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Job Matching, Family Gap and Fertility Choice

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Abstract: This paper concentrates on the role of job matching frictions in influencing the interactions between fertility choice and wage offers and show that job market frictions are a crucial factor in wage differentials among female workers. The goals of this paper are to examine how the home-stay alternative and asymmetric market frictions influence this wage differential and whether the well-documented negative correlation between fertility choice and female earnings still holds in the face of life-cycle choices. To address these questions, we develop a search-theoretic model that incorporates fertility and job decisions and assume that the workers with children face a lower job matching rate and a higher job quitting rate relative to the rates faced by the workers without children. As a result, we find that a wider wage differential is associated with more asymmetric market frictions and that the wage differential has a positive effect from output differential, a positive scale effect from the output of mothers and a negative effect from the utility from staying home. The wage differential is positively correlated with both fertility and home-stay rates and negatively correlated with market thickness and matching technology. Furthermore, by having both fertility and home-stay choices endogenously determined, we can show that a tighter market will enhance the fertility rate and the home-stay rate and a better job matching technology would not only increase the home-stay rate but decrease the fertility rate. In general equilibrium, the effects of both market tightness and the matching technology on the wage differential are indeterminate. These results indicate that the home-stay choice actually gives workers with children not only an alternative but also the power to negotiate higher wages. This shrinks the wage differential. Finally, the result that a higher fertility rate enlarges the wage differential also implies a negative relationship between the fertility rate and the earnings of workers with children. This result is consistent with standard empirical findings.

JEL Classification: J13, J31.

Keywords: job Search, female labor participation, fertility choice

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1 Introduction

The gender wage gap has received broad attention in economic research. After adopting the policy on equal opportunities across genders, the gender wage gap has shrunk significantly, but the family gap has been widened over time. The family gap is referred to the wage gap between non-mothers and mothers. It is shown in the literature that the family gap explains 40%-50% of the gender gap [Waldfogel (1998)]. Therefore, understanding the family gap may help shrinking the gender gap further.

How large is the family gap? This family penalty counts for 10-15% of mothers' wage rates, compared to non-mothers' wage rates [Korenman Neumark (1992), Waldfogel (1997, 1998a)]. Only 18% of mothers, compared to 45% of non-mothers, earns more than 25% of the earnings for college-educated men in 1980s, but the family gap between fathers and non-fathers is found insignificant [Goldin (1997)].

This may also explain why female earnings and fertility rate are negatively correlated in the data [Butz and Ward (1979), Rosenzweig and Schultz (1985), and Moffitt (1984)].

There are several explanations on the family gap. One explanation is fertility choice. Theoretically, Becker (1985) shows that the responsibility of spending time and efforts on child-rearing usually falls on mothers. This may crowd out mothers' other activities, which also require time and efforts, such as job searching and career development. Empirically,

Another explanation is women's labour force participation rate, which is shown significant on women's earnings. The labor force participation rate of single women is showb always higher than that of married women [Goldin (1990), and Van Der Klaauw (1996)]. Moreover, the full-time earnings of non-mothers is subsequently higher than that of mothers, who broke their employment at child-birth, in the data of Britain [Joshi, Paci and Waldfogel(1999), and Waldfogel (1995)], and of Anglo-American, Continental European, Nordic countries [Sigle-Rushton and Waldfogel (2007)], and United States¹. The mothers, who maintain employed at childbirth, earn as well as non-mothers [Joshi, Paci and Waldfogel (1999)].

¹The average wage rate of a single female worker is approximately 20% higher than that of a married female worker [Van Der Klaauw (1996)].

The individual's time constraint indicates that childrearing may crowd out other activities that also require massive time and efforts. Many women might leave the labor force for child-rearing under some circumstances temporarily or permanently or they might optimally choose jobs with a higher quitting rate. This explanation is borne out by empirical studies which show that women frequently leave the labor force for pregnancy.

In this paper, our analysis focuses on how much the market frictions can affect the family gap how market frictions, fertility choice, and women's labour force participation affect the family gap, and how the family gap may affect the fertility choice and women's labour force participation rate.

This paper aims to look insight the relationships of market frictions, fertility choice, women's labour force participation and family gap, the wage gap between mothers and non-mothers, especially on how they affect each other when market frictions and fertility decisions are endogenously determined.

For women who hold positions, child-rearing crowds out their career development. This is especially true for women on the professional jobs that require massive amount of time and efforts at work, such as research scholars, computer programmers, medical doctors, lawyers..etc. Therefore, if these employed women want to develop career, they may need to either postpone the child birth or choose being childless. This could be one of the reasons behind the finding that despite the rapid growth in the women labour force participation rate since the end of World War II ², the fertility rate has been falling in the developed world. Or if these employed women want to bear children, they may need to postpone the career development, either by switching to the positions or jobs that require less time and efforts at work, or by giving up working, focusing on child-rearing and maybe returning to the labour force after a significant period of time. This may explain the U-shaped married women's labor force participation rate³ [Goldin (1990), Moffit (1984), and Addison and Portugal (1992)].

²Goldin (1990) shows that the labor force participation rate of women grows from 18.9% to 51.1% in the period of 1890-1980.

³That is, married women show the lowest interest to join the labor force at the middle stage of marriage, meaning 4-6 years after getting married.

The opportunity costs also exist on women who don't hold positions and have a new-born baby, especially for those who were engaged in job searching activities before the babies were born. child-rearing may take mothers' time away from their planned job searching activities. This may reduce the likelihood for mothers to find a job. Or for mothers who quit their jobs for child-rearing, the interruption on career development may reduce the likelihood for them to find a job. In a tight job market for workers, these mothers may quit searching for vacancies, leave the labour force, and stay at home for child-rearing. So the asymmetric market frictions, namely, the job matching and quitting rates, for mothers and non-mothers, and are important in explaining the fertility choice, the labour force participation, and the family gap.

Moreover, the labour force participation matters to a mother's earnings subsequently. On an individual level, a woman's earnings indicate her opportunity cost of leaving the labour force to raise a child. This opportunity costs of rearing a child are increasing in women's earnings. On the aggregate level, breaking the employment or postponing career development (switch to jobs that require less time and efforts) may stop or decrease a mother's earnings. It is found that mothers are paid lower wage rates than non-mothers, and that full-time earnings of non-mothers is subsequently higher than that of mothers who broke their employment at child-birth in the data of Britain [Joshi, Paci and Waldfogel(1999), and Waldfogel (1995)], and of Anglo-American, Continental European, and Nordic countries [Sigle-Rushton and Waldfogel (2007)]. Similar results are also found in United States⁴. This may also explain why female earnings and fertility rate are negatively correlated in the data [Butz and Ward (1979), Rosenzweig and Schultz (1985), and Moffitt (1984)]. Butz and Ward (1979) also point out that the baby bust in the 1960s might have resulted from the increase in the earnings of female workers. In contrast, that Therefore, the labour force participation is important for a mother's earnings, and is crucial in explaining the family gap.

While the non-working mothers sacrifice their earnings for child-rearing, the working mothers are paying baby-sitting costs, which may include other costs associated with baby-sitting, from their own earnings. This means that the baby-sitting costs could be crucial for a woman's decision on fertility and labour force participation. If a woman's earnings are not sufficient to pay baby-sitting

⁴The average wage rate of a single female worker is approximately 20% higher than that of a married female worker [Van Der Klaauw (1996)].

costs additional to other required living expenses, she may decide not to have a child and stay in the labour force. If having a child, she may leave the labour force for child-rearing to save the baby-sitting costs. Therefore, the child-rearing costs may play a role on a mother's reservation wage rate.

All observations mentioned above show that it is not child-rearing activities alone that causes the family gap. Women do have choices on fertility and on labour force participation partially if not fully. The crucial factors, including market frictions, productivity, home-stay alternative, opportunity costs for mothers and non-mothers, are important for women's decisions on fertility and labour force participation, and are important in determining the family gap. It is important not to neglect the effects of these factors on each other, which might narrow or widen the family gap. The effects of these factors on women's decisions and family gap may not be unidirectional. The goal of this paper is to find how these factors affect each other, then affect women's decisions on fertility and labour force participation, and hence, the family gap. In particular, the questions we would like to address in this paper are as follows. First, how do the asymmetric market frictions affect women's decisions on fertility and on labour force participation? How much can the asymmetric market frictions faced by mothers and non-mothers explain the family gap and fertility rate? Second, how does that home-stay alternative affect women's decisions on fertility and labour force participations, and the family gap? Third, by allowing for endogenous market frictions and home-stay alternative, will the family gap be enlarged, and will the negative correlation between wage rates and fertility rate still be captured and remain as strong as the case with exogenous market frictions and home-stay alternative? How does the family gap affect the market frictions, women's decisions on fertility and labour force participation?

A few papers are related to these issues in different aspects. Galor and Weil (1996) link the capital stock with the fertility rate and capture the gender wage gap by assuming that child-rearing places higher demands on women's time, and conclude that the causation underlying the negative relationship between fertility and female earnings need not be unidirectional⁵. In the existing theoretical analysis related to fertility rate analysis, the home stay alternative of mothers

⁵Galor and Weil (1996) argue that an increase in the capital-labor ratio will increase the real wage of women and this will cause a decline in fertility, which in turn enhances capital accumulation and raises women's wage rates.

are usually ignored due to the full-employment assumption. However, this home-stay alternative could be attractive for mothers and causes them to leave the labour force. The search equilibrium framework constructed by Moen (1997) allows for market frictions and show that the social optimal equilibrium allocation can be achieved under certain circumstances. Taking one of these literatures alone is not sufficient in answering the proposed questions.

To shed light on these questions, we introduce the home-stay alternative into a search-equilibrium based framework with endogenous fertility and home-staying choices. The focus of the model is on the interactions between female workers and firms in a labor market. Each woman decides whether to bear a child in the future at the beginning of her life; however, whether to conceive depends on an exogenous probability. The non-mothers would join the labour force while the mothers will decide whether to stay in the labour force. The mothers who stay in the labour force will conduct job-relating activities and child-rearing. The job-relating activity is working for employed mothers, and it is job-searching for unemployed mothers. If deciding to leave the labour force, the mothers would stay at home and focus on child-rearing. The case in which mothers who chose to stay home for child-rearing can return to the labor force later, and those who chose to do both child-rearing and working can leave the labour force will be discussed. The asymmetric market frictions are assumed in the general case where mothers face a lower job matching rate and a higher job quitting rate relative to non-mothers.

Consequently, we find that in the absence of labor-market frictions, the family gap between mothers and non-mothers is exactly the output gap of mothers and non-mothers. In the presence of market frictions, the causalities between fertility rate and women's job contact rate and between mother's transition rate of leaving labour force and women's job contact rate need not be unidirectional. Moreover, the more asymmetric market frictions for mothers and non-mothers are, the wider the family gap will be. The family gap is positively correlated with the output gap of mothers and non-mothers, and with the degree of asymmetric market frictions between mothers and non-mothers. The more asymmetric the market frictions for mothers and non-mothers, whether the job contact rates or the job separation rates, the wider the family gap. However, the overall effects of this the degree of asymmetric market frictions on fertility rate is ambiguous. While a

more symmetric job contact rate between mothers and non-mothers may stimulate women's labour force participation and reduce the unemployment duration, a more symmetric job separation rate would discourage the labour force participation and lengthen the unemployment duration. The policies, such as benefits to home-stay mothers, or subsidies to baby-sitting and day-care costs, are shown effective in stimulating fertility rate and labour force participation, in shrinking the family gap, and in reducing the unemployment duration.

The flow of the paper is as follows. The environment of the general case is described in section 2, followed by the specification on matching and bargaining technology in section 3. The equilibrium is derived in section 4, and the discussion on different cases is in section 5 and conclusion in section 6.

2 The Environment

The focus of this paper is to analyze how women's decisions on fertility and labour force participation, and market frictions affect each other, and hence, the wage differential between mothers' and non-mothers', the so called family gap. The market frictions in this paper is referred to the job matching rate and quitting rate. We incorporate an additional "home-staying" state of women in the search theoretical framework. Without losing generality, we assume that women make the fertility decision prior to joining the labour force. However, whether to have children would depend on both their fertility decision and the probability of conception, which is exogenously determined. This setup allows us to determine the population of women in different states, the fertility rate, the family gap, the labour force participation rate, the unemployment rate, and the unemployment duration of women.

The economy is populated with a continuum of available workers F , and a continuum of vacancies, V . Total population of women is normalized to one. A woman starts making her decisions at the time of her own birth, when she decides whether to be fertile. It is assumed that non-mothers always join the labour force, while mothers have an option of staying at home for child-rearing. The unemployed mothers who choose to join the labour force would face a lower job matching rate than the unemployed non-mothers since both child-rearing and job searching activities are time-

consuming. Moreover, the mothers who choose to stay at home for child-rearing for a significant period of time may face lower likelihood in finding jobs when returning to the labour force than non-mothers, due to the interruption in learning-by-doing, especially in the professional positions, which require more updated knowledge over time. The employed mothers might face a higher job quitting rate than employed non-mothers due to the child-rearing activities. For example, the sickness of children and the difficulty of finding suitable baby sitters or caretakers for a long period of time may cause a mother to quit the job. Therefore, in general, mothers face a lower job matching rate and a higher job quitting rate than non-mothers. This face is also assumed in the model.

The number of vacancies is determined by the market tightness (θ), which is defined as the ratio of unemployed workers to vacancies. This market tightness indicates the existence of an implicit entry barrier for firms. In particular, a greater entry barrier for firms indicates less vacancies, and hence, a tighter market for workers (a higher θ)⁶. The vacancies are assumed non-worker specific. This means that any vacancy can be filled by a mother or a non-mother. More details of the environment are described in the following subsections.

2.1 Populations of subgroups and states

In this model, we do take into account the population growth rate and death rate. The tasks of this subsection are to describe different states of women in this model, to specify the probabilities and arrival rates that divide women into different subgroups and to determine the populations of women in different states. Figure 1 shows the effects of women's fertile decisions on the population of the next generation, and Figure 2 depicts the changes of populations of each subgroup and the flows of workers in various subgroups.

As shown in Figure 1, women's make their fertility decisions at the beginning of their lifetime. With probability λ , a woman chooses to be fertile, and with probability $(1 - \lambda)$, a woman chooses not to be fertile. This fertile decision λ , which will be endogenously determined later, would divide

⁶One way to endogenously determine the market tightness is to set the equalization entry condition in which the flow value of an available vacancy is equal to the entry cost in equilibrium [Laing, Palivos and Wang (1995)]. However, it is hard to analyze the effect of the market tightness in fertility and labor force participation rates in this paper. Thus, we adopt the environment with the market tightness exogenously chosen.

women into two groups: F_{ON} (non-mothers decide not to fertile) and F_{OF} (not-yet-mothers decide to fertile). Note that the fertile decision does not make a woman become a mother. It is assumed that women who choose not to be fertile will not have a child and remain at F_{ON} group. A woman, who chooses to be fertile, would depend on an exogenous probability of conception π , to become a mother. It is π that divides the group F_{OF} further into two subgroups: F_{OF} (non-mothers, who decide to be fertile, fail to conceive) with probability $(1 - \pi)$, and F_1 (mothers, who decide to be fertile, do conceive) with probability π . This reflects the fact that despite the advanced technology such as the stem cell research and the assistance by laboratory experiments, the of-conception is still not completely controlled by human beings.

Let F denote the population of all women, and let F_i ($i = ON, OF, \text{ and } 1$) denote the population of women of a subgroup i .

$$F_{ON} + F_{OF} + F_1 = F \quad (1)$$

Note that in each generation, only those of conception would bring the next generation. By assuming the average number of baby girls of each woman who gives birth is k , the population of the next generation of women is $k\pi F_{OF}$. Then the new population will decide whether to fertile at the time of birth, denoted by λ , which divides the new generation into two streams, F_{ON} and F_{OF} .

As shown in Figure 2, the women in F_{OF} would conceive with probability π and become the members of F_1 . Since F_{OF} is the only source of F_1 , given the average death rate d , the changes of F_1 over time is:

$$\dot{F}_1 = \pi F_{OF} - dF_1 \quad (2)$$

The inflow to F_{OF} from the new generation is $\lambda k\pi F_{OF}$, and the inflow to F_{ON} from the new generation is $(1 - \lambda)k\pi F_{OF}$. So the changes of population in the states F_{ON} and F_{OF} are:

$$\dot{F}_{ON} = k(1 - \lambda)\pi F_{OF} - dF_{ON} \quad (3)$$

$$\dot{F}_{OF} = k\lambda\pi F_{OF} - dF_{OF} - \pi F_{OF}. \quad (4)$$

The boundary condition is set as the initial population of the whole economy is normalized to one:

$$F(0) = F_{ON}(0) + F_{OF}(0) + F_1(0) = 1. \quad (5)$$

By combining equations (2)-(5), the initial population of each subgroups can be solved as functions of the endogenous variable λ [details in appendix]:

$$F_{OF}(0) = \frac{d+n}{\Phi} \equiv F_{OF}(\lambda) \quad (6)$$

$$F_{ON}(0) = \frac{k(1-\lambda)\pi}{\Phi} \equiv F_{ON}(\lambda) \quad (7)$$

$$F_1(0) = \frac{\pi}{\Phi} \equiv F_1(\lambda) \quad (8)$$

, where $n \equiv k\lambda\pi - d - \pi$ and $\Phi \equiv d + n + \pi [k(1-\lambda) + 1]$. Both F_{OF} and F_1 are increasing in λ while F_{ON} is decreasing in λ . Moreover, the steady state population requires outflow and inflow to be equal:

$$dF = k\pi F_{OF}. \quad (9)$$

By plugging in equations (5) and (6) into (9), the steady state average number of children starting at the initial period is a function of λ :

$$k(0^+) = k(\lambda) = \frac{d\Phi}{\pi(d+n)}, \quad (10)$$

where $k(0^+)$ is decreasing in λ . That is, the more women choose to be fertile, the less children each woman will have in a steady state level.

2.2 Workers

2.2.1 Mothers

Each woman is assume unemployed when entering the labour force. Moreover, an employment may not be guaranteed by staying in the labour force. Therefore, some women may leave the labour force. Comparing mothers and non-mothers, it is usually mothers who are more likely to leave the labour force and stay home for child-rearing. To highlight this fact, we gave this "stay-home" choice

(H) to mothers only. Thus, a mother (F_1) may be in either of the these three states: unemployed (U_1), employed (L_1), and home-staying (H).

$$F_1(\lambda) = H + U_1 + L_1. \quad (11)$$

As shown in Figure 3, there are inflows and outflows among these three states. With the job contact rate $\beta\mu$ ($\beta < 1$), an unemployment mother U_1 will match up with a vacancy and becomes employed (L_1), and with the job divorce rate δ , an employed mother may be separated from her job and become unemployed (U_1). With the transition rate σ , an unemployed mother may leave the labour force and stay at home for child-rearing (H). Note that for an employed mother (L_1) to change her state to stay at home for child-rearing, she would be divorced from her job first, and then transit from U_1 to H . With an arrival rate τ^7 , a home-staying mother would return to the labor force, and start her job searching (U_1). In steady state, the following two equations must be satisfied:

$$\tau H = \sigma U_1 \quad (12)$$

$$\beta\mu U_1 = \delta L_1 \quad (13)$$

By combining equations (8), and (11)-(13), the populations of mothers in different states can be solved:

$$U_1(\lambda; \mu, \sigma) = \frac{\tau\delta}{\tau\delta + \sigma\delta + \beta\mu\tau} \left(\frac{\pi}{\Phi} \right) \quad (14)$$

$$H(\lambda; \mu, \sigma) = \frac{\sigma\delta}{\tau\delta + \sigma\delta + \beta\mu\tau} \left(\frac{\pi}{\Phi} \right) \quad (15)$$

$$L_1(\lambda; \mu, \sigma) = \frac{\beta\mu\tau}{\tau\delta + \sigma\delta + \beta\mu\tau} \left(\frac{\pi}{\Phi} \right). \quad (16)$$

The advantages of being a stay-home mother include not to be restricted by the office hours, the deadlines of job applications and interview schedules, and to have more flexible schedule for

⁷In this framework, we can either have σ or τ endogenously determined, but not both. Each has its own significance in interpreting the transition of female labour force and female labour force participation. In the baseline model, we have σ as an endogenous variable. The case in which τ is endogenously determined will be discussed in a later section of the paper.

personal activities. Let b denote the utility from the advantages of staying home and let w_1 denote an employed mother's wage rate, which will be determined by Nash bargaining process with equal division. Let $J^i (i = H, U_1, L_1)$ denote the value of a woman at state i . The flow values of each state are:

$$rJ^H = b + \tau(J^{U_1} - J^H) \quad (17)$$

$$rJ^{U_1} = \beta\mu(J^{L_1} - J^{U_1}) + \sigma(J^H - J^{U_1}) \quad (18)$$

$$rJ^{L_1} = w_1 + \delta(J^{U_1} - J^{L_1}) \quad (19)$$

In equation (17), a mother in the state H receives the utility b from staying at home and may return to the labor force and change the state to U_1 with the arrival rate τ . Similarly, the flow value of a mother in the state U_1 is the expected value of changing states: one is to get employed and change to L_1 with the job contact rate $\beta\mu$, and another is to leave the labour force and change to H with the transition rate σ [equation (18)]. A mother in the state L_1 receives the wage rate w_1 and could be separated from the job and change to U_1 with probability δ [equation (19)].

2.2.2 Non-mothers

Both subgroups, F_{OF} and F_{ON} , are non-mothers (F_0), who are assumed always staying in the labor force without the "stay-home" option. This is the main difference between mothers and non-mothers. Therefore, there are only two states for non-mothers: employed (L_0), and unemployed (U_0). The total population of non-mothers are:

$$F_0(\lambda) = F_{ON}(\lambda) + F_{OF}(\lambda) = U_0 + L_0 \quad (20)$$

Comparing to unemployed mothers, unemployed non-mothers may be able to spend more time on job searching. Therefore, the likelihood of a unemployed non-mother to get employed is larger than that of an unemployed mother. Let μ denote the job contact rate for an unemployed non-mother (U_0) to match with a vacancy and to get employed (L_0). Let $\gamma\delta$ ($\gamma < 1$) denote the job separation rate for an employed non-mother (L_0) to be separated from her current job and becomes unemployed (U_0). In steady state, the inflows equal outflows for each state:

$$\mu U_0 = \gamma\delta L_0 \quad (21)$$

Combining equations (6)-(7) and (20)-(21) gives the populations of U_0 and L_0 :

$$U_0(\lambda; \mu, \sigma) = \frac{\gamma\delta}{\gamma\delta + \mu} \left(1 - \frac{\pi}{\Phi}\right), \quad (22)$$

$$L_0(\lambda; \mu, \sigma) = \frac{\mu}{\gamma\delta + \mu} \left(1 - \frac{\pi}{\Phi}\right). \quad (23)$$

The wage rate received by an employed non-mother is w_0 , which is also determined by Nash bargaining process with equal division. The flow value of non-mothers in the states of L_0 and U_0 are:

$$rJ^{U_0} = \mu(J^{L_0} - J^{U_1}) \quad (24)$$

$$rJ^{L_0} = w_0 + \gamma\delta(J^{U_0} - J^{L_0}) \quad (25)$$

Equation (24) shows that an unemployed non-mother in the state U_0 may get employed and change to state L_0 with the job contact rate μ . An employed non-mother in the state L_0 would receive the wage rate w_0 but could be separated from her job and change to state U_0 with probability $\gamma\delta$ [equation (25)].

2.3 Vacancies

The vacancies are assumed to be non worker-specific and it is indivisible. That is, a vacancy can be filled by either a mother or a non-mother. This gives a vacancy three states: empty (V), filled by a mother (L_1), and filled by a non-mother (L_0). In general, the child-rearing activities may distract mothers from their work. To shed light on this fact, it is assumed that the average productivity of a mother (y_1) is different from that of a non-mother (y_0)⁸. Both y_0 and y_1 are public information and exogenously determined. Since the mass of vacancies may be different from the mass of unemployed female workers, the job contact rate of a vacancy to an unemployed worker (η) may be different from the job contact rate of an unemployed worker to a vacancy (μ and $\beta\mu$). That is, a vacancy would be filled with the exogenous probability η . Let $\Pi^j(j = V, L_1, L_0)$ denote the flow value of a

⁸Another interpretation behind this assumption is that the average hourly productivities of mothers and non-mothers are the same, but mothers need to reserve some time for child-rearing activities, and tend to pick up part-time positions. Thus, the total monthly or annual productivity of a mother is lower than that of a non-mother.

vacancy in the state j .

$$r\Pi^{L_1} = (y_1 - w_1) + \delta(\Pi^V - \Pi^{L_1}), \quad (26)$$

$$r\Pi^{L_0} = (y_0 - w_0) + \gamma\delta(\Pi^V - \Pi^{L_0}), \quad (27)$$

$$r\Pi^V = \eta [\rho_1 (\Pi^{L_1} - \Pi^V) + \rho_0 (\Pi^{L_0} - \Pi^V)] \quad (28)$$

, where $\rho_1 \equiv \frac{U_1}{U}$, $\rho_0 \equiv \frac{U_0}{U}$, $U = U_0 + U_1$, and $\rho_1 + \rho_0 = 1$. As shown in equations (26) and (27), the flow value of a vacancy filled by a female worker is the sum of the net profit created by that particular female worker and the expected value of changing states when the employed worker leaves this position. The flow value of an empty vacancy (Π^V) [equation (28)] is the expected value of this vacancy to be filled by either an unemployed mother or an unemployed non-mother. By plugging equations (26) - (27) into equation (28), we get:

$$\Pi^V(\lambda; \mu, \sigma) = \frac{\eta [\rho_1 (r + \gamma\delta) (y_1 - w_1) + \rho_0 (r + \delta) (y_0 - w_0)]}{r \{(r + \delta) (r + \gamma\delta) + \eta (r + \rho_1\gamma\delta + \rho_0\delta)\}}. \quad (29)$$

Then $\Pi^{L_0}(\lambda; \mu, \sigma)$ and $\Pi^{L_1}(\lambda; \mu, \sigma)$ can be obtained by substituting equation (29) into equations (26) and (27).

2.4 Matching and Bargaining Technology

In this search-theoretical framework, unemployed workers and vacancies are the two parties searching for each other to find a match. It is assumed that one position fits only one worker, and vice versa. Thus, the flow match of unemployed workers must equal to the flow match of vacancies. It is assumed that this match process is based on a random matching function, $q_0Q(U, V)$, which exhibits constant return to scale:

$$\mu U_0 + \beta\mu U_1 = \eta V = q_0Q(U, V), \quad (30)$$

where q_0 represents a matching technology. For example, the web site creation (e.g. JOE) may drive up the matching rate between unemployed workers and vacancies, which would be reflected in a higher q_0 . [Diamond (1982a, 1982b, and 1984), and Laing, Palivos, and Wang (1995)].

Once does a vacancy and an unemployed worker meet, they would determine the wage rate based on a simple symmetric cooperative Nash bargaining rule with equal division of the surplus.

This means that the wage rates will be determined at a level where the joint surplus of the vacancy and the worker will be maximized [equations (31) and (32)]:

$$\max_{w_1} [\Pi^{L_1} - \Pi^V]^{1/2} [J^{L_1} - J^{U_1}]^{1/2} \quad (31)$$

$$\max_{w_0} [\Pi^{L_0} - \Pi^V]^{1/2} [J^{L_0} - J^{U_0}]^{1/2} \quad (32)$$

The optimization of equations (31) and (32) gives:

$$J^{L_1} - J^{U_1} = \Pi^{L_1} - \Pi^V \quad (33)$$

$$J^{L_0} - J^{U_0} = \Pi^{L_0} - \Pi^V \quad (34)$$

, which determines w_1 and w_0 , respectively. The equilibrium wage rates $w_0^* = w_0(\lambda; \mu, \sigma)$ and $w_1^* = w_1(\lambda; \mu, \sigma)$ can then be pinned down by the following two equations, respectively:

$$\left[\frac{1}{r + \mu + \gamma\delta} + \frac{r + \delta + \eta\rho_1}{B} \right] w_0^* - \frac{\eta\rho_1}{B} w_1^* = \frac{(r + \delta + \eta\rho_1) y_0 - \eta\rho_1 y_1}{B} \quad (35)$$

$$-\frac{\eta\rho_0}{B} w_0^* + \left[\frac{r + \tau + \sigma}{A} + \frac{r + \gamma\delta + \eta\rho_0}{B} \right] w_1^* = \frac{-\eta\rho_0 y_0 + (r + \gamma\delta + \eta\rho_0) y_1}{B} + \frac{\sigma b}{A} \quad (36)$$

, where $A \equiv \sigma(r + \delta) + (r + \tau)(r + \delta + \beta\mu)$, $B \equiv (r + \delta)(r + \gamma\delta) + \eta[\rho_1(r + \gamma\delta) + \rho_0(r + \delta)]$.

3 Equilibrium

In equilibrium, the values of λ , μ , and σ can be solved endogenously. Then all populations and equilibrium wage rates, which can be derived as functions of λ , μ , and σ , can all be determined. While μ can be solved by matching function [equation (30)], both λ and σ will be solved by the no-arbitrage conditions.

By plugging equations (14) and (22) into equation (30), we can rewrite V as:

$$V(\lambda; \mu, \sigma) = \frac{\mu}{\eta} \left[\frac{\gamma\delta}{\gamma\delta + \mu} \left(1 - \frac{\pi}{\Phi} \right) + \frac{\beta\tau\delta}{\tau\delta + \sigma\delta + \beta\mu\tau} \left(\frac{\pi}{\Phi} \right) \right]. \quad (37)$$

Let $\theta \equiv U/V$ denote the market tightness. An increase in θ indicates an increasing difficulty for unemployed workers to match with a vacancy. Equations (30) and (37) can be written as:

$$\theta = \theta(\lambda; \mu, \sigma) = \frac{U_0(\lambda; \mu, \sigma) + U_1(\lambda; \mu, \sigma)}{V(\lambda; \mu, \sigma)} \quad (38)$$

$$\eta = q_0(\theta, 1) \quad (39)$$

where equation (38) has the character $\partial\theta/\partial\mu < 0$. Both equations (38) and (39) can be plotted on a (η, μ) space and to determine $\mu^* = \mu(\lambda, \sigma)$ [Figure 5].

3.1 No-arbitrage Conditions

The no-arbitrage conditions are to reflect the situations faced by women when they make decisions. The conditions states that a woman feels indifferent in the choices she has when making decisions. In other words, the flow values of each option is the same. Equation (41) shows the no-arbitrage condition of a woman when making fertility decision, while equation (40) is the condition of a mother when making labour force participation decision. Note that when making her labour force participation decision, a mother would have to consider the baby-sitting and day-care costs, denoted by χ , which are not counted in the flow values of joining the labour force (J^{U1}). That is because even before becoming employed, a mother may need to pay for baby-sitting or day-care costs to attend job interviews or job training lessons. These baby-sitting and day-care costs are not avoided for a mother staying in the labour force.

$$J^H = J^{U1} - \chi, \tag{40}$$

$$J^{ON} = J^{OF}. \tag{41}$$

By substituting μ^* in equations (40) and (41), we could use Cremer's Rule to solve both λ and σ [details in appendix]. This case where λ , μ , and σ are endogenously determined, is called general case. The results of this general case are included in the next section. In order to evaluate how the crucial endogenous variables affect the equilibrium, we now turn our attention to discuss cases in which either one or all are exogenous.

4 Case Studies

In this section, the focus will be on the family gap, women's labour force participation rate, unemployment rate in equilibrium in three cases. These three cases are when both market tightness θ and the transition rate σ are exogenous (case I), when θ is exogenous and σ is endogenous (case II),

and when both θ and σ are endogenous (case III, general case). Then through the comparison of three cases, we could analyze the effects of θ and σ on fertility rate λ and Π^V , then on the family gap, women's labour force participation rate, and unemployment rate. The family gap is the wage gap of mothers and non-mothers, defined as $WD \equiv w_0 - w_1$. In this model, only mothers in the state H do not join the labour force, so the women's labour force participation rate is $LPR \equiv 1 - \frac{H}{F}$. Unemployment rate, defined as $UER \equiv \frac{U}{F-H}$, is the ratio of women in the labour force but not yet employed.

There have been several policies proposed to stimulate fertility rate in developed countries. One of the policies widely adopted is child benefits. In this model, this policy has to be discussed in two different aspects. One aspect is to improve the utility of staying home for mothers (an increase in b), and another aspect is to subsidize baby sitting costs (an increase in χ). The effects of these two variables will be our focus in each case. More of other possible policies will be analyzed in the general case (case III).

4.1 Case I: exogenous market tightness(θ) and transition rate(σ)

According to equation (30), both μ and η can be written as $\mu^*(\theta)$ and $\eta^*(\theta)$. After combining the equations (26)-(28), and (33)-(34), the values of λ , w_0 and w_1 can be determined. Furthermore, both w_0 and w_1 react to Π^V negatively, and w_0 reacts to Π^V more strongly than w_1 does [see equations (A17) and (A18) in appendix]. Then equations (A17), (A18), and (41) jointly provide the equilibrium equations (A19), called $\Pi^U(Value)$ locus, and (A20), called $\Pi^U(NA)$ locus, to determine both Π^V and λ ⁹. As shown in Figure 6, while $\Pi^U(NA)$ locus is independent of λ , $\Pi^U(Value)$ locus is decreasing in λ .

The family gap WD and unemployment rate UER are both increasing in λ and LPR is decreasing in λ . That could be caused by the positive effect of λ on H . When a vacancy (Π^V) becomes more valuable (an increase in Π^V), women would respond by reducing the fertility rate (λ) to compete on matching with that vacancy. Therefore, LPR increases, UER decreases, and WD shrinks.

⁹Note that λ endogenously determines ρ_i ($i = 0, 1$), which appears on equation (A19), the $\Pi^U(Value)$ locus. $\Pi^U(NA)$ is independent of λ .

If there is an increase in the utility of home staying b , would move both $\Pi^U(Value)$ and $\Pi^U(NA)$ downward. Thus, λ would increase in response to an increase in b since the effects of b via $\Pi^U(NA)$ locus tend to dominate. This means that an increase in home-staying utility for mothers would encourage the fertility rate (λ). This will in turn, enlarge the family gap, decrease women's labour force participation rate, and increase women's unemployment rate.

However, the change of baby sitting and day care costs χ has no effects in this case. Therefore, when the market tightness and the transition rate are exogenous, improving home-stay mothers' utility (an increase in b) is more effective in stimulating the fertility rate (λ) than subsidizing baby sitting and day care costs.

4.2 Case II: exogenous market tightness(θ), and endogenous transition rate(σ)

In this case, the transition rate (σ) of mothers to leave the labour force is endogenous determined. To do so, equation (40) is introduced, and together with equations (A19)-(A21), the equilibrium values of Π^V , λ , and σ can be determined.

An increase in the utility of home staying b would lead more mothers to leave the labour force, and cause σ and λ to increase, but Π^V to decrease. Note that an increase in b would reduce Π^V as case one and enlarge the family gap. Additionally, an increase in σ caused by a higher b , on one hand, would reduce (Π^V) further and widen the family gap (WD). On the other hand, an increase in σ drives up mother's wage rate (w_1) by increasing mothers' reservation wage. This would shrink the family gap (WD). If the effect of b via σ dominate that via Π^V , the family gap in case two is smaller than that in case one, $WD^{II} < WD^I$.

Note that the decrease in Π^V increases both w_0 and w_1 , and an increase in σ would drive up w_1 further. This pecuniary advantage for all women, mothers and non-mothers, would attract women to join the labour force and to demotivate them to have children (a decrease in λ). However, the increase in σ would stimulate more women to bear children (an increase in λ). When the negative effect of b on Π^V is sufficiently small, the overall effect of an increase in b would drive up fertility rate (λ). Therefore, this increase in both σ and λ caused by a higher b would reduce LPR . The LPR in case two would be lower than that in case one, $LPR^{II} < LPR^I$. However, the increase in λ and σ affect UER in opposite directions and the positive effects of λ on UER is stronger than

the negative effects of σ on UER . Therefore, the overall effect of b on UER is positive, and UER in case two is lower than that in case one, $UER^{II} < UER^I$.

A subsidy on baby-sitting and day care costs for mothers (a decrease in χ) would increase σ , Π^V , and λ . This, in turn, would shrink WD , increase LPR , and decrease UER due to the stronger effects of χ via λ on UER than via σ . Therefore, in this case, both improving the utility of home-stay mothers (an increase in b) and subsidizing in baby-sitting and day care costs (χ) would both be effective in enlarging the family gap.

4.3 Case III (general case): endogenous market tightness(θ) and transition rate(σ)

The case in which both market tightness (θ) and transition rate of leaving the labour force (σ) are endogenously determined is discussed. Equation (37) is introduced to pin down the value of θ . Equation (37) can be written as $\theta = \theta(\eta)$, where θ is increasing in η . Together with equations (A19)-(A21), we can now solve the equilibrium values of σ , λ , Π^V , and μ and determine the effects of the important factors, such as the values of staying home (b), the baby-sitting and day care costs (χ), and the degree of symmetry on the job frictions of mothers and non-mothers (β, γ), on the equilibrium values, and hence on the family gap (WD), women's labour force participation rate (LPR), and the unemployment rate of women (UER).

According to equation (A22), in this general case, an increase in b would drive up both λ and σ , then affect θ indirectly via σ and λ . While a higher λ increases θ and μ , a higher σ decreases θ and μ . In turn, the decrease in μ would reduce σ but would push up λ to a higher level, and hence, increases θ and μ to higher values. Therefore, an increase in b would stimulate μ . This increase in μ would increase λ , Π^V , but would decrease σ .

An increase in μ caused by an increase in b would drive up all women's wage rates, both w_0 and w_1 , and widen the family gap (WD) since the impacts of μ on w_0 is stronger than on w_1 . However, the strong negative direct effects of b on WD together with the indirect negative effects of μ on WD via Π^V , an increase in b would shrink WD overall. Meanwhile, LPR would increase, and UER would decrease if the direct effect of μ on UER dominate.

This case has shown that the causalities between fertility rate (λ) and women's job contact rate

(μ) and between mother's transition rate of leaving labour force (σ) and women's job contact rate (μ) need not be unidirectional. The women's contact rate (μ) affects WD , LPR , and UER directly and indirectly via λ , σ , and Π^V . Moreover, some of the indirect effects might offset the direct effects of μ on WD , LPR and UER . More interestingly, if direct effects dominate, the overall effects of b on LPR and UER in case III become opposite to those in case II. It would require the indirect effects of b on LPR and on UER via λ then μ to be sufficiently strong to dominate the direct effects, in order for the overall effects of b on LPR and UER consistent with case I and II. Then the comparison on three cases would give $LPR^{II} < LPR^{III}$, and $UER^{III} < UER^{II} < UER^I$.

Moreover, the endogenously determined market tightness θ in this job matching model also allows us to discuss the duration of unemployment for women. The unemployment duration is defined as: $T(\mu, \lambda, \sigma) \equiv \frac{\theta}{\eta}$ and has the following characteristics: $\partial T/\partial \mu < 0$, $\partial T/\partial \lambda > 0$, $\partial T/\partial \sigma < 0$ [see the appendix]. That means that an increase in job contact rate (μ) would reduce the market tightness (θ) and make it easier for an unemployed worker to match with a vacancy; hence, the unemployment duration will be reduced. In response to an increase in b , the unemployment duration (T) would be lengthen.

The effects of a subsidy in χ on θ , σ , and λ , then on WD , LPR , UER and T are similar to that of an increase in b . It is worth to note that the change in χ has only indirect effects via both Π^V and σ on λ , and both indirect effects affect λ in the same direction. The change in b , however, has direct effects on λ , but its indirect effects via Π^V and σ are opposite. Therefore, it is unclear whether it is an increase in b or a decrease in χ that would have stronger effects on the equilibrium values, and hence on WD , LPR , UER and T .

Additional to the effects of b and χ , the variables that cause the asymmetry of mothers and non-mothers are worth discussion. They are γ and β . An increase in γ indicates an increase in the job separation rate for non-mothers, and an increase in β means that mothers are facing an increase in the job contact rate. At $\gamma = 1$, mothers and non-mothers face the same job separation rates, and at $\beta = 1$, mothers and non-mothers face the same job contact rates. Both An increase in β and a decrease in γ affect λ , Π^V and σ , and hence, μ , LPR , UER and T in the same direction. To be more specific, an increase in β decreases σ and Π^V . Although it has direct positive effects on λ ,

both of its indirect effects on λ via σ and Π^V are in the opposite direction. This leaves the overall effects of an increase in β on λ ambiguous, while the dominant direct effects of β would result in an increase in LPR , and a decrease in UER , θ , and T . Meanwhile, both β and γ affect WD negatively and directly, and this direct effects of β and γ on WD tend to dominate the indirect effects.

Above all, we can conclude that the policies towards to symmetric job frictions for mothers and non-mothers, either on the job contact rate ($\beta = 1$) or on the job separation rate ($\gamma = 1$) would shrink the family gap but leave the overall effects on fertility rate ambiguous. Meanwhile, while the policies that increase β would stimulate women's LPR , and reduce the UER and T , the policies increase γ would do the opposites on LPR , UER and T . The policies that target on benefitting mothers, such as subsidies on baby sitting and day care costs or on home-stay mothers, may stimulate the fertility rate, but might widen the family gap as well.

5 Conclusion and Extensions

The features of the search theoretical framework allows us to look insights the links of family gap, fertility rate, labour force participation rate, unemployment rate and duration, and market frictions and to evaluate the effectiveness of various policies on shrinking the family gap and stimulating the fertility rate. We find that the policies towards to symmetric job frictions for mothers and non-mothers may shrink the family gap, but may not stimulate the fertility rate. Moreover, while an increase in mothers' job contact rate might increase women's labour force participation rate, and reduce the unemployment rate and unemployment duration, an increase on non-mothers' job separation rate would do the opposites to the labour force participation rate, unemployment rate and unemployment duration. The policies, that encourage mothers to stay-home, and subsidize baby sitting and day care costs, may be effective in both stimulating fertility rate and women's labour force participation rate, shrinking the family gap, and reducing the unemployment duration.

There are several possible extension from this model. The first possible extension is to introduce heterogeneity of vacancies and workers, and allow for segmented job markets. To be more specific, vacancies could target on either mothers or non-mothers. Mothers and non-mothers could have different preferences over positions. The second extension is to allow for multi-matching on

vacancies and workers. This extension could address the issues on part-time and full-time positions. It is possible for a worker to work for several part-time positions, as well as for one position filled in with several part-time workers taking turns to work. The third extension is to allow for more states and to allow women to change between different states. For example, this model has restrictions in evaluating the effectiveness of maternity leave on the family gap and fertility rate. All these possible extensions might provide insights of the links between job markets, women's decision making on fertility and labour force participation. The understanding on these links would be helpful for policy makers who are interested in stimulating the fertility rate, and shrinking the family gap, and hence, possibly the gender gap, at the mean time.

Appendix

Steady state population flow:

Solving the differential equations, the general solution for F_{OF} can be written as:

$$\begin{aligned} F_{OF}(t) &= F_{OF}(0)e^{(k\lambda\pi-d-\pi)t} \\ &= F_{OF}(0)e^{nt} \end{aligned} \tag{A1}$$

where $n \equiv k\lambda\pi - d - \pi$. Similarly, one can write:

$$F_1(t) = F_1(0)e^{nt} \tag{A2}$$

$$F_{ON}(t) = F_{ON}(0)e^{nt} \tag{A3}$$

Equation (2) can now be written as:

$$\frac{dF_1(t)}{dt} = F_1(0)ne^{nt} = \pi F_{OF} - dF_1 \tag{A4}$$

Therefore, once can solve

$$F_1(0) = \frac{\pi F_{OF}(0)}{d+n} \tag{A5}$$

Following the same steps, we find

$$F_{ON}(0) = \frac{k(1-\lambda)\pi F_{OF}(0)}{d+n} \tag{A6}$$

Combining with equation (5), and letting $\Phi \equiv d + n + \pi[k(1-\lambda) + 1]$, the population of all subgroups can be found.

$$F_{OF}(0) = \frac{d+n}{\Phi} \equiv N_{OF}(\lambda) \tag{A7}$$

$$F_1(0) = \frac{\pi}{\Phi} \equiv N_1(\lambda) \tag{A8}$$

$$F_{ON}(0) = \frac{k(1-\lambda)\pi}{\Phi} \equiv N_{ON}(\lambda). \tag{A9}$$

The value of F: Solved from population

From Figure 2, the flows of the population should be equal for each status, and based on market clearing condition, 2 flow population equations are sufficient. Thus, the flow population equations for statuses EC and UD are chosen:

$$(EC) : \delta L^C = \beta\mu(1-\phi)\lambda F \tag{A10}$$

$$(UD) : \gamma\delta L^D = \mu(1-\lambda)F \tag{A11}$$

Plus the equation of total population normalization: $F + L^C + L^D = 1$. Then all population can be solved:

$$F = \frac{1}{1 + \frac{\beta\mu\lambda(1-\phi)}{\delta} + \frac{\mu(1-\lambda)}{\gamma\delta}}; L^C = \frac{\beta\mu(1-\phi)\lambda F}{\delta}; L^D = \frac{\mu(1-\lambda)F}{\gamma\delta} \quad (\text{A12})$$

Section 4:

Given the population of HC and SC and the Poisson arrival rates from HC to SC , τ , and from SC to HC , σ , the following equation can be obtained: $\tau\lambda\phi F = \sigma\lambda(1-\phi)F$. Thus, $\phi = \sigma/(\tau + \sigma)$.

Given the calculation details in the previous sections, the wage rates of workers with and without children when the exchangeability is allowed are given by:

$$w^C(\lambda) = y^C - \frac{\{A(\lambda) + r + \delta + \eta\lambda(1-\phi)\}M\{(r + \tau + \sigma)y^C - \sigma b\} + \eta(1-\lambda)By^D}{\Delta(\lambda)} \quad (\text{A13})$$

$$w^D(\lambda) = y^D - \frac{[(r + \tau + \sigma)A(\lambda) + NB]y^D + \eta\lambda(1-\phi)M\{(r + \tau + \sigma)y^C - \sigma b\}}{\Delta(\lambda)} \quad (\text{A14})$$

where $M \equiv r + \gamma\delta + \mu$, $N \equiv r + \gamma\delta + \eta(1-\lambda)$, $B \equiv (r + \delta)(r + \tau + \sigma) + \beta\mu(r + \tau)$, $A(\lambda) \equiv (r + \delta)N + \eta\lambda(1-\phi)(r + \delta)$, $\Delta(\lambda) \equiv (r + \tau + \sigma)\{A(\lambda) + [r + \delta + \eta\lambda(1-\phi)]M\} + B[M + N]$.

No-arbitrage conditions: By substituting $\mu^* w_1^*(\lambda, \sigma)$, $w_0^*(\lambda, \sigma)$ into equations (40) and (41), we get:

$$\sigma(r + \delta)\chi = \beta\mu^* w_1^*(\lambda, \sigma) - (r + \delta + \beta\mu^*)[(r + \tau)\chi + b] \quad (\text{A15})$$

$$\begin{aligned} & \sigma(r + \delta)[\pi(r + \mu^* + \gamma\delta)(b + r\varepsilon) - (1 + \pi)\mu w_0^*(\lambda, \sigma)] \\ &= (1 + \pi)\mu^*(r + \tau)(r + \delta + \beta\mu^*)w_0^*(\lambda, \sigma) \\ & \quad - \pi(r + \mu^* + \gamma\delta)[\beta\mu^*(r + \tau)w_1^*(\lambda, \sigma) + r\varepsilon(r + \tau)(r + \delta + \beta\mu^*)]. \end{aligned} \quad (\text{A16})$$

Section 5

Case I: exogenous θ and σ

Combining equations (35), (36) and (41) to determine the values of λ , w_0 , and w_1 . We first derive w_0 and w_1 as functions of Π^V :

$$w_0(\Pi^V) = \frac{(r + \mu + \gamma\delta)(y_0 - r\Pi^U)}{\mu + 2(r + \gamma\delta)} \quad (\text{A17})$$

$$w_1(\Pi^V) = \frac{A(y_1 - r\Pi^U) + \sigma b(r + \delta)}{A + (r + \tau + \sigma)(r + \delta)}, \quad (\text{A18})$$

which shows that $\partial w_i / \partial \Pi^V < 0$ ($i = 0, 1$), and $|\partial w_0 / \partial \Pi^V| > |\partial w_1 / \partial \Pi^V|$.

Then plugging both equations (A17) and (A18) into equations (29) and (41) gives the following equilibrium conditions to determine Π^V and λ .

$$\left\{ 1 - \frac{\eta}{B} \left\{ + \rho_0 \left[\frac{\frac{A(r+\gamma\delta)}{A+(r+\tau+\sigma)(r+\delta)}}{\mu+2(r+\gamma\delta)} - \frac{A(r+\gamma\delta)}{A+(r+\tau+\sigma)(r+\delta)} \right] \right\} \right\} \Pi^V \quad (\text{A19})$$

$$= \frac{\eta(r+\gamma\delta)(r+\delta)}{rB} \left\{ \frac{(r+\tau+\sigma)y_1 - \sigma b}{A+(r+\tau+\sigma)(r+\delta)} + \rho_0 \left[\frac{y_0}{\mu+2(r+\gamma\delta)} - \frac{(r+\tau+\sigma)y_1 - \sigma b}{A+(r+\tau+\sigma)(r+\delta)} \right] \right\}$$

$$\left[\frac{\mu r(1+\pi)}{\mu+2(r+\gamma\delta)} - \frac{r\pi\beta\mu(r+\tau)}{A+(r+\tau+\sigma)(r+\delta)} \right] \Pi^V \quad (\text{A20})$$

$$= \frac{(1+\pi)\mu y_0}{\mu+2(r+\gamma\delta)} - \frac{\pi}{A} \left\{ rA\varepsilon + (r+\delta)\sigma b + \beta\mu(r+\tau) \left[\frac{Ay_1 + (r+\delta)\sigma b}{A+(r+\tau+\sigma)(r+\delta)} \right] \right\}.$$

Case II: exogenous θ and endogenous σ

Rearranging both equations (40) and (A20) gives:

$$\Pi^V = \frac{\beta\mu [Ay_1 + (r+\delta)\sigma b] - \{(r+\delta)\sigma\chi + (r+\delta+\beta\mu)[b + (r+\tau)\chi]\}C}{r\beta\mu A} \quad (\text{A21})$$

$$= \frac{\frac{(1+\pi)\mu y_0}{E} - \frac{\pi}{A} \left\{ rA\varepsilon + (r+\delta)\sigma b + \beta\mu(r+\tau) \left[\frac{Ay_1 + (r+\delta)\sigma b}{C} \right] \right\}}{\left[\frac{\mu r(1+\pi)}{E} - \frac{r\pi\beta\mu(r+\tau)}{C} \right]}$$

where $C \equiv A + (r + \tau + \sigma)(r + \delta)$, and $E \equiv \mu + 2(r + \gamma\delta)$. Equation (A21) helps pin down equilibrium values of σ^* and $(\Pi^V)^*$, which can be substituted into equation (A19) to determine equilibrium value of λ^* . In turn, both equilibrium values of w_0^* and w_1^* can be determined.

Case III: endogenous θ and σ

By combining equations (37) to (39), we can re-write θ as follows:

$$\theta \equiv \frac{U}{V} = \left(\frac{\eta}{\mu} \right) \left[\frac{\frac{\gamma\delta\Phi}{\gamma\delta+\mu} + \frac{[\mu\tau\delta(1-\beta\gamma) - \gamma\sigma\delta^2]\pi}{(\gamma\delta+\mu)(\tau\delta+\sigma\delta+\beta\mu\tau)}}{\frac{\gamma\delta\Phi}{\gamma\delta+\mu} + \frac{\{\tau\delta[\mu\beta(1-\gamma) - \gamma\delta(1-\beta)] - \gamma\sigma\delta^2\}\pi}{(\gamma\delta+\mu)(\tau\delta+\sigma\delta+\beta\mu\tau)}} \right] = \theta(\eta) \quad (\text{A22})$$

where θ is increasing in η , and equilibrium μ can be determined by equations (38) and (39), as shown in Figure 5. It is through the job contact rate μ that the factors affect WD , LPR , and UER .

Unemployment duration $T(\mu, \lambda, \sigma) \equiv \frac{\theta}{\eta}$, and by applying equation (A22), the unemployment duration can be written as:

$$T(\mu, \lambda, \sigma) \equiv \frac{\theta}{\eta} = \left(\frac{1}{\mu} \right) \left[\frac{\frac{\gamma\delta\Phi}{\gamma\delta+\mu} + \frac{[\mu\tau\delta(1-\beta\gamma) - \gamma\sigma\delta^2]\pi}{(\gamma\delta+\mu)(\tau\delta+\sigma\delta+\beta\mu\tau)}}{\frac{\gamma\delta\Phi}{\gamma\delta+\mu} + \frac{\{\tau\delta[\mu\beta(1-\gamma) - \gamma\delta(1-\beta)] - \gamma\sigma\delta^2\}\pi}{(\gamma\delta+\mu)(\tau\delta+\sigma\delta+\beta\mu\tau)}} \right]$$

where T has the following characteristics: $\partial T/\partial\mu < 0, \partial T/\partial\lambda > 0, \partial T/\partial\sigma < 0$.

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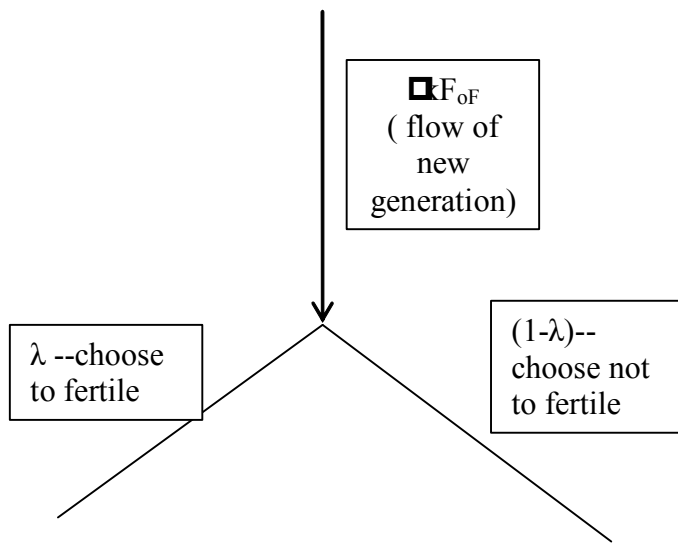


Figure 1: The flow of new generation

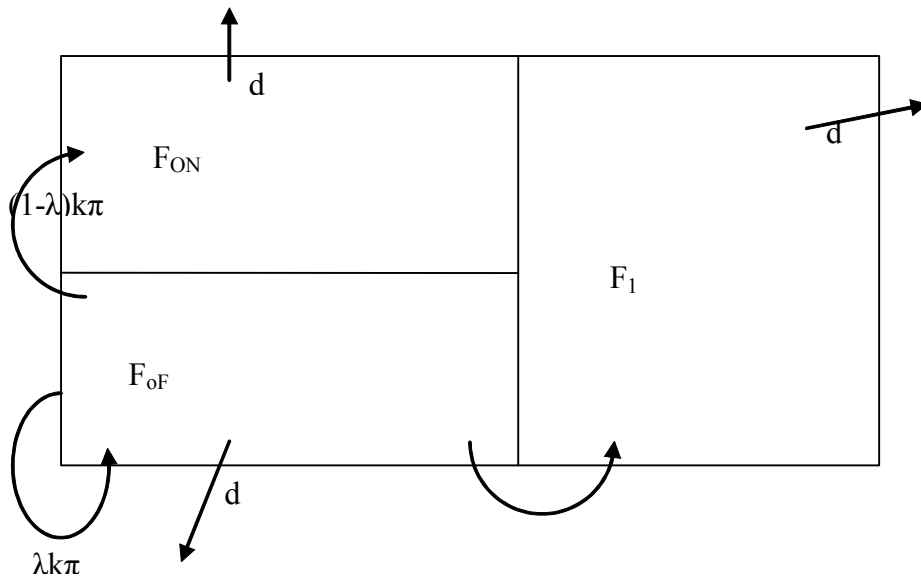


Figure 2: Population flows

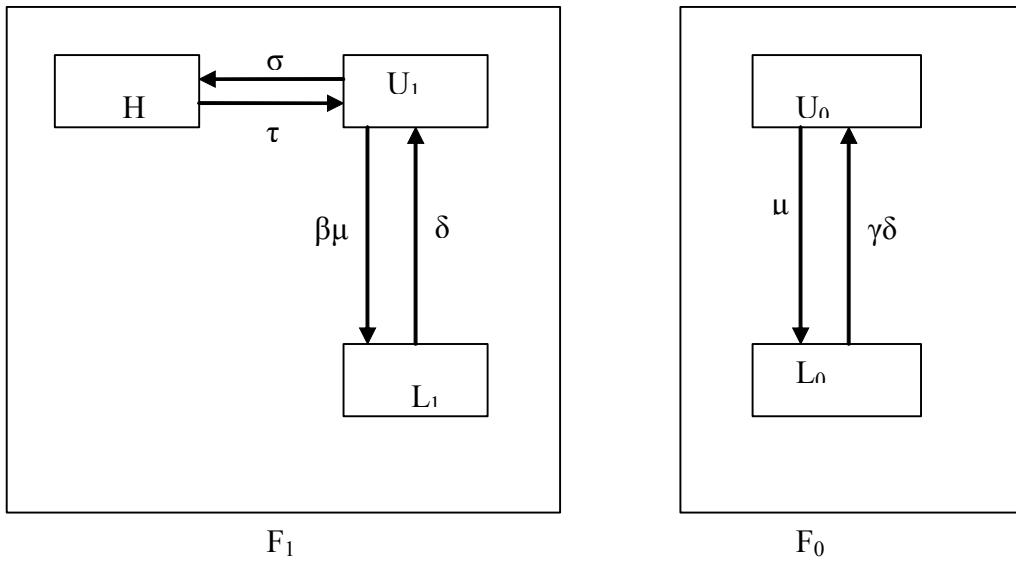


Figure 3: State Flows

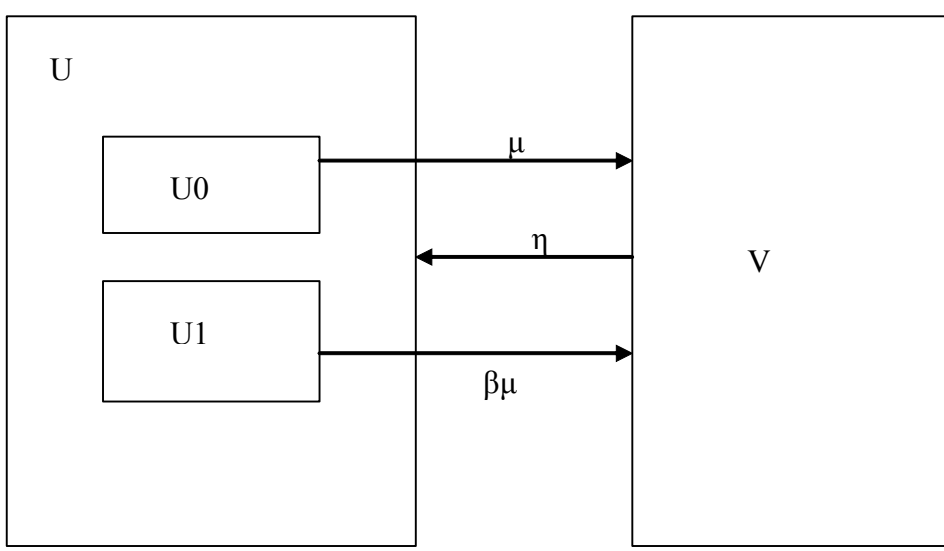


Figure 4: Job Contact Rates

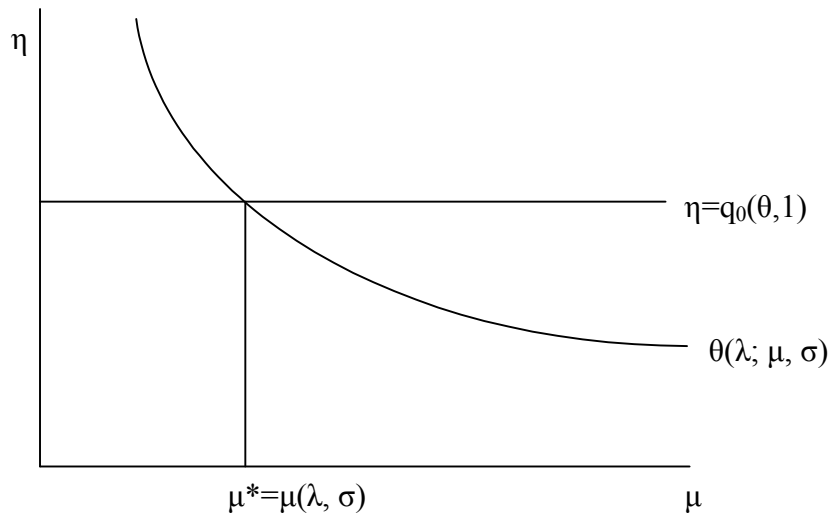


Figure 5: (μ, η) Locus

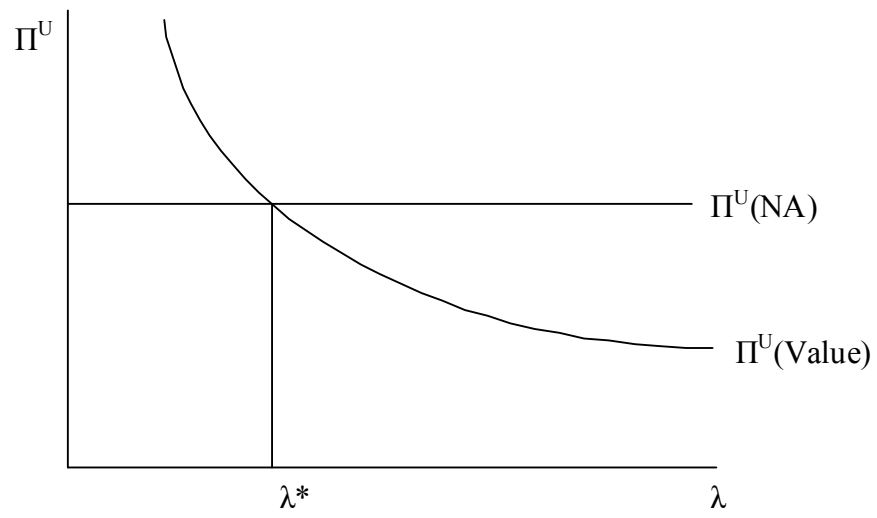


Figure 6: (Π^U, λ) Locus