0.13)
Non-residential	10.95 (1.97)	7.88	0.75 (0.07)	0.61 (0.18)	-	8.83	-	0.56 (0.17)
Other	9.14 (1.98)	6.58	0.58 (0.08)	0.57 (0.13)	-	7.37	-	0.61 (0.15)
Employment	1.34 (0.21)	0.96	0.89 (0.05)	0.56 (0.16)	-	1.08	-	0.51 (0.14)

Notes: Numbers in parentheses for volatility, persistence and cross correlations are robustly estimated standard errors  
Relative volatility is relative to GDP(expenditure) volatility; persistence is represented by first order serial correlation  
Maximum bivariate cross correlations are contemporaneous, except for employment which is at  $x_{t+2}$   
"-" denotes magnitude unchanged, as series for variable  $x$  not adjusted for supply shock

**TABLE 2**  
*Stylised Business Cycle Facts: 1987(2) - 2010(4)*  
*Comparative Volatilities (SD%), Relative Volatilities and Persistence*

Variable x\ De-trend method	Volatility (%)			Relative Volatility			Persistence		
	HP1600	HP100	loess	HP1600	HP100	loess	HP1600	HP100	loess
GDP (expenditure)	1.39 (0.21)	0.86 (0.09)	0.65 (0.07)	1.00	1.00	1.00	0.76 (0.06)	0.46 (0.09)	0.17 (0.10)
Consumption (private)	1.50 (0.17)	0.75 (0.07)	0.55 (0.06)	1.08	0.87	0.85	0.81 (0.06)	0.41 (0.09)	0.09 (0.10)
Non-durables	1.39 (0.13)	0.91 (0.09)	0.75 (0.08)	1.00	1.06	1.15	0.65 (0.08)	0.28 (0.10)	0.02 (0.10)
Durables	3.09 (0.46)	1.87 (0.22)	1.35 (0.16)	2.22	2.17	2.08	0.76 (0.06)	0.46 (0.09)	0.09 (0.10)
Gross Fixed Capital Formation	6.11 (0.81)	3.63 (0.35)	2.53 (0.25)	4.40	4.22	3.89	0.78 (0.06)	0.49 (0.09)	0.13 (0.10)
Investment (private)	7.50 (0.96)	4.71 (0.56)	3.05 (0.36)	5.40	5.48	4.69	0.76 (0.07)	0.52 (0.09)	0.13 (0.10)
Residential	8.67 (1.02)	6.06 (0.61)	3.74 (0.33)	6.24	7.05	5.75	0.76 (0.07)	0.58 (0.08)	0.18 (0.10)
Non-residential	10.95 (1.97)	6.56 (0.63)	4.93 (0.40)	7.88	7.63	7.58	0.75 (0.07)	0.32 (0.10)	-0.10 (0.10)
Other	9.14 (1.98)	6.57 (0.78)	4.91 (0.63)	6.58	7.64	7.55	0.58 (0.08)	0.34 (0.10)	0.04 (0.10)
Govt. Expenditure (total)*	2.59 (0.26)	1.85 (0.16)	1.53 (0.14)	1.86	2.15	2.35	0.54 (0.09)	0.22 (0.10)	-0.10 (0.10)
Govt. Consumption*	1.30 (0.13)	1.05 (0.09)	0.91 (0.09)	0.94	1.22	1.40	0.44 (0.09)	0.17 (0.10)	-0.07 (0.10)
Govt. Investment	10.55 (1.25)	7.39 (0.65)	5.90 (0.47)	7.59	8.59	9.08	0.61 (0.08)	0.27 (0.10)	-0.11 (0.10)
Net Govt. Exp./GDP**	1.30 (0.09)	0.97 (0.08)	0.87 (0.07)	0.94	1.13	1.34	0.40 (0.09)	0.05 (0.10)	-0.19 (0.10)
Gross Govt. Debt/GDP**	2.89 (0.68)	1.29 (0.26)	1.00 (0.22)	2.08	1.50	1.54	0.85 (0.06)	0.43 (0.09)	0.15 (0.10)
Net Exports Share*	1.54 (0.20)	1.16 (0.17)	0.79 (0.09)	1.11	1.35	1.22	0.71 (0.07)	0.55 (0.09)	0.20 (0.10)
Exports goods & services	2.37 (0.26)	2.08 (0.18)	1.76 (0.17)	1.71	2.42	2.71	0.46 (0.09)	0.26 (0.10)	0.01 (0.10)
Imports goods & services*	4.90 (0.57)	3.49 (0.48)	2.15 (0.24)	3.53	4.06	3.31	0.77 (0.07)	0.64 (0.08)	0.30 (0.10)
Terms of Trade	3.29 (0.49)	2.54 (0.37)	1.62 (0.23)	2.37	2.95	2.49	0.74 (0.07)	0.64 (0.08)	0.33 (0.10)
Real TWI	6.24 (0.93)	3.41 (0.48)	2.38 (0.32)	4.49	3.97	3.66	0.85 (0.05)	0.64 (0.08)	0.42 (0.09)
CPI***	0.90 (0.20)	0.51 (0.07)	0.34 (0.04)	0.65	0.59	0.52	0.86 (0.05)	0.64 (0.08)	0.35 (0.10)
Non-tradables***	1.09 (0.28)	0.41 (0.06)	0.32 (0.05)	0.78	0.48	0.49	0.91 (0.04)	0.55 (0.09)	0.28 (0.11)
Tradables***	1.27 (0.16)	0.74 (0.08)	0.56 (0.07)	0.91	0.86	0.86	0.79 (0.06)	0.51 (0.09)	0.22 (0.11)
Real 90-day Bank Bill****	1.10 (0.19)	0.76 (0.09)	0.51 (0.06)	0.79	0.88	0.78	0.82 (0.06)	0.67 (0.08)	0.49 (0.09)
Real M3*****	2.40 (0.35)	1.16 (0.09)	0.96 (0.06)	1.73	1.35	1.48	0.83 (0.06)	0.37 (0.10)	0.09 (0.10)
Employment	1.34 (0.21)	0.58 (0.06)	0.40 (0.03)	0.96	0.67	0.62	0.89 (0.05)	0.54 (0.09)	0.21 (0.10)
Unemployment	0.65 (0.15)	0.34 (0.05)	0.22 (0.02)	0.47	0.40	0.34	0.88 (0.05)	0.62 (0.08)	0.26 (0.10)

**TABLE 2 (continued)**  
*Stylised Business Cycle Facts: 1987(2) - 2010(4)*  
*Comparative Volatilities (SD%), Relative Volatilities and Persistence*

Variable x\ De-trend method	Volatility (%)			Relative Volatility			Persistence		
	HP1600	HP100	loess	HP1600	HP100	loess	HP1600	HP100	loess
Labour Productivity	1.09 (0.12)	0.87 (0.07)	0.66 (0.05)	0.78	1.01	1.02	0.66 (0.08)	0.49 (0.09)	0.25 (0.10)
Real Av. Hourly Earnings*****	0.86 (0.12)	0.59 (0.05)	0.48 (0.04)	0.62	0.69	0.74	0.67 (0.08)	0.31 (0.10)	0.07 (0.11)
Real Labour Cost*****	0.99 (0.11)	0.77 (0.08)	0.57 (0.05)	0.71	0.90	0.88	0.64 (0.09)	0.45 (0.11)	0.07 (0.12)
Real Unit Labour Cost*****	1.85 (0.27)	1.14 (0.13)	0.79 (0.07)	1.33	1.33	1.22	0.81 (0.07)	0.56 (0.10)	0.17 (0.12)

Notes: Numbers in parentheses for volatility and short-term persistence are robustly estimated standard errors.

Relative volatility is relative to GDP(expenditure) volatility; persistence is represented by first order serial correlation

\* SNZ National Accounts series, adjusted (as for NZ Treasury series) for frigate purchases recorded in 1997q2 and 1999q4.

The series not adjusted in this way show somewhat greater volatilities and less persistence, e.g. for HP1600 filtered series, total government expenditure and government consumption volatilities are 3.08% and 2.22% and persistences are 0.41 and 0.12; for imports of goods & services and for net exports share, the volatility magnitudes are 4.88% and 1.63%, and the persistences are 0.71 and 0.67.

\*\* NZ Treasury series

\*\*\* Sample period 1988(1)-2010(4); \*\*\*\* Sample period 1987(3)-2010(4); \*\*\*\*\* Sample period 1988(2)-2010(4)

\*\*\*\*\* Sample period 1989(1)-2010(4); \*\*\*\*\* Sample period 1992(4)-2010(4)

**TABLE 3**  
*Stylised Business Cycle Facts: 1987(2) - 2010(4)*  
*Comparative cross correlations with Real GDPE*

Variable $x \setminus$ De-trend method	Contemporaneous Cross Correlation			Most significant Non-contemporaneous Cross Correlation		
	HP1600	HP100	loess	HP1600	HP100	loess
Consumption (private)	<b>0.75 (0.15)</b>	<b>0.58 (0.13)</b>	<b>0.43 (0.15)</b>	-	-	-
Non-durables	<b>0.60 (0.13)</b>	<b>0.35 (0.11)</b>	<b>0.23 (0.09)</b>	-	-	-
Durables	<b>0.82 (0.17)</b>	<b>0.62 (0.15)</b>	<b>0.52 (0.19)</b>	-	-	-
Gross Fixed Capital Formation	<b>0.82 (0.16)</b>	<b>0.62 (0.14)</b>	<b>0.55 (0.15)</b>	-	-	-
Investment (private)	<b>0.78 (0.15)</b>	<b>0.59 (0.15)</b>	<b>0.51 (0.17)</b>	-	-	-
Residential	<b>0.73 (0.13)</b>	<b>0.64 (0.14)</b>	<b>0.47 (0.13)</b>	-	-	-
Non-residential	<b>0.61 (0.18)</b>	0.19 (0.14)	0.22 (0.14)	-	-	-
Other	<b>0.57 (0.13)</b>	<b>0.42 (0.14)</b>	<b>0.37 (0.16)</b>	-	-	-
Govt. Expenditure (total)*	0.14 (0.12)	0.00 (0.09)	0.14 (0.09)	<b>0.56 (0.13) <math>X_{t+5}</math></b>	<b>0.23 (0.11) <math>X_{t+5}</math></b>	0.23 (0.13) $X_{t+5}$
Govt. Consumption*	0.08 (0.09)	-0.00 (0.07)	0.11 (0.06)	<b>0.51 (0.14) <math>X_{t+5}</math></b>	<b>0.27 (0.11) <math>X_{t+5}</math></b>	<b>0.18 (0.09) <math>X_{t+5}</math></b>
Govt. Investment	0.17 (0.14)	0.02 (0.10)	0.11 (0.11)	<b>0.50 (0.14) <math>X_{t+5}</math></b>	0.15 (0.10) $X_{t+5}$	0.18 (0.12) $X_{t+5}$
Net Govt. Exp./GDP** (Expend. - Net Tax)	-0.51 (0.12)	-0.24 (0.10)	-0.06 (0.08)	<b>-0.52 (0.12) <math>X_{t-1}</math></b>	<b>-0.23 (0.10) <math>X_{t-1}</math></b>	-0.08 (0.11) $X_{t-1}$
Gross Govt. Debt/GDP**	-0.45 (0.15)	-0.21 (0.09)	-0.08 (0.07)	<b>-0.61 (0.15) <math>X_{t+3}</math></b>	-0.29 (0.16) $X_{t+3}$	-0.32 (0.22) $X_{t+3}$
Net Exports Share*	-0.31 (0.11)	-0.01 (0.11)	0.17 (0.11)	<b>-0.53 (0.14) <math>X_{t+2}</math></b>	<b>-0.35 (0.15) <math>X_{t+2}</math></b>	<b>-0.33 (0.13) <math>X_{t+2}</math></b>
Exports goods & services	<b>0.30 (0.12)</b>	<b>0.46 (0.14)</b>	<b>0.45 (0.16)</b>	-	-	-
Imports goods & services*	0.51 (0.12)	0.25 (0.11)	0.09 (0.09)	<b>0.58 (0.14) <math>X_{t+2}</math></b>	<b>0.37 (0.16) <math>X_{t+2}</math></b>	0.26 (0.14) $X_{t+2}$
Terms of Trade	0.14 (0.12)	-0.05 (0.12)	-0.14 (0.08)	<b>0.29 (0.14) <math>X_{t+3}</math></b>	0.29 (0.15) $X_{t+3}$	<b>0.21 (0.10) <math>X_{t+3}</math></b>
Real TWI	0.56 (0.13)	0.40 (0.14)	0.12(0.09)	<b>0.57 (0.12) <math>X_{t+1}</math></b>	-0.47 (0.14) $X_{t-5}$	-0.31 (0.11) $X_{t-5}$
CPI***	-0.16 (0.13)	-0.22 (0.15)	-0.14 (0.11)	<b>0.55 (0.15) <math>X_{t+5}</math></b>	<b>0.36 (0.14) <math>X_{t+5}</math></b>	-0.17 (0.13) $X_{t-2}$
Non-tradables***	-0.20 (0.14)	-0.23 (0.14)	-0.28 (0.16)	<b>-0.61 (0.18) <math>X_{t-4}</math></b>	0.38 (0.13) $X_{t+4}$	-
Tradables***	<b>-0.22 (0.11)</b>	-0.20 (0.13)	-0.18 (0.14)	-	0.24 (0.14) $X_{t+4}$	0.19 (0.11) $X_{t+4}$

**TABLE 3 (continued)**  
*Stylised Business Cycle Facts: 1987(2) - 2010(4)*  
*Comparative cross correlations with Real GDPE*

Variable $x \setminus$ De-trend method	Contemporaneous Cross Correlation			Most significant Non-contemporaneous Cross Correlation		
	HP1600	HP100	loess	HP1600	HP100	loess
Real 90-day Bank Bill****	0.39 (0.16)	0.08 (0.15)	-0.04 (0.12)	<b>0.75 (0.15) <math>x_{t+3}</math></b>	<b>0.52 (0.14) <math>x_{t+3}</math></b>	<b>0.36 (0.13) <math>x_{t+3}</math></b>
Real M3*****	0.19 (0.13)	0.05 (0.13)	0.16 (0.13)	<b>0.62 (0.14) <math>x_{t+5}</math></b>	-0.34 (0.13) $x_{t-3}$	-0.32 (0.10) $x_{t-3}$
Employment	0.56 (0.16)	0.19 (0.13)	0.11 (0.09)	<b>0.60 (0.15) <math>x_{t+2}</math></b>	0.22 (0.12) $x_{t+2}$	0.10 (0.10) $x_{t+2}$
Unemployment	-0.66 (0.17)	-0.27 (0.12)	-0.17 (0.10)	<b>-0.68 (0.16) <math>x_{t+1}</math></b>	<b>-0.34 (0.13) <math>x_{t+1}</math></b>	-0.23 (0.12) $x_{t+1}$
Labour Productivity	<b>0.54 (0.15)</b>	<b>0.64 (0.14)</b>	<b>0.57 (0.13)</b>	-	-	-
Real Av. Hourly Earns*****	-0.32 (0.13)	0.06 (0.12)	0.11 (0.12)	<b>-0.45 (0.12) <math>x_{t+2}</math></b>	<b>-0.32 (0.11) <math>x_{t+2}</math></b>	-0.22 (0.15) $x_{t+2}$
Real Labour Cost*****	-0.09 (0.12)	0.14 (0.13)	0.04 (0.11)	<b>-0.40 (0.14) <math>x_{t+5}</math></b>	<b>-0.43 (0.14) <math>x_{t+5}</math></b>	<b>-0.33 (0.11) <math>x_{t+5}</math></b>
Real Unit Labour Cost*****	<b>-0.62 (0.16)</b>	<b>-0.42 (0.15)</b>	<b>-0.38 (0.13)</b>	-	-	-

Notes: Numbers in parentheses are robustly estimated standard errors

"-" Most significant cross correlation is Contemporaneous

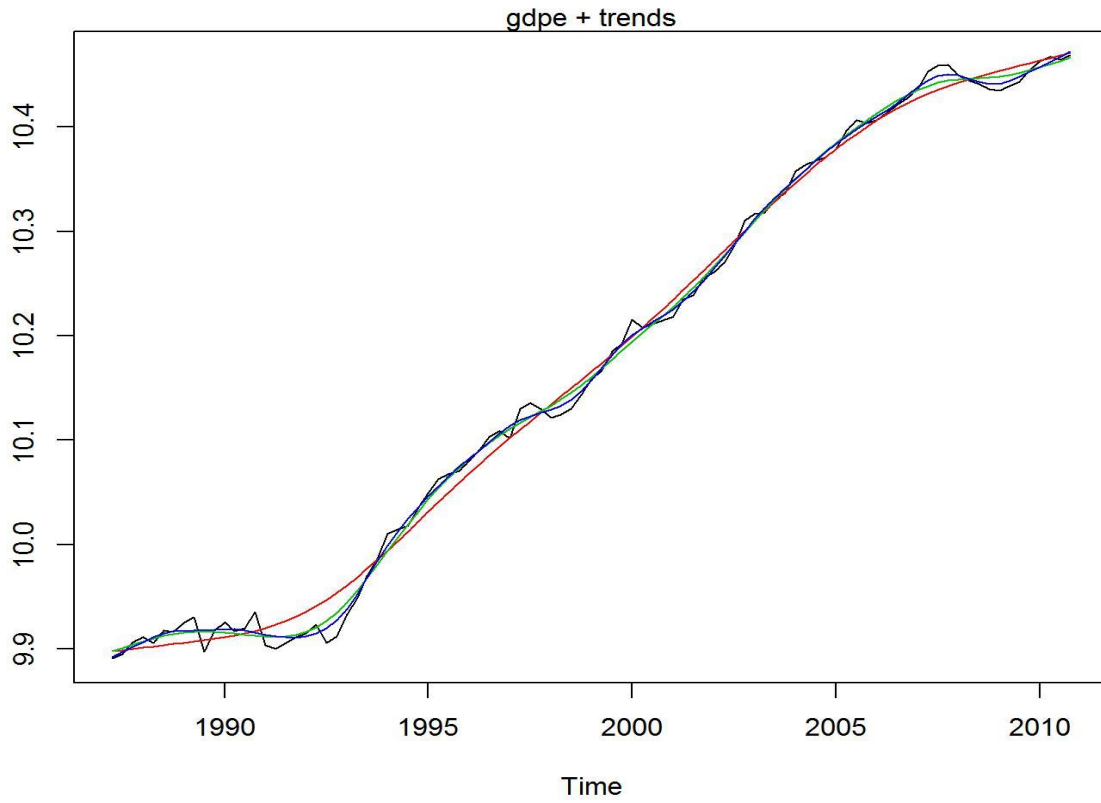
\* SNZ National Accounts series, adjusted (as for NZ Treasury series) for frigate purchases recorded in 1997q2 and 1999q4.

The series not adjusted in this way show somewhat weaker (or relatively similar) cross correlations with real GDPE, e.g. for HP1600 filtered series, those statistically significant total government expenditure and government consumption correlations are 0.45 ( $x_{t+5}$ ) and 0.26 ( $x_{t+5}$ ); for imports of goods and services and for net exports share, the correlations are 0.57 ( $x_{t+2}$ ) and -0.52 ( $x_{t+2}$ ).

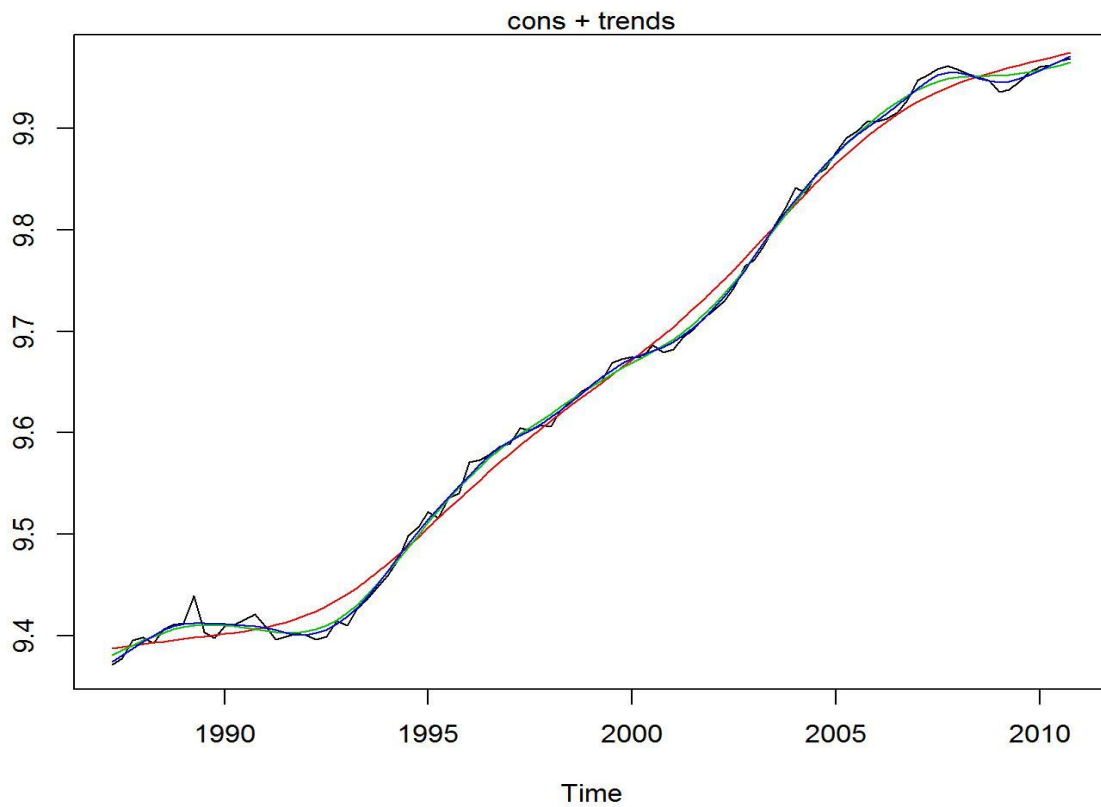
\*\* New Zealand Treasury Series

\*\*\* Sample period 1988(1)-2010(4); \*\*\*\* Sample period 1987(3)-2010(4); \*\*\*\*\* Sample period 1988(2)-2010(4)

\*\*\*\*\* Sample period 1989(1)-2010(4); \*\*\*\*\* Sample period 1992(4)-2010(4)



**Figure 1: log gdpe, hp1600 log gdpe, hp100 log gdpe, loess log gdpe**



**Figure 2a: log cons, hp1600 log cons, hp100 log cons, loess log cons**



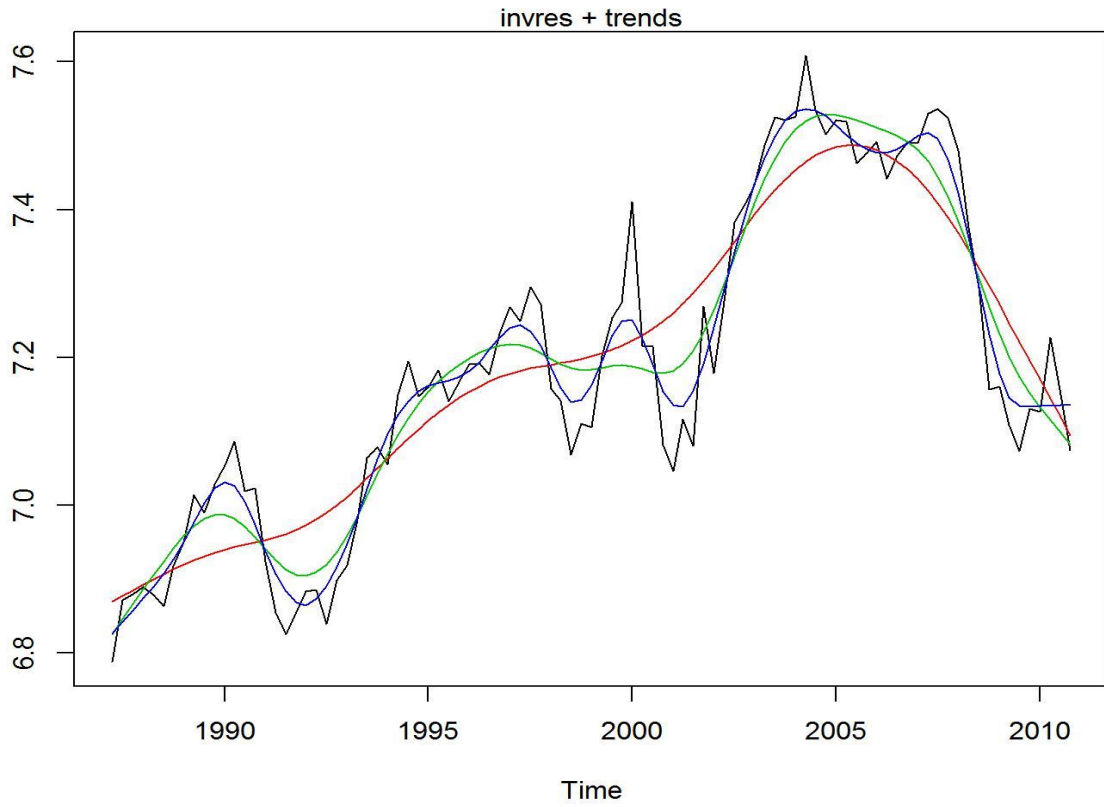


Figure 2b: log invres, **hp1600 log invres**, **hp100 log invres**, **loess log invres**

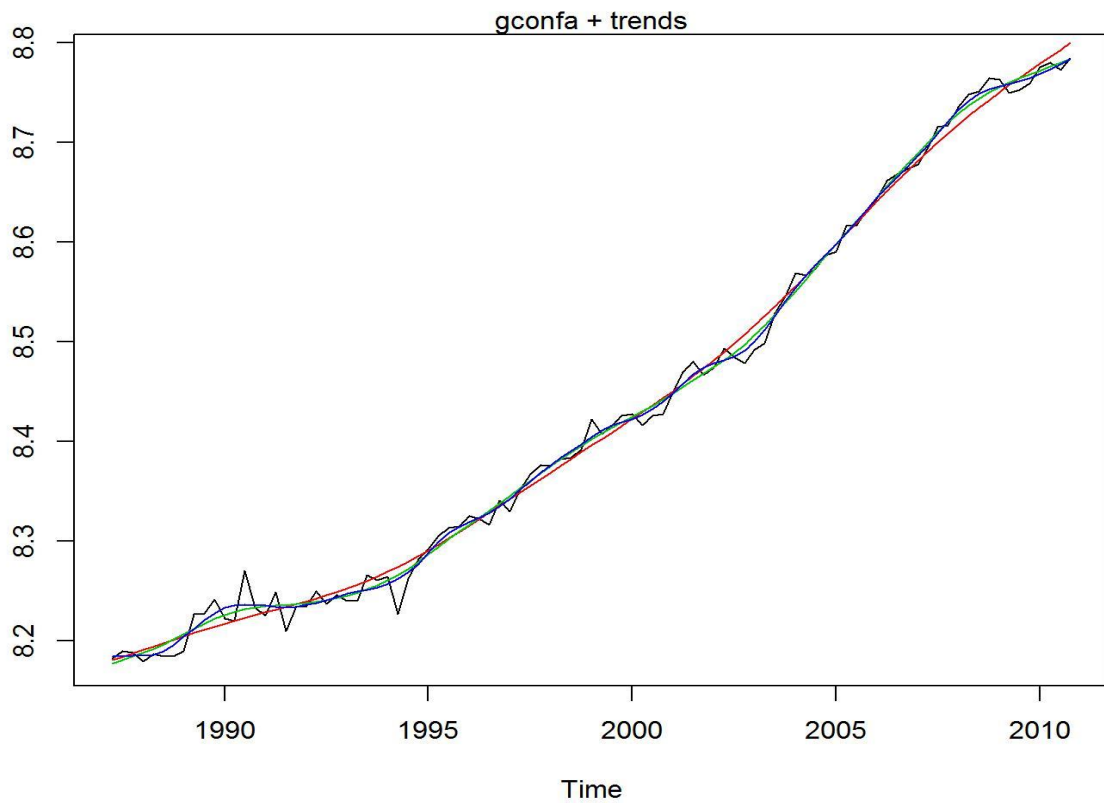


Figure 2c: log gconfa, **hp1600 log gconfa**, **hp100 log gconfa**, **loess log gconfa**

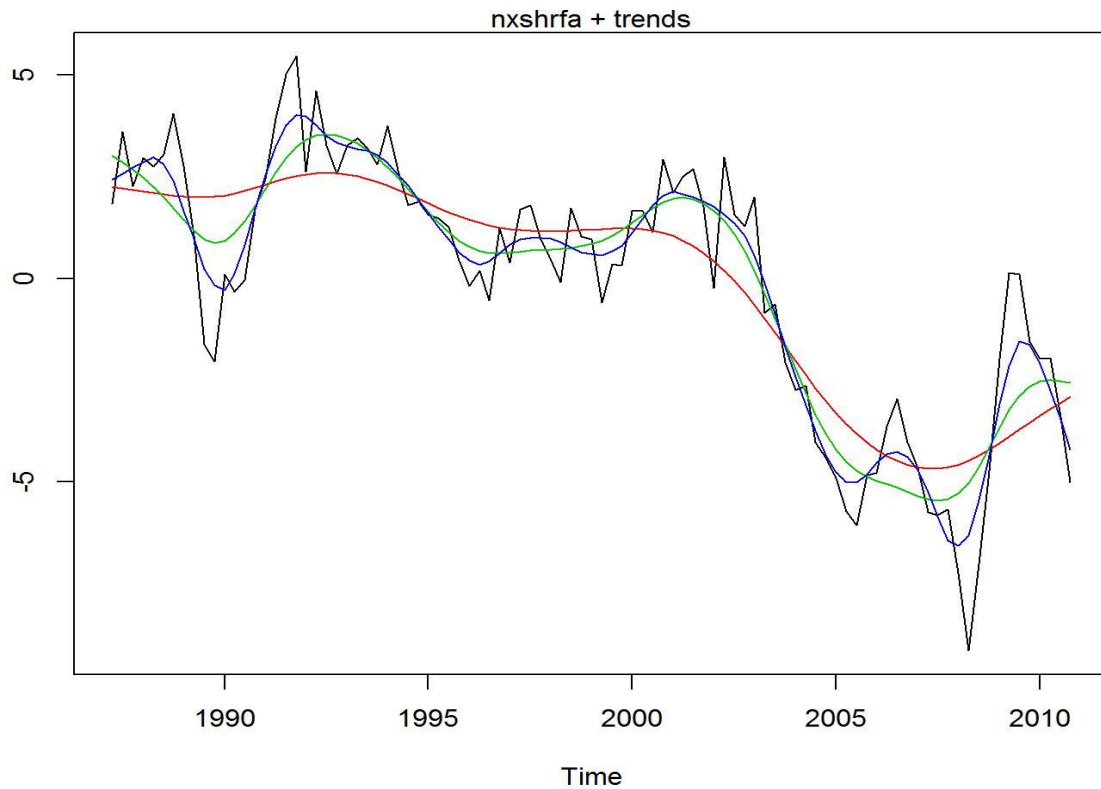


Figure 2d: log nxshrfa, **hp1600 log nxshrfa**, **hp100 log nxshrfa**, **loess log nxshrfa**

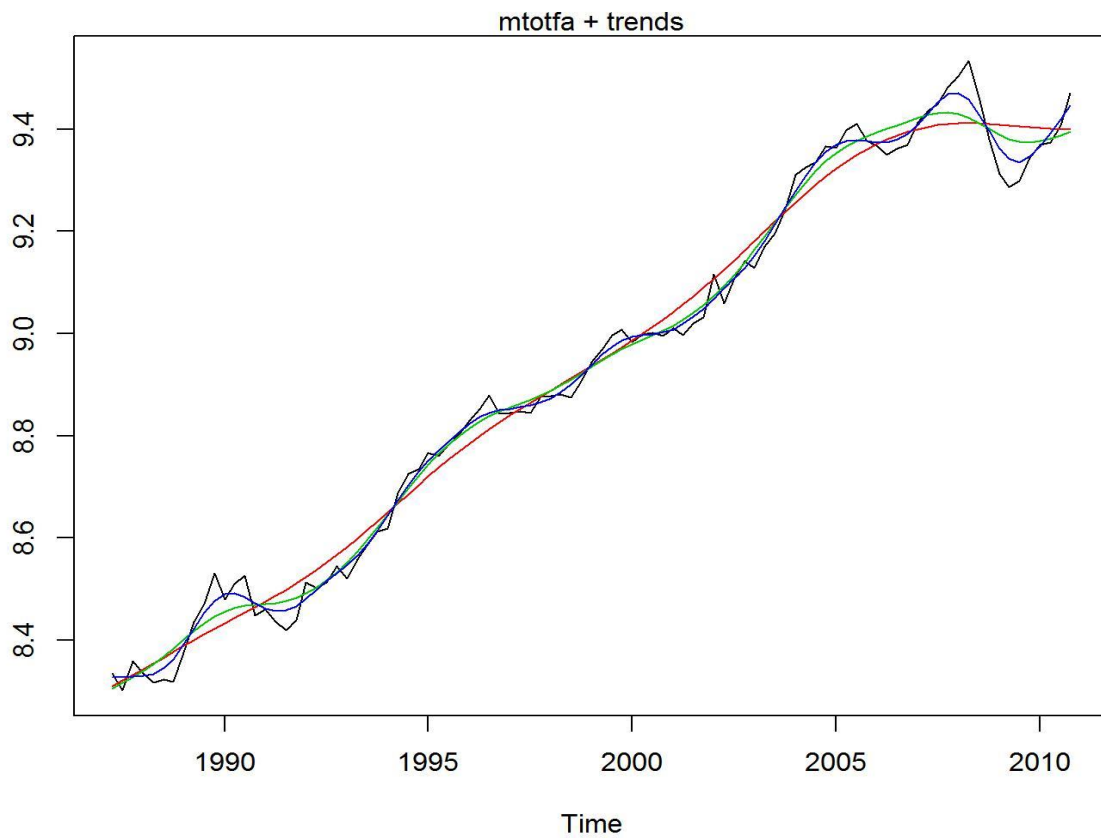


Figure 2e: log mtotfa, **hp1600 log mtotfa**, **hp100 log mtotfa**, **loess log mtotfa**

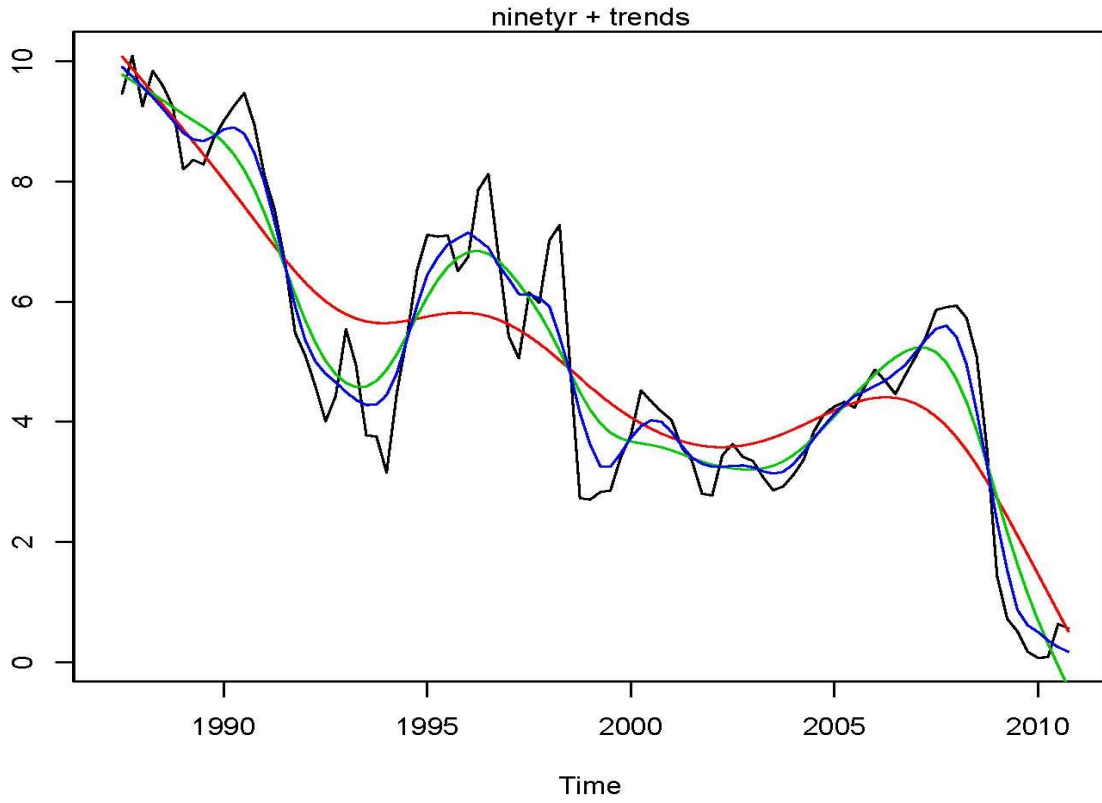


Figure 2f: log ninetyr, **hp1600 log ninetyr**, **hp100 log ninetyr**, **loess log ninetyr**

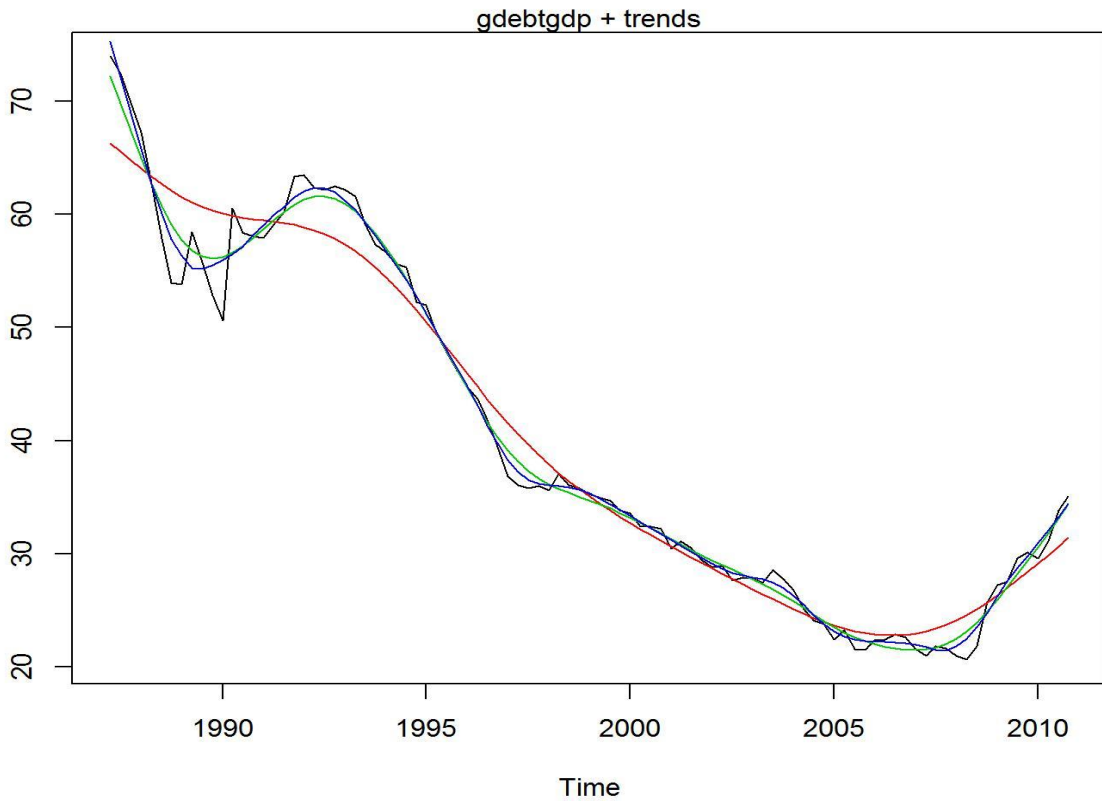


Figure 2g: log gdebtgdp, **hp1600 log gdebtgdp**, **hp100 log gdebtgdp**, **loess log gdebtgdp**

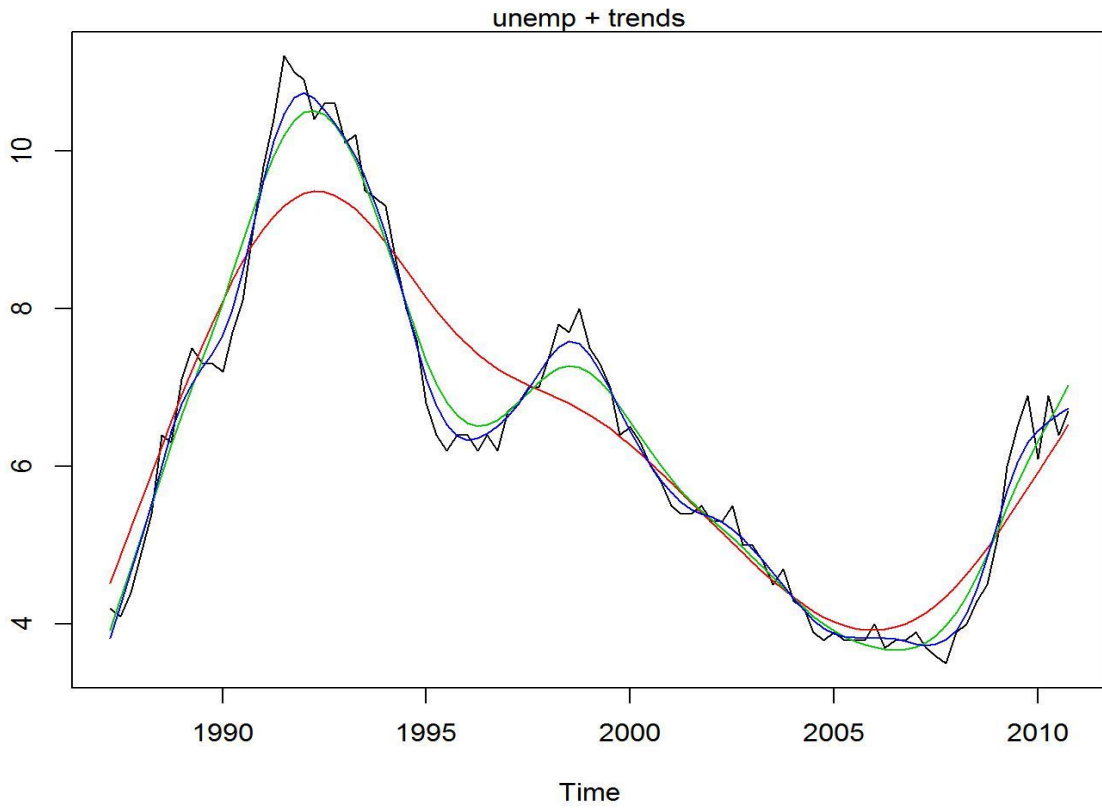


Figure 2h: log unemp, **hp1600 log unemp**, **hp100 log unemp**, **loess log unemp**

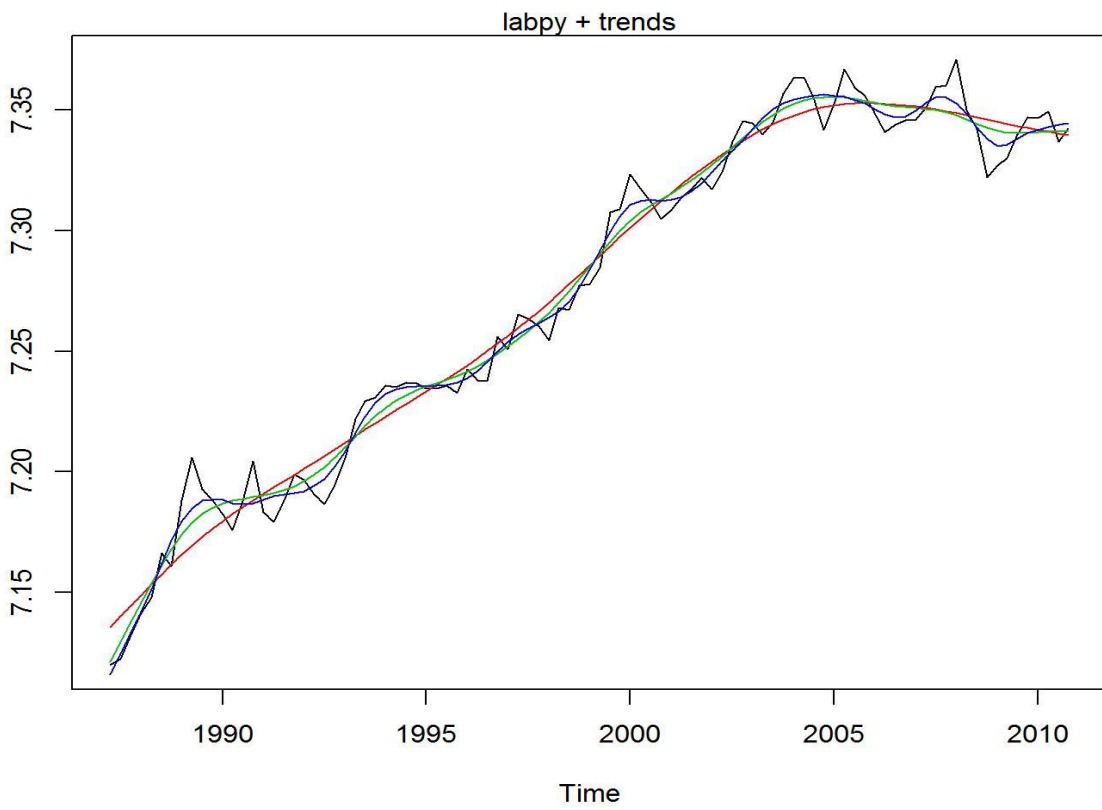


Figure 2i: log labpy, **hp1600 log labpy**, **hp100 log labpy**, **loess log labpy**

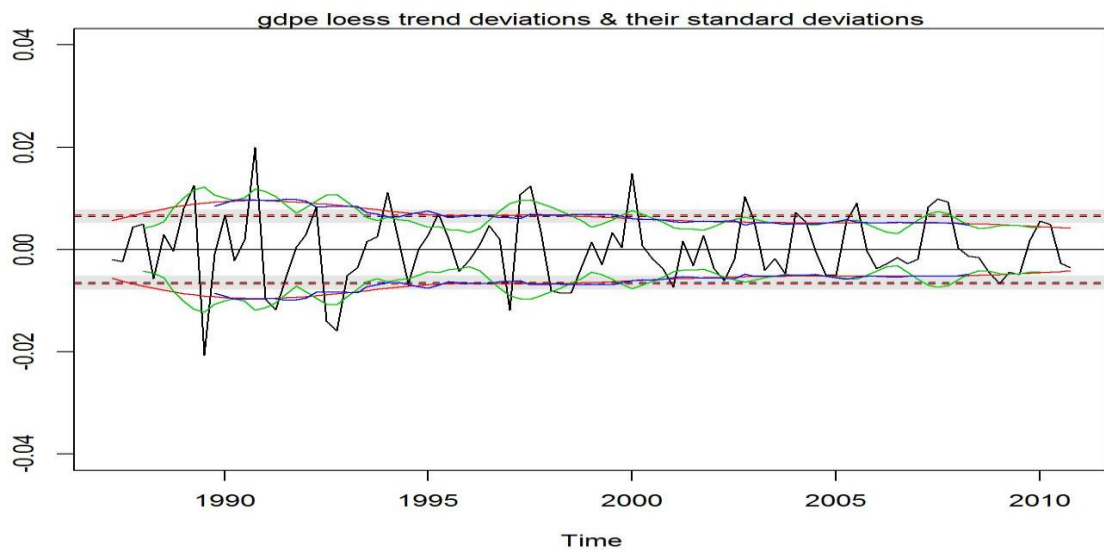
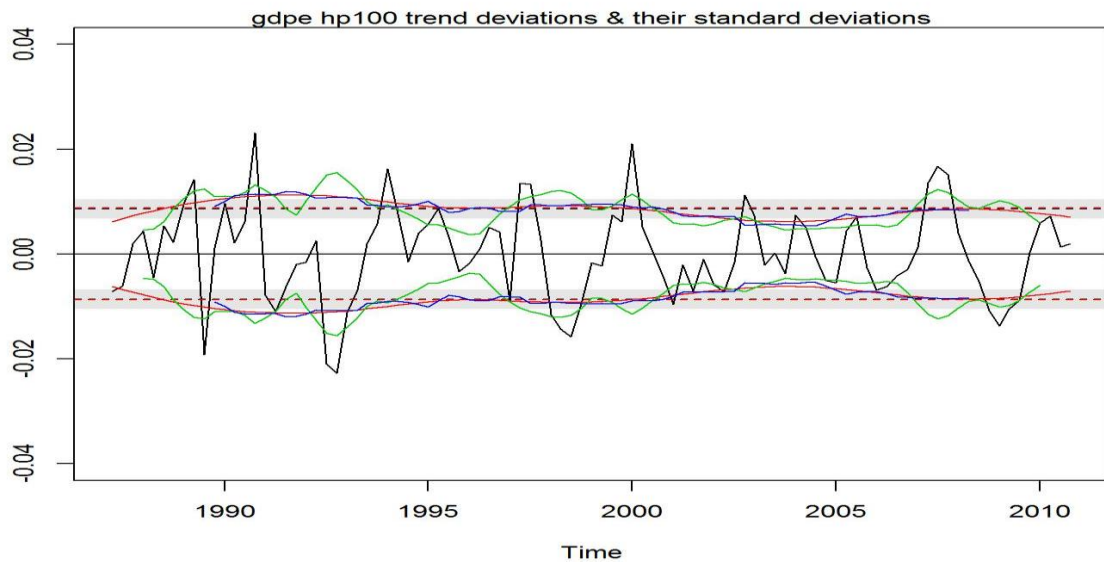
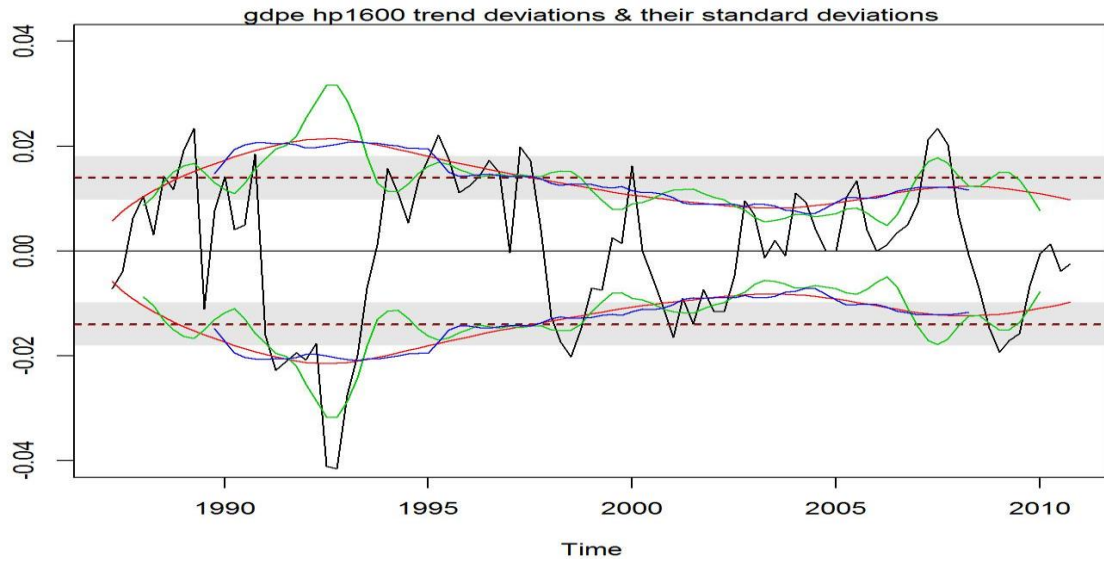


Figure 3a: **hp1600**, **7 point triangular**, **21 qtr uniform** trend deviations gdpe

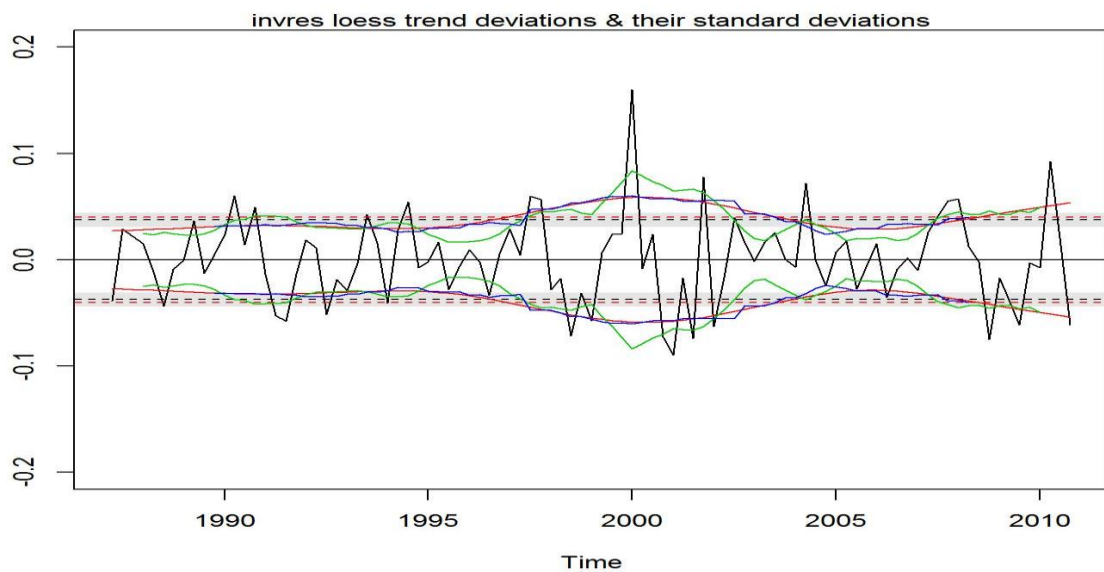
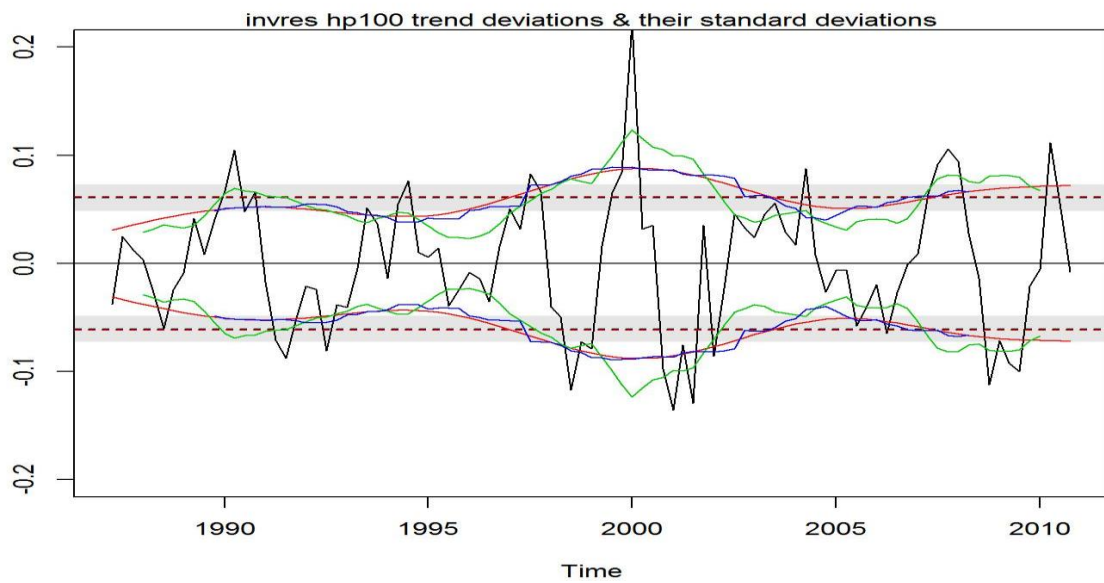
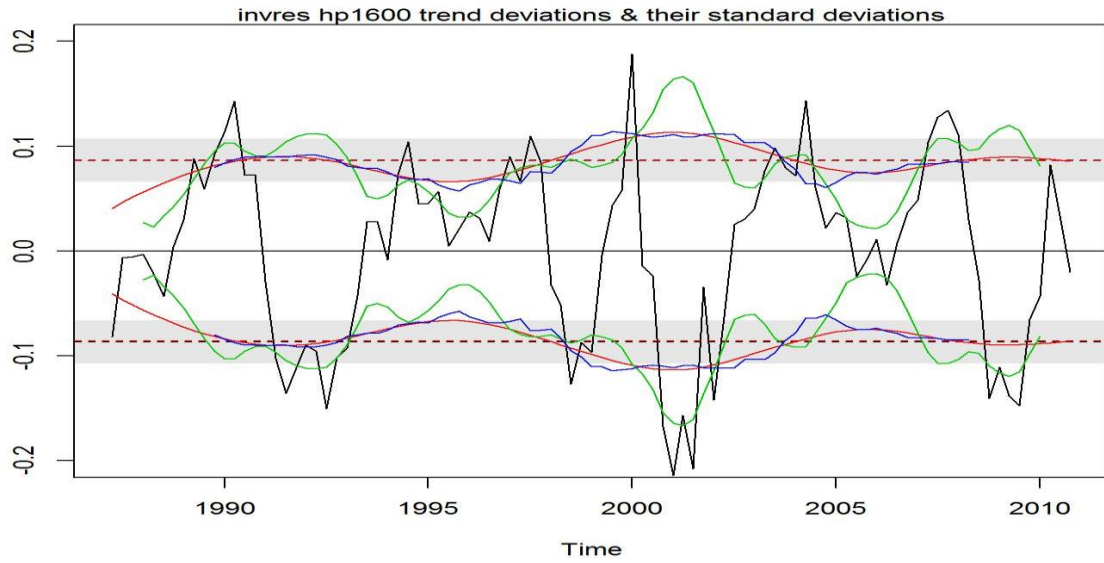


Figure 3b: hp1600, 7 point triangular, 21 qtr uniform trend deviations invres

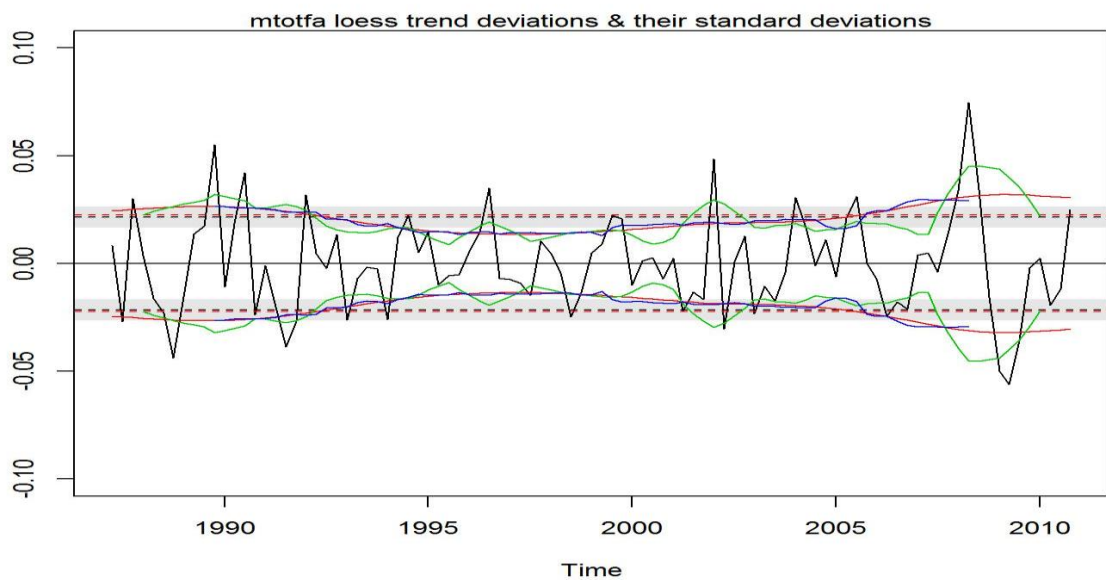
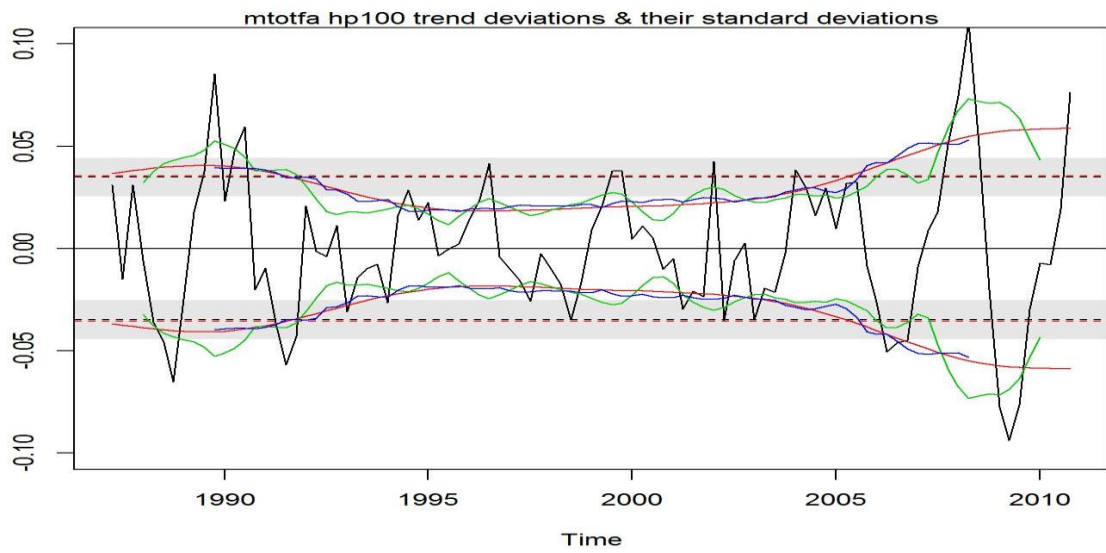
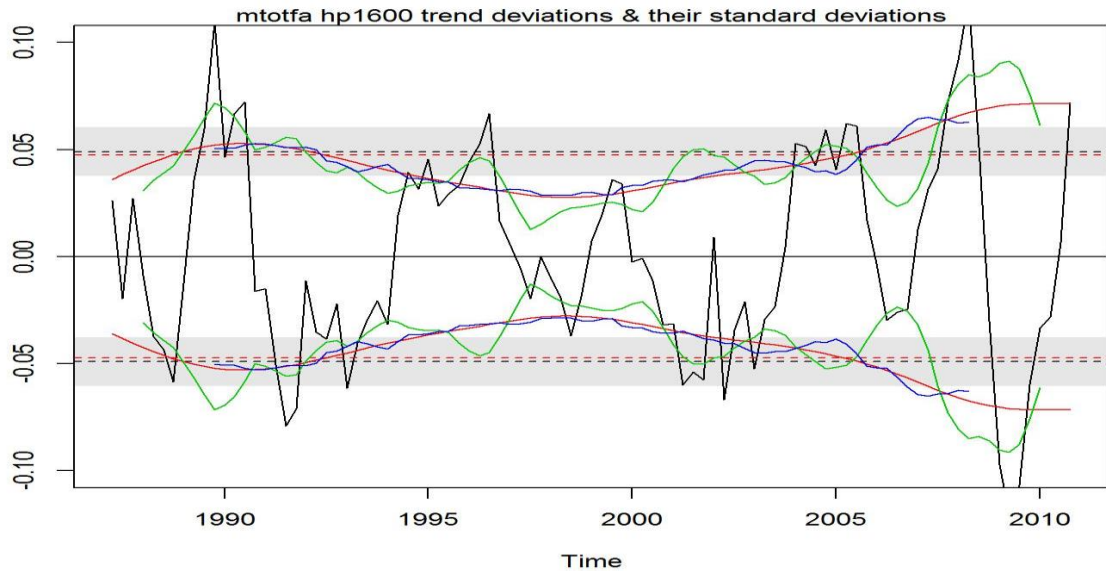
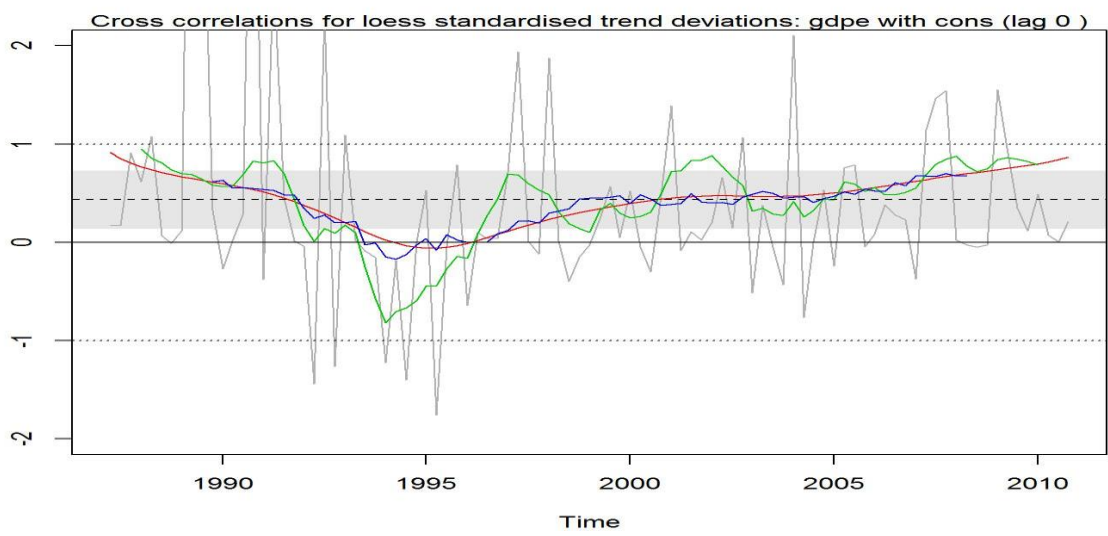
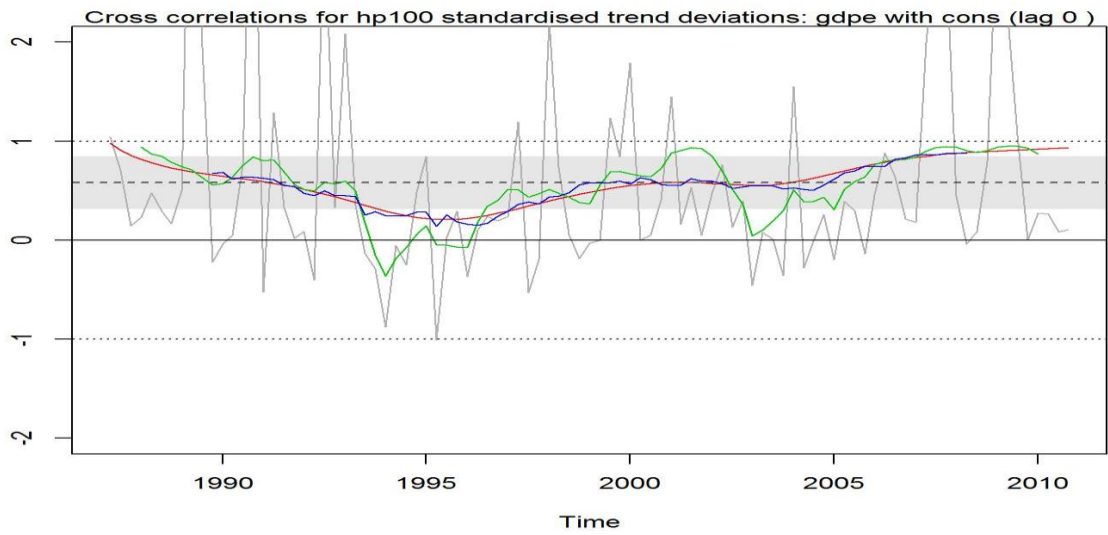
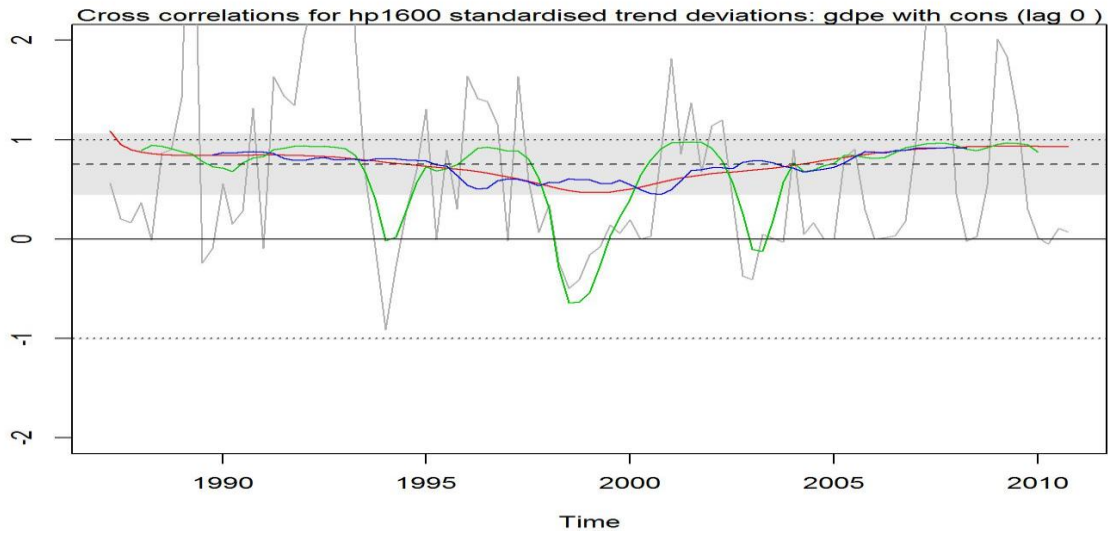
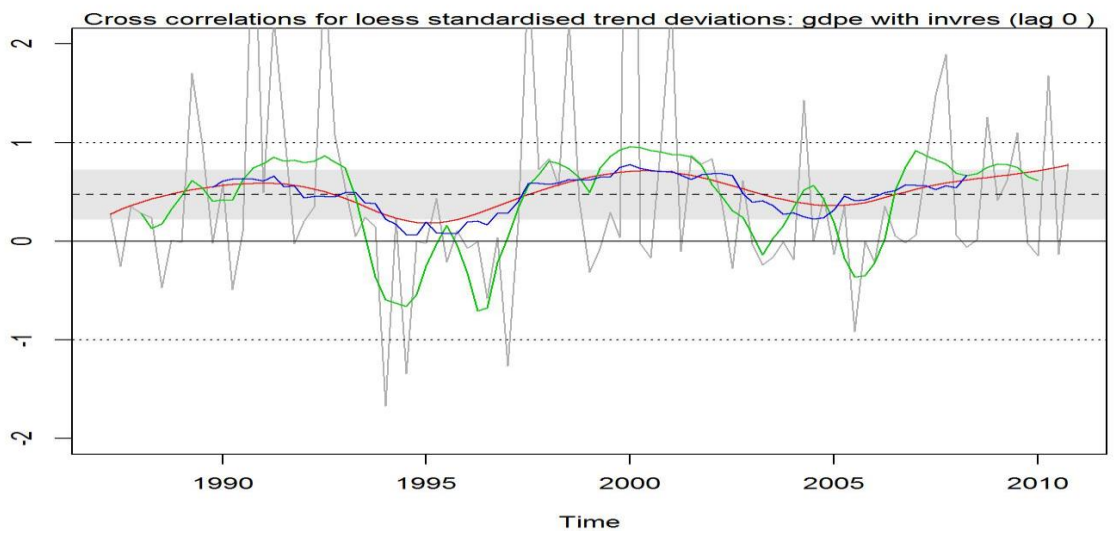
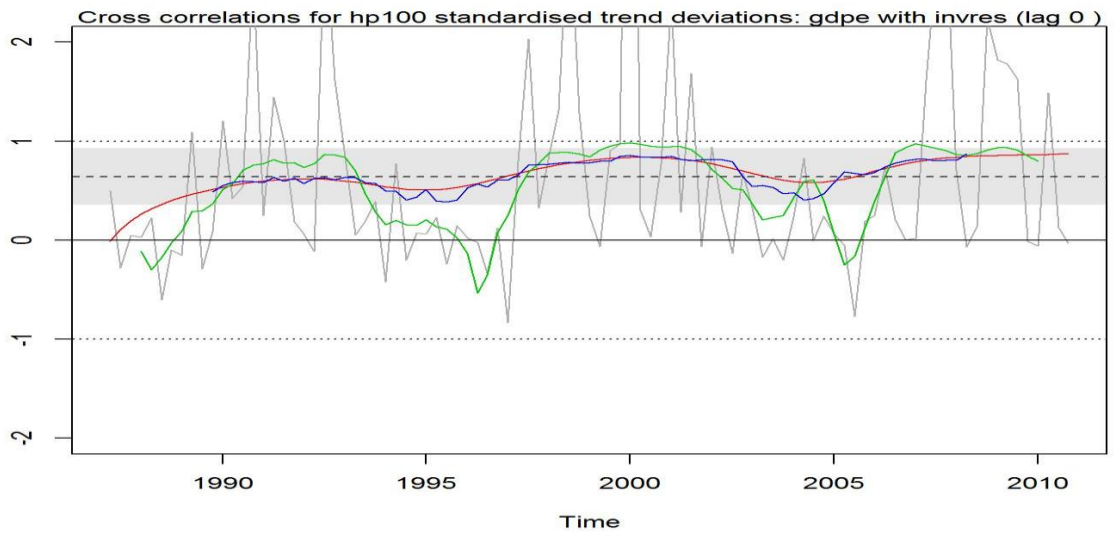
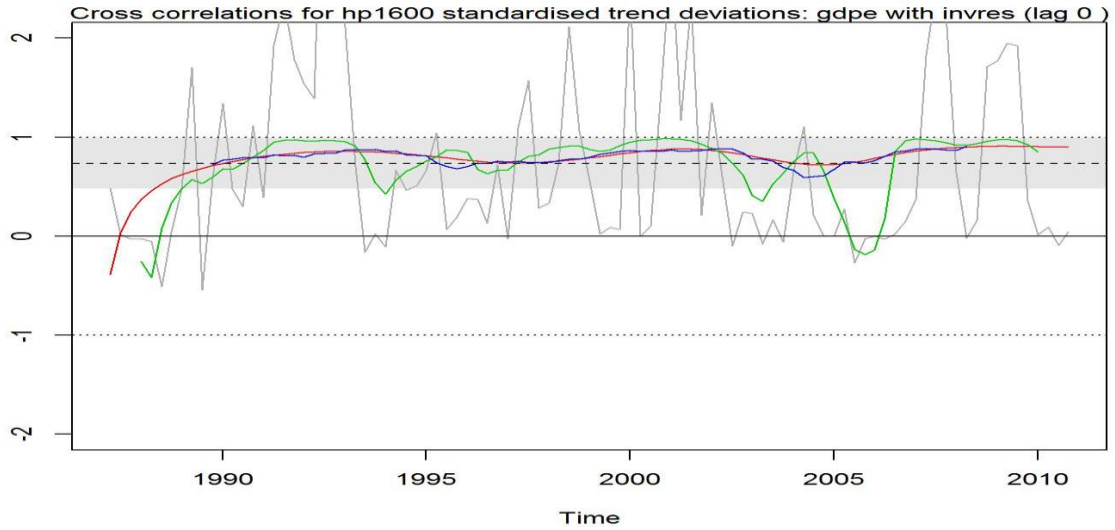


Figure 3c: **hp1600**, **7 point triangular**, **21 qtr uniform** trend deviations mtotfa



**Figure 4a: cross correlations for standardised deviations, gdpe with cons(0) cross products, hp1600, 7 pt triangular, 21 qtr uniform variability over time**





**Figure 4b: cross correlations for standardised deviations, gdpe with invres(0) cross products, hp1600, 7 pt triangular, 21 qtr uniform variability over time**

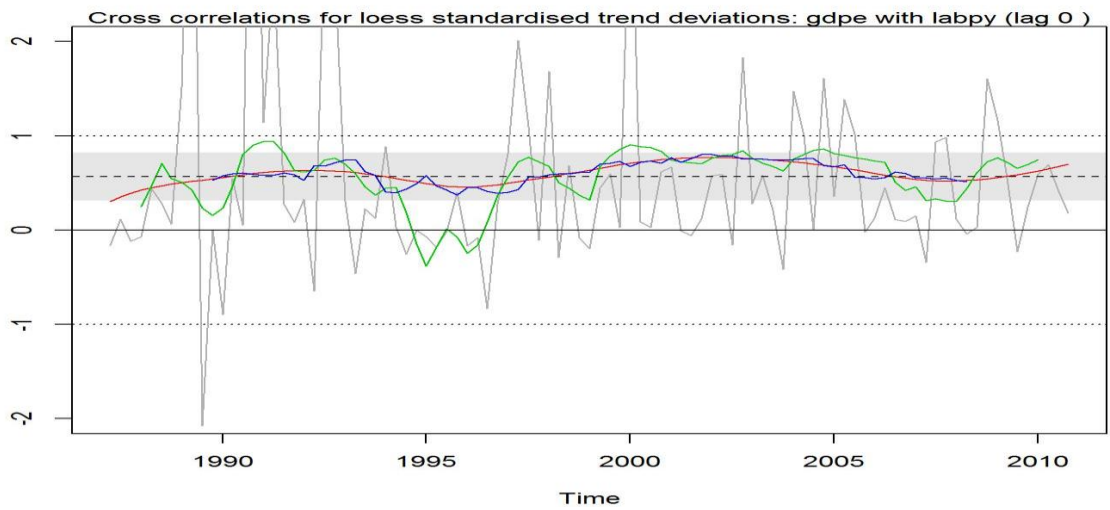
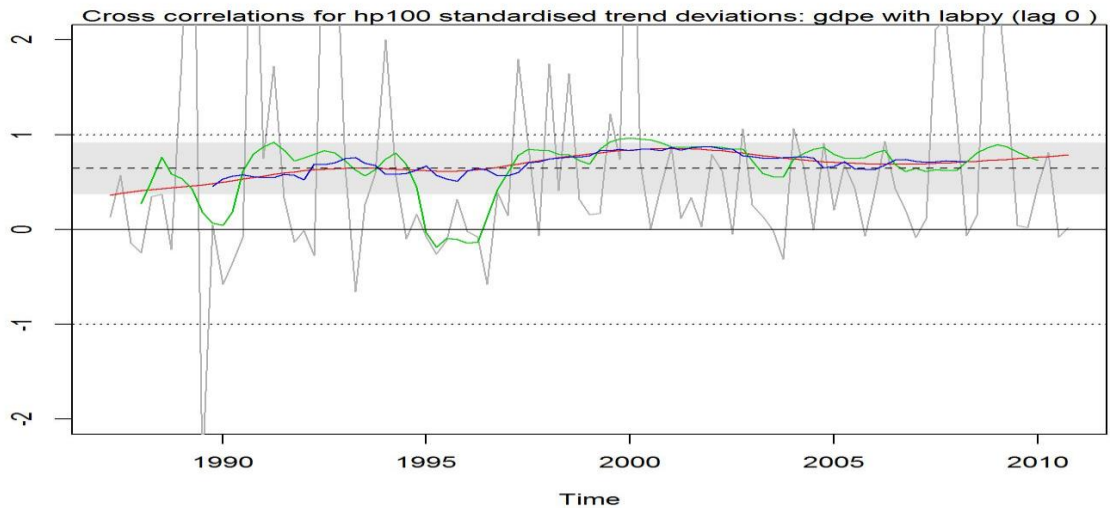
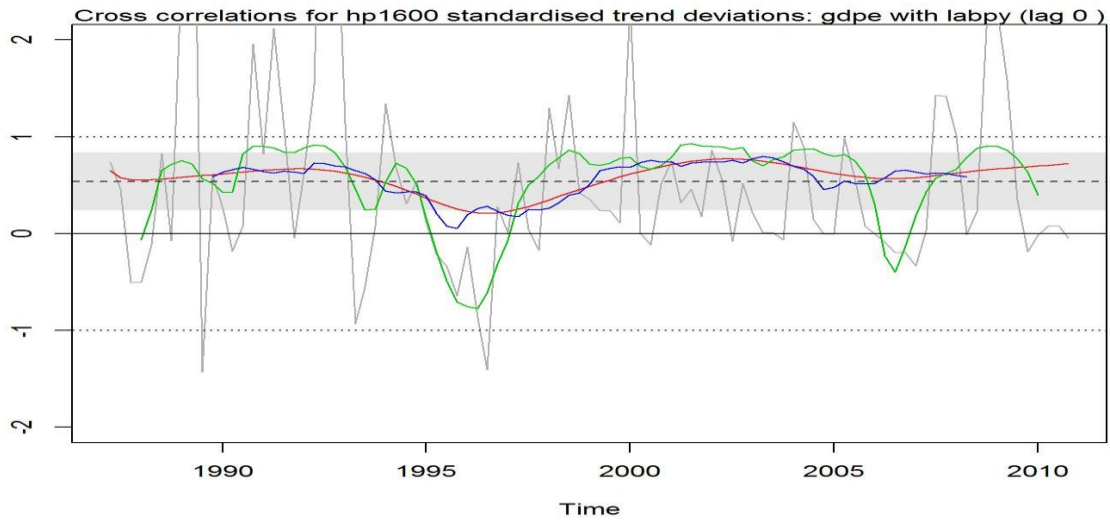


Figure 4c: cross correlations for standardised deviations, gdpe with labpy(0) cross products, **hp1600**, **7 pt triangular**, **21 qtr uniform** variability over time

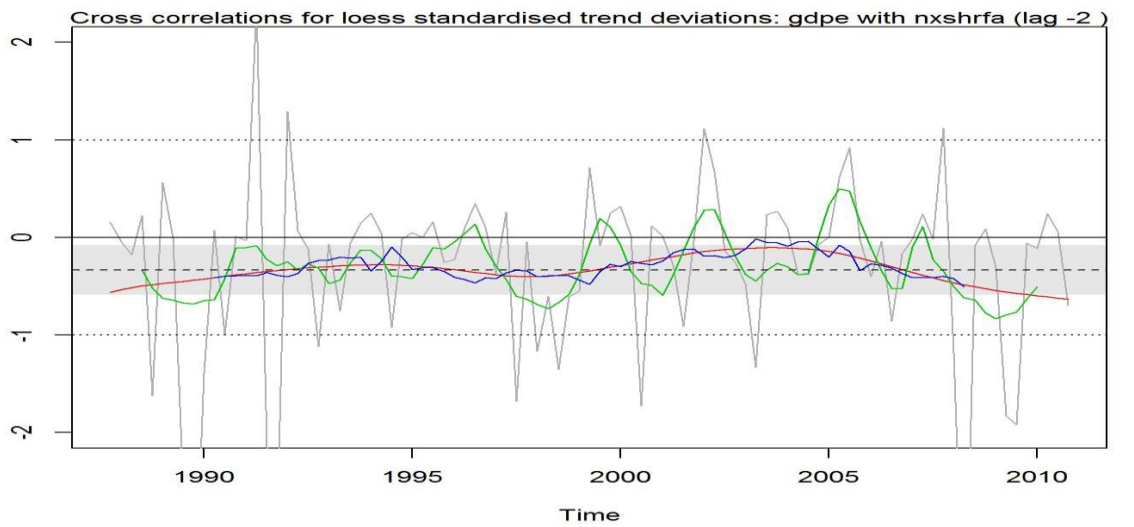
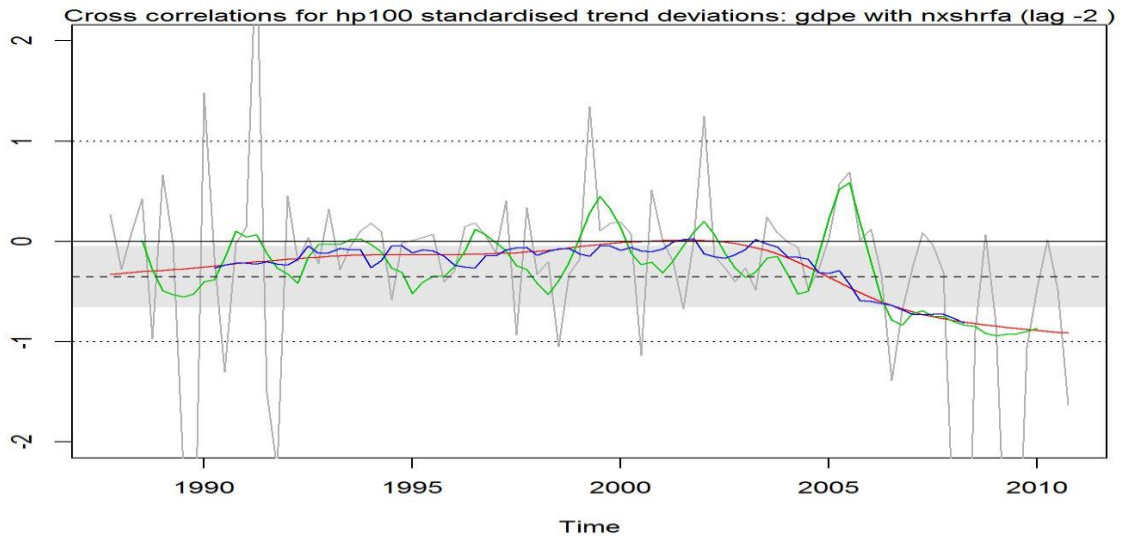
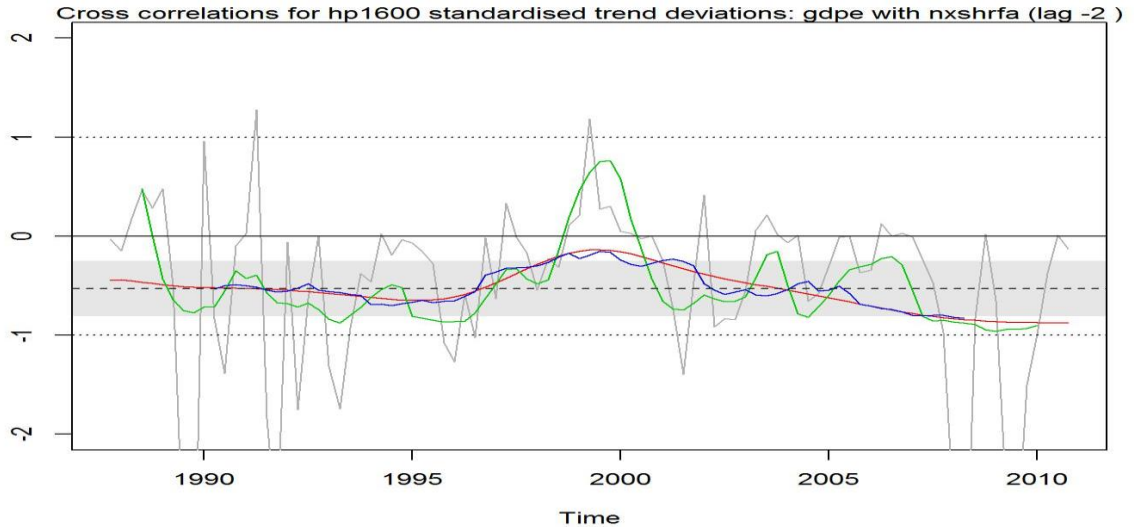


Figure 5a: cross correlations for standardised deviations, gdpe with nxshrfa lag(-2)  
 cross products, **hp1600**, **7 pt triangular**, **21 qtr uniform** variability over time

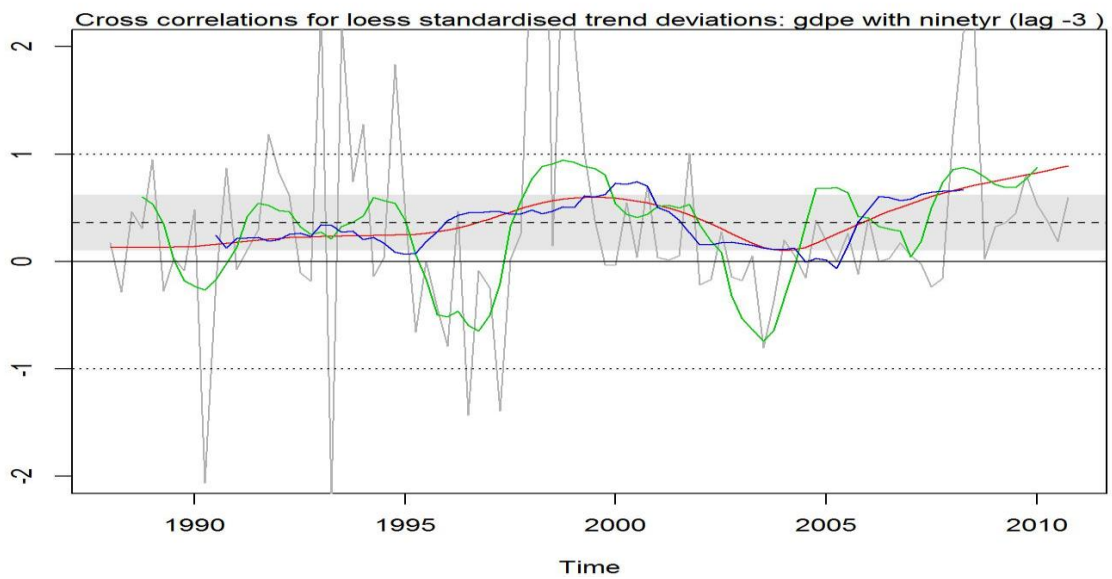
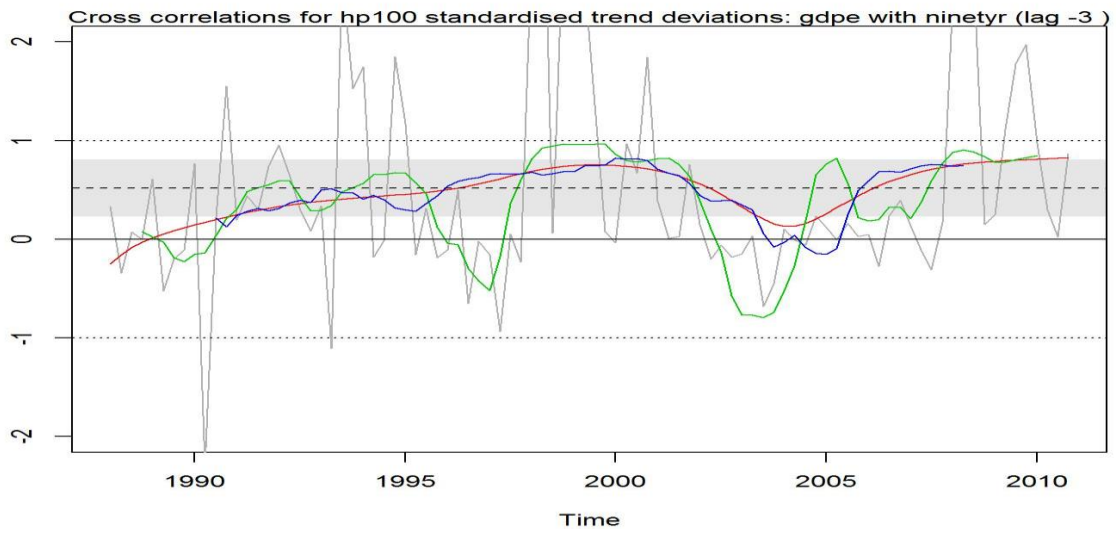
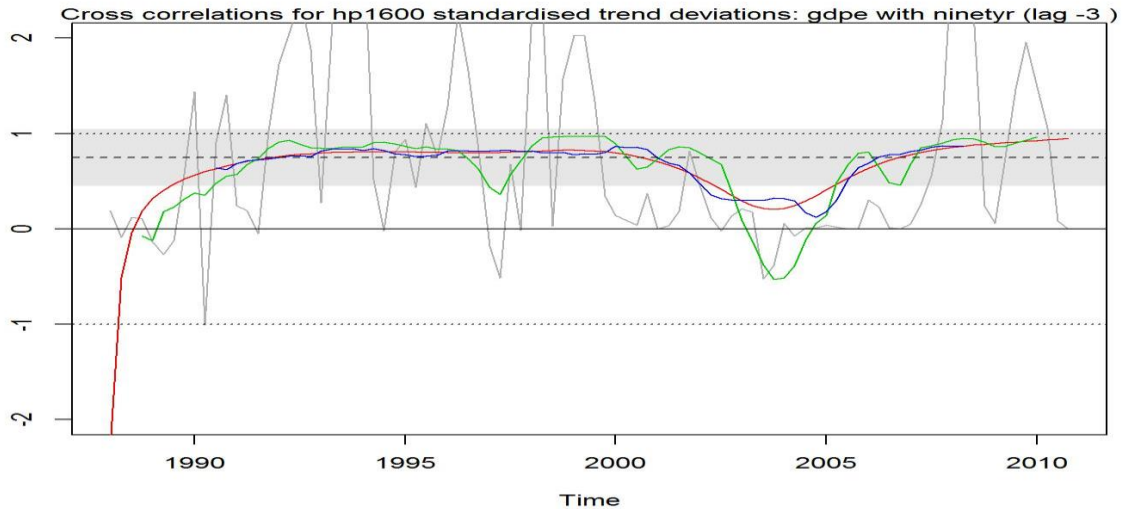


Figure 5b: cross correlations for standardised deviations, gdpe with ninetyr lag(-3)  
 cross products, **hp1600**, **7 pt triangular**, **21 qtr uniform** variability over time

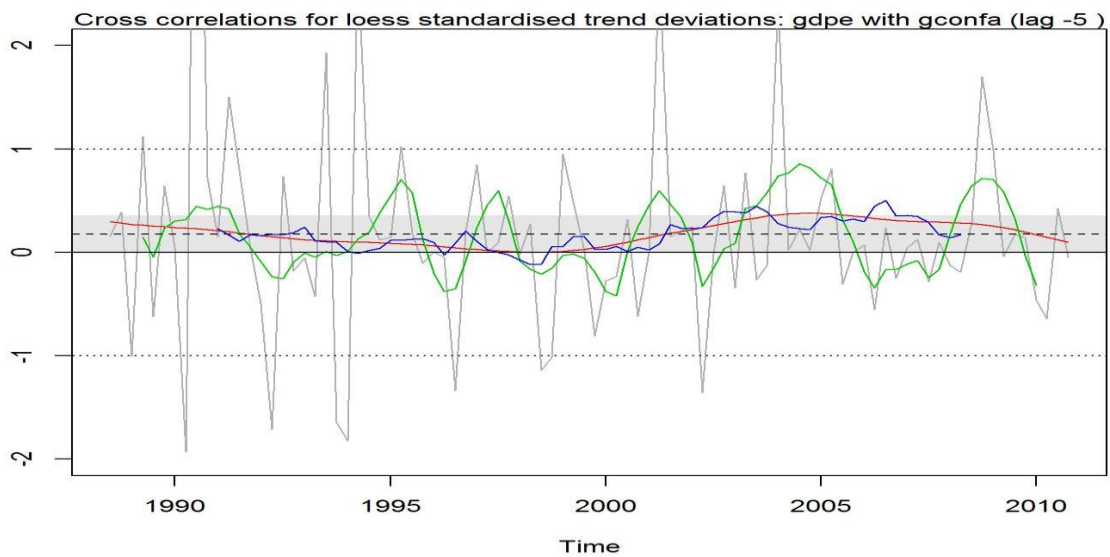
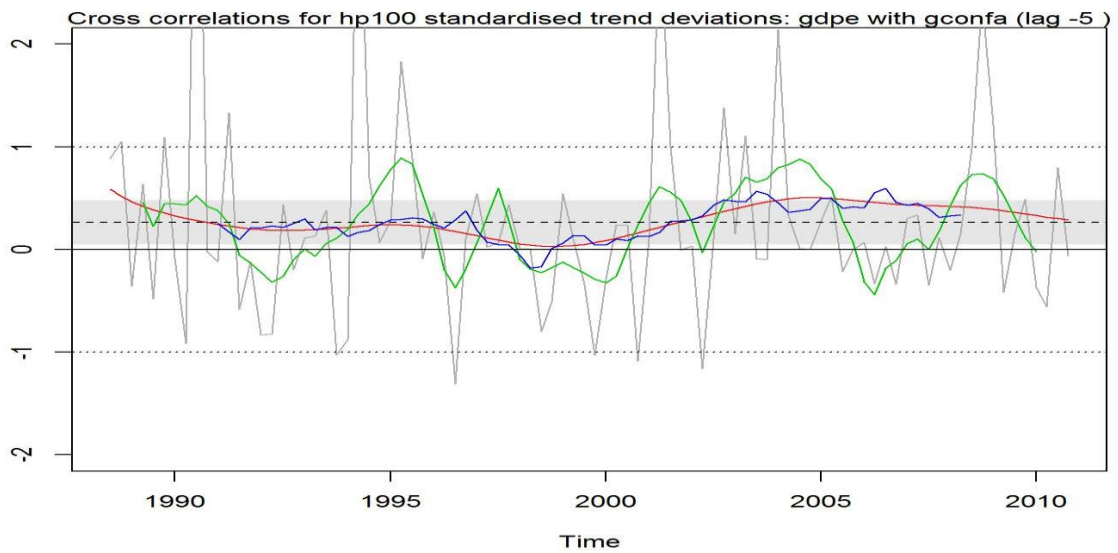
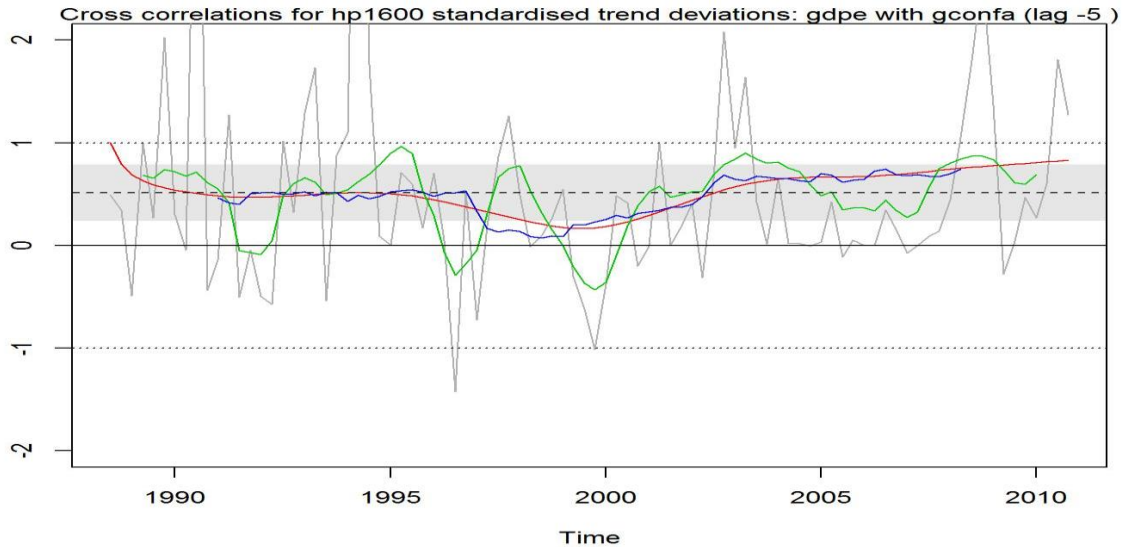


Figure 5c: cross correlations for standardised deviations, gdpe with gconfa lag(-5)  
 cross products, hp1600, 7 pt triangular, 21 qtr uniform variability over time

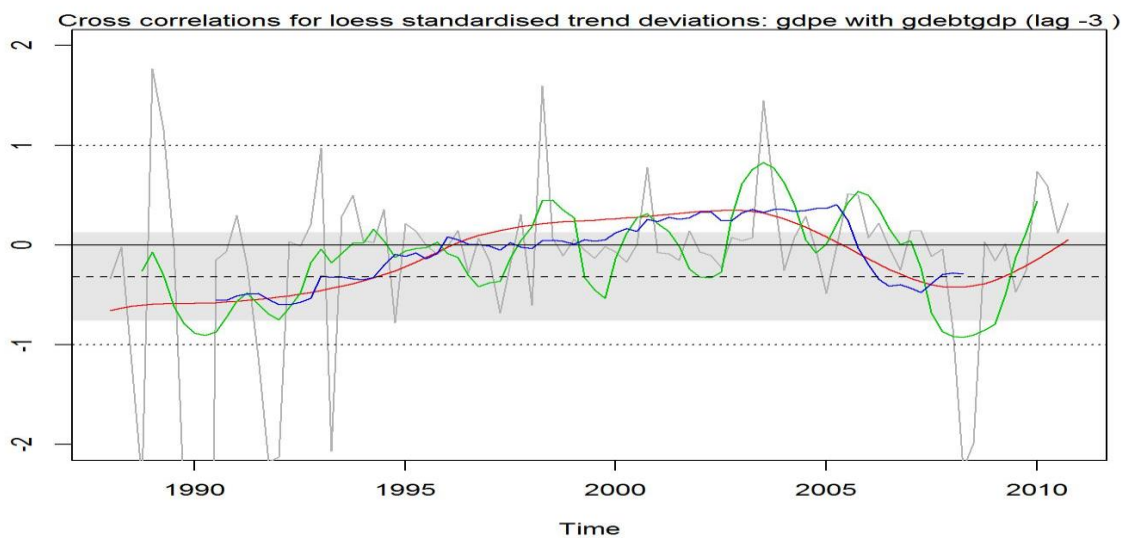
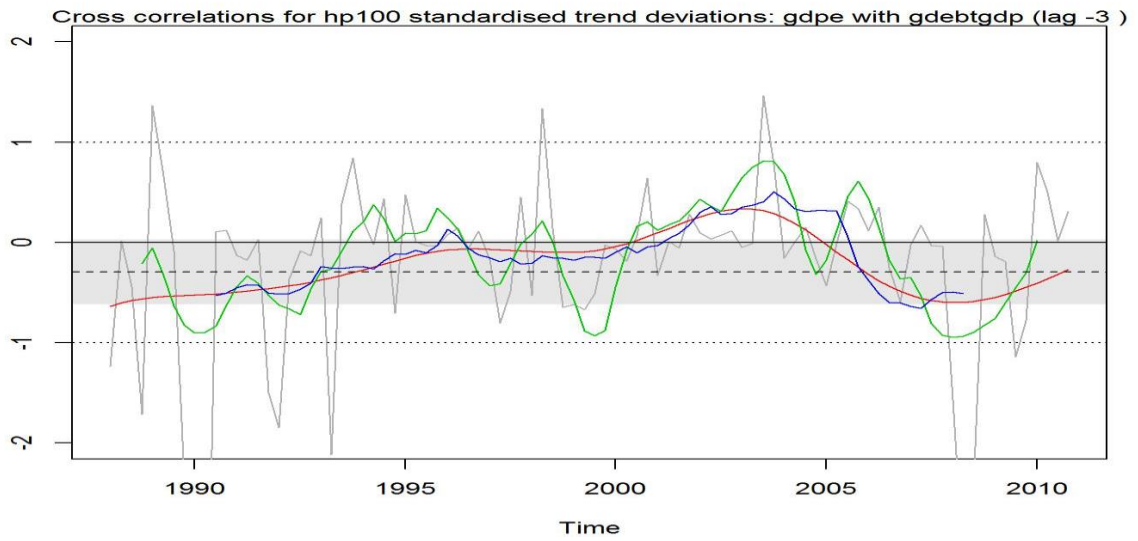
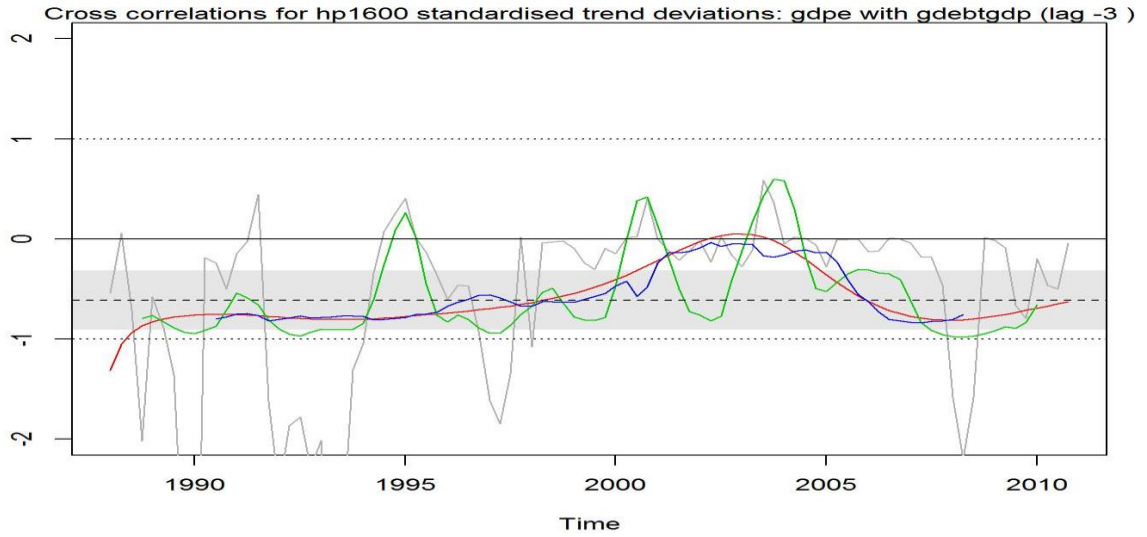


Figure 5d: cross correlations for standardised deviations, gdpe with gdebtgdp lag(-3)  
cross products, hp1600, 7 pt triangular, 21 qtr uniform variability over time

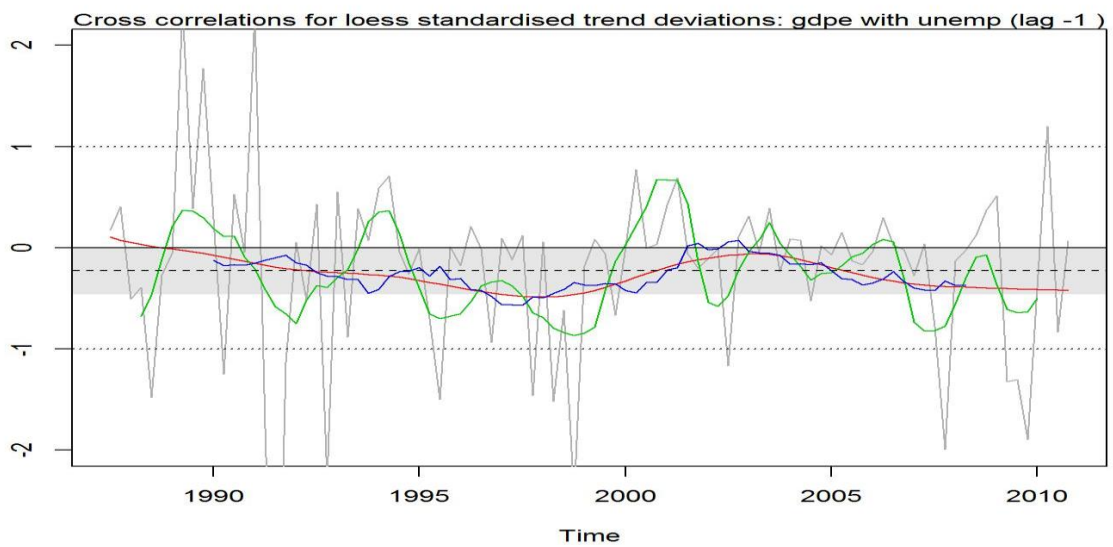
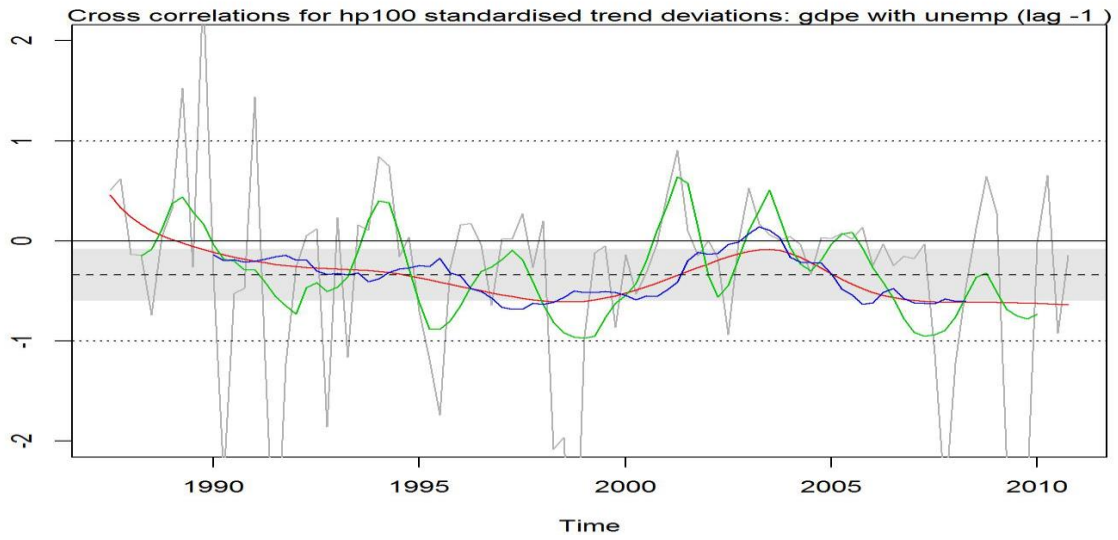
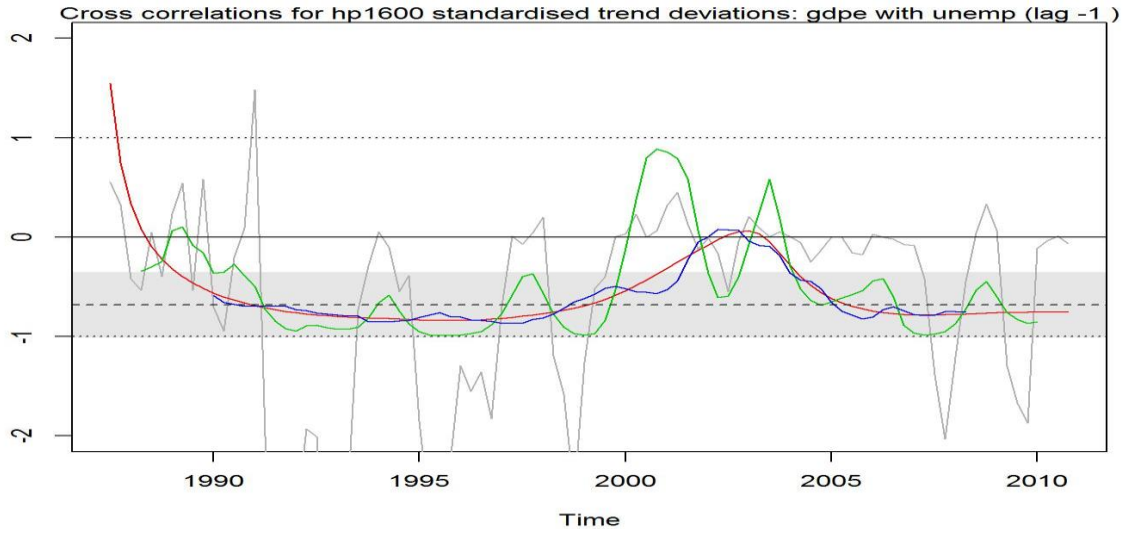


Figure 5e: cross correlations for standardised deviations, gdpe with unemp lag(-1)  
 cross products, hp1600, 7 pt triangular, 21 qtr uniform variability over time

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