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### New Entries and Economic Growth

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The main goal of this paper is to construct a theoretical model that provides an explanation for the relationship between growth and new entry that is consistent with empirical evidence. The model is a four sector endogenous growth model in which there is a technologically advanced and a technologically laggard consumption goods which are imperfect substitutes. The production of each good requires its own stock of human capital and physical capital. The accumulation of physical capital and human capital in each industry is modelled by a Cobb-Douglas production function. The main result of the model is that new entries have a positive effect on the fraction of the existing stock of human capital devoted to the accumulation of human capital in both the advanced and laggard sectors. However, this effect is stronger in the advanced sectors than in the laggard sectors. This result is consistent with empirical evidence.

*Key Words:* new entry, growth, advanced, laggard, technology JEL: O11, O41

#### 1. INTRODUCTION

New entry is widely recognized as an important influence on economic growth. The coexistence of both the advanced and laggard sectors in the same industry has become more and more common in different industries. Despite the broad discussion on the effects of new entries on economic growth, there is still lack of readily available theoretical studies on the impacts of the new entries on economic growth in the presence of both advanced and laggard sectors. This paper revisits this issue. The contribution of this paper, therefore, is to provide a theoretical analysis on investigating the effects of new entries on economic growth via physical and human capital investment and accumulation when both advanced and laggard sectors coexist.

The contribution of this paper, therefore, is to provide a detailed theoretical explanation of the effects on economic growth that is consistent with current empirical evidence. The theoretical explanation is based on a four sector endogenous growth model with Cobb-Douglas production functions. The model describes a closed economy with two consumption goods that are imperfect substitutes, with one technology advanced and the other technology laggard. New entries contribute via human capital devotion. That is, a new entry would devote to accumulating human capital, and affect the optimal human capital in a systematically way. The effects of the new entries to the outcome of the sector would depend on the level and the type of technology adopted in that particular sector.

Note that this exercise is not about the new and old firms competing over rents in the advanced and the laggard sectors, nor is about the merger between firms. When the new entries arrive the particular sector, either the advanced or the laggard sector, their knowledge and creativity devote to the sector's human capital pool, which contributes to the sector's goods production. Therefore, the goods produced in the same sector are identical. There is no difference between new and old goods, and there are only two types of goods in this economy. Each type of good is produced in a particular sector: the advanced good produced in the advanced sector, and the laggard good produced in the laggard sector.

The main result of the model is that new entries have a positive effect on the human capital accumulation in both the advanced and the laggard sectors. Under certain circumstance, this effect is stronger in the advanced sectors, leading to a higher growth rate in the advanced sector than in the laggard sector. The intuition behind could be because a more competitive advanced sector might attract more consumers. This may lead to more investment in capital and a higher growth rate in the advanced sector than in the laggard sector. In equilibrium, due to the relative price of the advanced and the laggard goods, and individuals view these two goods as imperfect substitutes, there are demands for both types of goods.

These results may have implications for country by country comparisons. If we regard the economies with different technologies as different sectors in the model. The economies that start off as advanced will invest more and grow faster, thus enlarging the initial gap between economies. To help explain the model, the recent television industry with different flat panel technologies could be used as a relevant example.

Two years ago, before LED becomes available, the flat panel technologies could be divided into two: liquid crystal display (LCD) and plasma. LCD televisions create images by passing light through molecular structures of liquid crystals, whereas plasma televisions generate images by passing a high voltage through a low-pressure, electrically neutral, highly ionized atmosphere using the polarizing properties of light. The low power consumption allows the LCD technology to be viewed as more eco-friendly than the plasma technology. Also, some may view LCD technology offers a better quality of picture. For these reasons, we regard a LCD television as the advanced good, and a plasma television as the laggard good.

In the television market, plasma TVs have been dominating the market of larger screen sizes (42 inches and higher) due to consistently lower prices than LCD TVs<sup>1</sup>. This price gap has led billions of dollars investment in research and development in the sector producing LCD TVs. For example, Sony, Sharp, and Samsung consider reducing the price of large screen LCD TVs through massive production. Meanwhile, the sector producing plasma TVs are investing in research and development in improving plasma technology. There are new entries devoting to accumulating human capital in both sectors. Although Market researcher estimates that LCD TVs display revenues might balloon to \$91 billion globally by 2010, while plasma TVs might limp along with just under \$16 billion<sup>2</sup>, it does not have the details on the sales for different screen sizes. With continuing contribution by the new entries toward human capital pool in both sectors, it is possible that the coexistence of both sectors may last for a long while, but with a wider growth rate gap between sectors.

Empirically, there has been literature considering the effects of new entries. A broad survey conducted by Geroski (1995) shows that a high rate of new entry may be responsible for

 $<sup>^1\</sup>mathrm{In}$  the 50-inch class, LCD TVs are about 50% more expensive than plasma TVs.

<sup>&</sup>lt;sup>2</sup>This result is addressed in the Quarterly Worldwide FPD Forecast Report of DisplaySearch.

stimulating economic growth. However, Aghion et al (2006) point out that a high entry rate may not always induce a high growth rate. The micro-data suggests that there is a consistent heterogeneity across sectors in the industry in the effects of the new entries on average growth. One may argue a very large cross-section variation in entry. In fact, the differences in entry between industries do not persist for very long, and the entry rates are rarely high or persistently low over time in any particular industry. This relatively similar entry rates across industries, however, have a strongly positive effect on growth in some sectors, while depressing the growth rate in other sectors. To be specific, the positive effects tend to be found in the technologically advanced industries relative to the world technology frontier, and the negative effects are often found in the technologically laggard industries. The main results of this paper are consistent with these empirical findings. That is, an increased entry in both advanced and laggard sectors would lead to more investment in human capital, and stimulate higher growth, and this effect is stronger in the advanced sector than in the laggard sector.

In the same paper, Aghion et al (2006) construct a theoretical framework to focus on the links of a firm's technology level to its incentive in R&D investment without discussion on their effects on economic growth. In contract, by extending the endogenous growth models of Romer (1986), Rebelo (1991), and Lucas (1988) to incorporate the technologically heterogeneity between the advanced and the laggard sectors, this paper will fill in the gap in the literature by describing a balanced growth path for the economy's output, consumption, and capital growth rates in the sectors with different technologies. In our model, both advanced and laggard sectors operate in the same industry and investment continues to be made in both sectors. This is similar to the descriptions of the LCD (advanced) and plasma (laggard) sectors in the television industry.

The result that new entry has a stronger positive effect on the fraction of existing human capital devoted to the accumulation of human capital in the advanced sectors might have direct implications for policy debates. These findings are relevant to the consideration of the costs and benefits of globalization and the discussion on entry regulation in different countries and industries. The analysis suggests that policies aiming at decreasing or removing entry barriers may foster productivity growth in the economy on average, but the effects will be stronger in the technologically developed sectors.

This paper is structured as follows: Section 2 follows with a description of the theoretical setting; an endogenous growth model with four Cobb-Douglas production functions. This is then followed by section 3, which introduces the full employment constraint. The next section covers a special case of the main model, where the education production does not require physical capital. Section 5 then analyzes the steady state values. The next section discusses the main results of the theoretical models. Section 7 concludes with a summary of the main results and possible directions of future research.

#### 2. THE ENVIRONMENT

The economy consists of a continuum of households, and the population is assumed constant. There are two types of consumption goods, advanced goods (A), and laggard goods (L). These two types of goods are imperfect substitutes for households. The differences between these two goods are that advanced goods are produced by advanced technology while the laggard goods are produced by laggard technology. The amounts of goods, whether advanced or laggard, provide households utilities. Let  $C^A$  and  $C^L$  denote advanced goods and laggard goods consumption, respectively. The utility function of a household has the property of constant intertemporal elasticity and is in the form of:

$$U(C^{A}, C^{L}) = \int_{t=0}^{\infty} e^{-\rho t} \frac{(C^{A} + \lambda C^{L})^{1-\sigma} - 1}{1-\sigma},$$
(1)

where the discount rate  $\rho$  and the coefficient of (relative) risk aversion  $\sigma$  are both positive. The parameter  $\lambda$  is assumed less than 1 and is given exogenously. The marginal rate of substitution between  $C^A$  and  $C^L$  is therefore  $MRS_{AL} = 1/\lambda$ . An agent would be indifferent to giving up  $1/\lambda (> 1)$  units of the laggard good to obtain one additional unit of the advanced good. The technologically advanced good provides higher utility than the laggard good. In the example of the LCD and plasma televisions, the LCD television (the advanced good) is preferred to the plasma television (the laggard good); one LCD television provides more utility than a plasma television.

There are two types of production: advanced good production  $(Y^A)$ , and laggard good production  $(Y^L)$ . Let  $K^A$  and  $K^L$  denote the total stock of physical capital for the advanced and laggard sectors respectively. The total production of each type of good consists of consumption  $(C^i, i = A, L)$ , the change of physical capital accumulation  $(\dot{K}^i, i = A, L)$ , and depreciated physical capital  $(\delta K^i, i = A, L)$ . The total output of advanced and laggard goods, therefore, can be written as:  $Y^A = C^A + \dot{K}^A + \delta K^A$  and  $Y^L = C^L + \dot{K}^L + \delta K^L$ , respectively. Due to the technology differences,  $K^A$  and  $K^L$ , are assumed not transferable between the advanced and laggard sectors. That is to reflect the fact that the physical capital used to produce advanced good, such as a factory or machinery, cannot be used to produce the laggard good and vice versa. For example, machinery used to produce LCD screens cannot be used to make a plasma screen.

This economy also has a cumulative stock of human capital  $(H^i, i = A, L)$  specific to the production of each good. The stock of human capital reflects the general skill level of the workers. Individual workers in each sector allocate their time between producing consumption goods and investing in education which affects their productivity and skills in the future. Since the production of advanced and laggard goods requires specific knowledge, human capital stocks, similar to physical capital, are assumed non-transferable between sectors.

To simplified the model, the amount of time available to work and to get education for each worker is normalized to one. The time allocation between work and education could be different across advanced and laggard sectors. Each worker in the advanced sector devotes a fraction, u, of his available time to production, and the remaining fraction, 1-u, to human capital accumulation (get education), e.g. attending training sessions and studying to improve skills and increase the knowledge. This efforts, the time spent on human capital accumulation, 1-u, would contribute to the change of human capital stock of the sector,  $\dot{H}^A$ . Similarly, an individual worker in the laggard sector devote a fraction v of his available time to production, and the remaining fraction, 1-v, to human capital accumulation. The time allocation of workers in each sector is depicted in Figure 1.

Physical capital is required for both good production and education (e.g. providing equipment for advanced study such as a library or computers). In the advanced sector, a fraction s of total physical stock,  $K^A$ , is devoted to good production, and the remaining fraction, (1 - s), is spent on the education of advanced technology. Similarly, in the laggard sector, the fraction, q, of  $K^L$ is devoted to good production and the remaining fraction (1 - q) is contributed to the education of laggard technology.

Production of advanced and laggard goods depends on the sector-specific productivity level  $B^i$  (i = A, L), the physical and human capital devoting to each sector. The technologies of both advanced and laggard sectors are constant return to scale, and the production functions are in

the form of Cobb-Douglas.

$$Y^{A} = C^{A} + \dot{K}^{A} + \delta K^{A} = B^{A} (sK^{A})^{\beta} (uH^{A})^{1-\beta}$$
(2a)

$$Y^L = C^L + \dot{K}^L + \delta K^L = B^L (qK^L)^\theta (vH^L)^{1-\theta}$$
(2b)

where  $0 < \beta < 1$  and  $0 < \theta < 1$ . The values of parameters  $\beta$  and  $\theta$  are are constants, which are determined by the available technology and are exogenous to this model.

In either the advanced or the laggard sector, there are good production and human capital production. The production functions for good and for human capital are both in the form of Cobb-Douglas with constant return to scale, but with different shares of physical and human capital.

The economy is assumed starting with an positive level of human capital specific to each type of goods, i.e.  $H^i(0) > 0(i = A, L)$ . The rate of change in the human capital stock depends on the production of human capital and the new entries into the economy. There are N new entries to the economy at any given point of time. A fraction  $\alpha$  of the new entries enter the advanced sector, where  $1 > \alpha > 0$ , while the remainder  $(1 - \alpha)$  joins the laggard sector. Each new entry joins the market with an already established amount of human capital. It takes time for the new entries to adopt the environment and to learn necessary knowledge before starting contributing their human capital to the existing human capital pool, and the contribution may not be one-to-one. Let  $\xi^i$  (i = A, L) denote the proportion of the contribution of each new entry. The extra contributions to the human capital pools of advanced and laggard sectors are  $\alpha \xi^A N H^A$  and  $(1 - \alpha) \xi^L N H^L$ , respectively [Figure 1]. Thus, the rates of change in the level of the human capital stocks in advanced and laggard sectors are:

$$\dot{H}^{A} = [(1-s)K^{A}]^{\phi}[(1-u)H^{A}]^{1-\phi} + \alpha\xi^{A}NH^{A} - \delta H^{A},$$
(3a)

$$\dot{H}^{L} = [(1-q)K^{L}]^{\mu}[(1-v)H^{L}]^{1-\mu} + (1-\alpha)\xi^{L}NH^{L} - \delta H^{L},$$
(3b)

where  $0 < \varphi < 1$  and  $0 < \mu < 1$ . The values of parameters are assumed:  $\beta > \phi$  and  $\theta > \mu$  to reflect the empirically relevant case. That is, education (human capital production) tends to be more human capital intensive than goods production. The constant return to scale production function for both goods and human capital production would help in delivering a balanced growth path for the steady state values of  $C^i$ ,  $K^i$ , and  $H^i$ .

The resource allocation problem in this economy is to choose a time path  $C^A(t)$  and  $C^L(t)$ for per-capita consumption, the fractions u and v of the human capital pools, and the fractions q and s of the physical capital pools devoted to production. Let  $\eta^A$  and  $\eta^L$  be the shadow prices of physical capital, and let  $\varphi^A$  and  $\varphi^L$  be the shadow prices of human capital devoted to the good production in the advanced and laggard sectors, respectively. Then by maximizing the utility function [equation (1)] subject to the rates of changes of physical and human capital [equations (2a)-(3b)], the current-value Hamiltonian J can be written as:

$$J = U(C^{A}, C^{L})e^{\rho t} + \eta^{A} \{B^{A}(sK^{A})(uH^{A})^{1-\beta} - C^{A} - \delta K^{A}\} + \eta^{L} \{B^{L}(qK^{L}).(vH^{L})^{1-\theta} - C^{A} - \delta K^{A}\} + \varphi^{A} \{[(1-s)K^{A}]^{\phi}[(1-u)H^{A}]^{1-\phi} + \alpha \xi^{A} N H^{A} - \delta H^{L}\} + \varphi^{L} \{[(1-q)K^{L}]^{\mu}[(1-v)H^{L}]^{1-\mu} + (1-\alpha)\xi^{L} N H^{L} - \delta H^{L}\},$$
(4)

An optimal allocation must maximize expression J at each date t, provided the shadow prices are chosen correctly. The first order conditions with respect to the choice variables  $C^A$ ,  $C^L$ , s, q, u, and v are provided in the appendix. According to definitions:  $\dot{\varphi}^A = -\frac{\partial J}{\partial H^A}$ ,  $\dot{\varphi}^L = -\frac{\partial J}{\partial H^L}$ ,  $\dot{\eta}^A = -\frac{\partial J}{\partial K^A}$ , and  $\dot{\eta}^L = -\frac{\partial J}{\partial K^L}$ , one can easily obtained the rates of changes of each shadow price  $(\dot{\varphi}^A/\varphi^A, \dot{\varphi}^L/\varphi^L, \dot{\eta}^A/\eta^A, \dot{\eta}^L/\eta^L)$ . Differentiating the first order condition with respect to  $C^A$  and  $C^L$  gives another expressions for  $\dot{\eta}^A$  and  $\dot{\eta}^L$ , which can be substituted into  $\dot{\eta}^A/\eta^A$  and  $\dot{\eta}^L/\eta^L$  [details in appendix]. Then by rearranging the equations, one can obtain:

$$\frac{\dot{C}^A}{C^A} = \left(\frac{C^A + \lambda C^L}{\sigma C^A}\right) \left[\beta B^A (sK^A)^{\beta - 1} (uH^A)^{1 - \beta} - \delta - \rho\right],$$
(5a)

$$\frac{\dot{C}^L}{C^L} = \left(\frac{C^A + \lambda C^L}{\sigma C^L}\right) \left[\theta B^L (qK^L)^{\theta - 1} (\upsilon H^L)^{1 - \theta} - \delta - \rho\right].$$
(5b)

An optimal allocation must maximize expression J at each date t, provided the shadow prices are chosen correctly. The first order conditions with respect to the choice variables  $C^A$ ,  $C^L$ , s, q, u, and v are provided in the appendix.

Both equations (5a) and (5b) show that the growth rate of consumption, whether advanced or laggard good consumption, is proportional to the marginal product of physical capital less  $(\delta + \rho)$ . The efficient production decisions are characterized by two conditions. The first one is static. It defines the optional allocation of the existing physical capital stock and the efficiency unit of human capital across two activities. In equilibrium, the marginal product of physical capital capital across sectors are equated, and so does the marginal product of human capital. Let  $P^i$  be defined as the relative value of human capital in terms of physical capital in sector i,  $P^i \equiv \varphi^i / \eta^i$ . The first order conditions of J with respect to s, u, q and v can be re-written as functions of  $P^i$  (i = A, L) [see equations (A6a)-(A7b) in appendix]. Then eliminating  $P^i$  (i = A, L) from these equations yields the requirement of efficiency in production. The marginal rate of transformation must be equated in the two sectors:

$$\left(\frac{\phi}{1-\phi}\right)\left(\frac{s}{1-s}\right) = \left(\frac{\beta}{1-\beta}\right)\left(\frac{u}{1-u}\right) \tag{6a}$$

$$\left(\frac{\mu}{1-\mu}\right)\left(\frac{q}{1-q}\right) = \left(\frac{\theta}{1-\theta}\right)\left(\frac{\nu}{1-\nu}\right)$$
(6b)

By combining equations (6a), (6b), (A6a) and (A6b),  $P^i(i = A, L)$  can be solved as functions of  $(sK^A/uH^A)$  and  $(qK^L/vH^L)$ :

$$P^{A} = B^{A} \left(\frac{\beta}{\phi}\right)^{\phi} \left[\frac{1-\beta}{1-\phi}\right]^{1-\phi} \left(\frac{sK^{A}}{uH^{A}}\right)^{\beta-\phi},$$
(7a)

$$P^{L} = B^{L} \left(\frac{\theta}{\mu}\right)^{\mu} \left[\frac{1-\theta}{1-\mu}\right]^{1-\mu} \left(\frac{qK^{L}}{vH^{L}}\right)^{\theta-\mu}.$$
(7b)

The second efficiency condition is dynamic on the investment decision in physical versus human capital. The net return by investing one unit of physical capital  $(r^i, i = A, L)$  must equal to the net return by investing  $1/P^i$  units of human capital  $(r^{i*}, i = A, L)$  [details in appendix]. This gives:  $r^i = r^{i^*}$  [equations (A8a) and (A8b)], which indicates that the interior solutions for  $P^i$  (i = A, L) requires the slope of  $(r^i)^*, i = A, L$  flatter than that of  $r^i, i = A, L^3$ .

<sup>&</sup>lt;sup>3</sup>In this model, we cannot rule out the possibility of the corner solution. The focus of the paper, however, is on the interior solutions, which requires the slope of  $(r^i)^*$  flatter than that of  $r^i$ .

#### 3. THE FULL EMPLOYMENT CONSTRAINT

To solve the growth rates in this economy with two sectors, based on Bond et al (1996) requires a full employment constraint. Let  $Y^i$  (i = A, L) denote the output of the goods production, and let  $X^i$  (i = A, L) denote the output of the education production. This gives:  $Y^A = uHB^A(k_y^A)^{\beta}$ ,  $Y^L = vHB^L(k_y^L)^{\theta}$ ,  $X^A = (1 - u)H(k_x^A)^{\phi}$ , and  $X^L = (1 - v)H(k_x^L)^{\mu}$ , where  $k_y^A \equiv sK^A/uH^A$  and  $k_x^A \equiv [(1 - s)K^A] / [(1 - u)H^A]$  are the ratios of physical to human capital in goods production and in education production for the advanced sector, respectively, while  $k_y^L \equiv sK^L/uH^L$  and  $k_x^L \equiv [(1 - s)K^L] / [(1 - u)H^L]$  are the corresponding ratios for the laggard sector. In equilibrium, the real rate of return of physical and human capital must be equal across sectors. The market rental rates of capital and the market real wage rates in both advanced and laggard sectors can be rewritten in terms of  $k_y^i$   $(i = A, L)^4$ .

Let  $k^i \equiv K^i/H^i(i = A, L)$  denote the ratio of aggregate physical to human capital in each sector. The full employment constraint says that  $uk_x^A(P) + (1-u)k_y^A(P) = k^A$  for the advanced sector, and  $vk_x^L(P) + (1-v)k_y^L(P) = k^L$  for the laggard sector. By differentiating  $k_y^i$ ,  $k_x^i$ ,  $r^i$ , and  $w^i$  (i = A, L), applying Cremer's rule, and combining with the full employment constraint, one can obtain the allocation of human capital between sectors and the scaled goods production in each sector [see appendix section 5 for details]:

$$u(P^{A}, k^{A}) = \frac{\left[k^{A} - k_{x}^{A}\right]}{\left[k_{y}^{A} - k_{x}^{A}\right]}, \qquad v(P^{A}, k^{L}) = \frac{\left[k^{L} - k_{x}^{L}\right]}{\left[k_{y}^{L} - k_{x}^{L}\right]}, \tag{8}$$

$$x^{A}(P^{A},k^{A}) = \frac{B^{A}\left(k_{x}^{A}\right)^{\beta}\left[k^{A}-k_{x}^{A}\right]}{k_{y}^{A}-k_{x}^{A}}, \qquad x^{L}(P^{L},k^{L}) = \frac{B^{L}\left(k_{x}^{L}\right)^{\theta}\left[k^{L}-k_{x}^{L}\right]}{k_{y}^{L}-k_{x}^{L}}, \tag{9}$$

$$y^{A}(P^{A},k^{A}) = \left[1 - \left(\frac{k^{A} - k_{x}^{A}}{k_{y}^{A} - k_{x}^{A}}\right)\right] \left(k_{x}^{A}\right)^{\phi}, \qquad y^{L}(P^{L},k^{L}) = \left[1 - \left(\frac{k^{L} - k_{x}^{L}}{k_{y}^{L} - k_{x}^{L}}\right)\right] \left(k_{x}^{L}\right)^{\mu} (10)$$

According to Bond et al (1996), three conditions are required to guarantee the existence and uniqueness of a non-degenerated balanced growth path for  $C^i$ ,  $H^i$ , and  $K^i$  to grow at a common rate  $\gamma^i$ . The first condition requires the maximal attainable growth rate of consumption to satisfy  $\rho > (1 - \sigma)\dot{C}^i/C^i_{\max}$  (i = A, L). This means that  $\sigma$  has to be sufficiently large. The second condition, which requires non-zero profits on both physical and human capital investment to ensure the existence of an equilibrium, is automatically met in this model. That is because of the Cobb-Douglas production functions for both goods and education, together with the positive initial conditions for both capitals, which provide positive rate of returns for the investment in both physical and human capital investment. The third condition is for non-degenerate growth. This condition requires  $r^{*i} - \delta > \rho$  to ensure the sectors sufficiently productive so that  $\dot{C}^i/C^i$ is positive at the price consistent with balanced growth. With all three conditions met, the balanced growth paths of the economy's advanced and laggard sectors exist and are unique in our model. This balanced growth path for each sector can be obtained by using equations (5a)

<sup>&</sup>lt;sup>4</sup>The market rental rates of capital can be rewritten as  $r^A \equiv \beta B^A (k_y^A)^{\beta-1}$  for the advanced sector, and as  $r^L \equiv B^L \theta (k_y^L)^{\theta-1}$  in the laggard sector. The market real wage rate can be rewritten as  $\omega^A \equiv B^A (k_y^A)^{\beta} (1-\beta)$  for the advanced sector, and as  $\omega^L \equiv B^L (k_y^L)^{\theta} (1-\theta)$  for the laggard sector.

and (5b):

$$\gamma^{A} = \left(\frac{C^{A} + \lambda C^{L}}{\sigma C^{A}}\right) \left\{ \beta B^{A} \left[\frac{P^{A}}{B^{A}} \left(\frac{\phi}{\beta}\right)^{\phi} \left(\frac{1-\phi}{1-\beta}\right)^{1-\phi}\right]^{\frac{\beta-1}{\beta-\phi}} - \delta - \rho \right\}, \quad (11a)$$

$$\gamma^{L} = \left(\frac{C^{A} + \lambda C^{L}}{\sigma C^{A}}\right) \left\{ \theta B^{L} \left[\frac{P^{L}}{B^{L}} \left(\frac{\mu}{\theta}\right)^{\mu} \left(\frac{1-\mu}{1-\theta}\right)^{1-\mu}\right]^{\frac{\theta-1}{\theta-\mu}} - \delta - \rho \right\}.$$
 (11b)

These two equations implies that  $Y^i$ ,  $C^i$ ,  $H^i$ , and  $K^i$  all grow in the steady state. The absolute levels of  $C^i$ ,  $H^i$ , and  $K^i$  will not influence the growth rates because the system can be written in terms of ratios.

#### 4. SPECIAL CASE

In this section, we examine the case in which the education production in both advanced and laggard sectors requires no physical capital,  $\varphi = \mu = 0$ . According to Lucas (1988), this setup allows us to linearize the education production. The changes of human capital in the advanced and the laggard sectors become:

$$\dot{H}^A = [(1-u) + \alpha \xi^A N] H^A - \delta H^A$$
(12a)

$$\dot{H}^{L} = [(1-v) + (1-\alpha)\xi^{L}N]H^{L} - \delta H^{L}$$
 (12b)

Applying similar procedure to the previous sections, the growth rates of  $K^i$  (i = A, L) and  $H^i$ (i = A, L) as well as  $\kappa^i \equiv K^i/H^i$  (i = A, L), can be obtained. Then combining with the first order conditions, one can derive the growth rates of consumption on both advanced and laggard sector  $(\dot{C}^i/C^i, i = A, L)$  as well as the growth rates of  $P^i$   $(\dot{P}^i/P^i, i = A, L)$  [see section 6 for details]:

$$\frac{\dot{C}^A}{C^A} = \left(\frac{C^A + \lambda C^L}{\sigma C^A}\right) \left[\beta B^A u^{1-\beta} (\kappa^A)^{\beta-1} - \delta - \rho\right], \tag{13a}$$

$$\frac{\dot{C}^{L}}{C^{L}} = \left(\frac{C^{A} + \lambda C^{L}}{\sigma C^{L}}\right) \left[\theta B^{L} v^{1-\theta} (\kappa^{L})^{\theta-1} - \delta - \rho\right],$$
(13b)

$$\frac{\dot{P}^A}{P^A} \equiv \frac{\dot{\varphi}^A}{\varphi^A} - \frac{\dot{\eta}^A}{\eta^A} = \beta B^A u^{1-\beta} (\omega^A)^{\beta-1} - \left[1 + \alpha \xi^A N\right],$$
(14a)

$$\frac{\dot{P}^L}{P^L} \equiv \frac{\dot{\varphi}^L}{\varphi^L} - \frac{\dot{\eta}^L}{\eta^L} = \theta B^L \upsilon^{1-\theta} (\omega^L)^{\theta-1} - \left[1 + (1-\alpha)\xi^L N\right].$$
(14b)

#### 5. STEADY STATE ANALYSIS

Equations (A16a),(A16b),(A21a),(A21b),(A22a), and (A22b) can be used to form a system of six differential equations for the variables:  $\{\kappa^A, \kappa^L, \chi^A, \chi^L, u, v\}$ . Let  $\kappa^A$  (0) and  $\kappa^L$  (0) be the initial values of  $\kappa^A$  and  $\kappa^L$ , the steady state values of  $\{\kappa^A, \kappa^L, \chi^A, \chi^L, u, v\}$  can be solved [see section 7 in appendix for details].

In a competitive economy, capital and labour are each paid their marginal products in goods productions, and the factor income shares are constant. The rental rates of physical capital in the advanced and the laggard sectors are:

$$(r^{A})^{*} = 1 + \frac{\alpha \xi^{A} N}{\beta} - \delta, \qquad (r^{L})^{*} = 1 + \frac{(1-\alpha)\xi^{L} N}{\theta} - \delta.$$
 (15)

The wage rates of human capital in the advanced and the laggard sectors are:

$$(w^{A})^{*} = B^{A} (1-\beta) \left(\frac{1+\alpha\xi^{A}N}{\beta B^{A}}\right)^{\frac{1}{\beta-1}}, \qquad (w^{L})^{*} = B^{L} (1-\theta) \left[\frac{1+(1-\alpha)\xi^{L}N}{\theta B^{L}}\right]^{\frac{1}{\theta-1}}.$$
(16)

Plugging equations (15) and (16) into (13a) and (13b) gives the steady state growth rate of  $Y^i$ ,  $C^i$ ,  $K^i$ , and  $H^i$  (i = A, L):

$$\gamma^{A} = \pi^{A} \left[ 1 + \frac{\alpha \xi^{A} N}{\beta} - \delta - \rho \right], \qquad \gamma^{L} = \pi^{L} \left[ 1 + \frac{(1-\alpha)\xi^{L} N}{\theta} - \delta - \rho \right].$$
(17)

#### 6. TRANSITIONAL DYNAMICS

To find transitional dynamics, we first define  $z^i (i = A, L)$  as the gross average product of physical capital in the goods production:  $z^A \equiv B^A u^{1-\beta} (\kappa^A)^{\beta-1} = Y^A/K^A$ ,  $z^L \equiv B^L v^{1-\theta} (\kappa^L)^{\theta-1} = Y^L/K^L$  before rewriting a system of equations [equations (A16a), (A16b), (A21a), (A21b), (A22a), and (A22b)] with  $\kappa^i (i = A, L)$  in terms of  $z^i (i = A, L)$  for the variables  $\{\kappa^A, \kappa^L, \chi^A, \chi^L, u, v\}$ . The advantage of doing so is that, similar to  $\chi^i (i = A, L)$ , the value  $z^i (i = A, L)$  approaches to constant in the steady state [see section 8 in appendix for this system of equations]. Combining this system of equations with equations (A23a)-(A25b), one can obtain the steady state values of  $z^i (i = A, L)$ :

$$\left(z^{A}\right)^{*} = \frac{1}{\beta} \left(1 + \alpha \xi^{A} N\right), \qquad \left(z^{L}\right)^{*} = \frac{1}{\theta} \left[1 + (1 - \alpha) \xi^{L} N\right]$$
(18)

The dynamics of  $z^i(i = A, L)$  can be found by solving the differential equations (A26a) and (A26b). Given the initial values of  $z^i$ ,  $z^i(0)(i = A, L)$ , one can find that the time paths of  $z^i(i = A, L)$  are linear and can be shown in a closed form<sup>5</sup>. By integrating the closed form, one can solve the equations for  $z^i(i = A, L)$ :

$$z^{A} = \frac{(z^{A})^{*} z^{A}(0)}{(z^{A})^{*} e^{-(1-\beta)(z^{A})^{*}t} + z^{A}(0) \left[1 - e^{-(1-\beta)(z^{A})^{*}t}\right]}$$
(19a)

$$z^{L} = \frac{(z^{L})^{*} z^{L}(0)}{(z^{L})^{*} e^{-(1-\theta)(z^{L})^{*}t} + z^{L}(0) \left[1 - e^{-(1-\theta)(z^{L})^{*}t}\right]}$$
(19b)

<sup>5</sup>Closed forms of  $z^i (i = A, L)$ :

$$\frac{z^{A} - (z^{A})^{*}}{z^{A}} = \left[\frac{z^{A}(0) - (z^{A})^{*}}{z^{A}(0)}\right] e^{-(1-\beta)(z^{A})^{*}t}, \quad \frac{z^{L} - (z^{L})^{*}}{z^{L}} = \left[\frac{z^{L}(0) - (z^{L})^{*}}{z^{L}(0)}\right] e^{-(1-\theta)(z^{L})^{*}t}.$$
 (53a)

Equations (19a) and (19b) imply that  $z^i$  (i = A, L) will adjust monotonically over time from  $z^i(0)$  (i = A, L) to  $(z^i)^*$  (i = A, L), the steady state value<sup>6</sup>. Figure 3 shows this stability property.

The rates of return of both physical and human capital in the advanced and the laggard sectors can also be re-written as the functions of  $z^{i7}$ . If  $z^i(0) < (z^i)^*$  (i = A, L) and  $w^i(0) > (w^i)^*$  (i = A, L), then  $r^i(0) < (r^i)^*$  (i = A, L) and  $w^i(i = A, L)$  will fall monotonically over time towards the steady-state value. If  $z^i(0) > (z^i)^*$  (i = A, L) and  $w^i(0) < (w^i)^*$  (i = A, L), then  $r^i(0) > (r^i)^*$  (i = A, L) and  $w^i(i = A, L)$  will rise monotonically over time towards the steady-state. That means that  $z^i(i = A, L)$  and  $r^i(i = A, L)$  would rise(or fall) monotonically in the same direction over time toward its steady-state value.

As for the dynamics of  $\chi^A$  and  $\chi^L$ , we construct a two-dimensional phase diagram in  $(z^i, \chi^i)$ (i = A, L) space for each of the advanced and the laggard sectors based on equations (A27a) and (A27b) and discuss two cases for the advanced and the laggard sectors. The first case is  $\pi^A\beta < 1$  for the advanced sector, and  $\pi^L\theta < 1$  for the laggard sector. The second case is the reverse:  $\pi^A\beta > 1$  for the advanced sector and  $\pi^L\theta > 1$  for the laggard sector. The detailed discussion of the transitional dynamics for these two cases are in section 9 of appendix.

In case one [Figures 8 and 9], both the policy functions of  $\chi^A(\chi^L)$  and u(v) in the advanced (laggard) sector are downward sloping functions of  $\kappa^A(\kappa^L)$ . If an economy begins with a relative scarcity of human capital, i.e.  $\kappa^i(0) < (\kappa^i)^*$ , then  $\kappa^i$  will fall over time and the values of  $\chi^i$  and u and v will rise. However, if he economy allocates a relatively small proportion of its resources to consumption (low  $\chi^i$ ) at the beginning, then it might spend a large proportion of its time on education (high (1-u) or (1-v) is high).

In case two [Figures 10 and 11], both policy functions are upward sloping with respect to  $\kappa^i$ . The transitional dynamics of the growth rates of consumption, human capital, physical capital, u and v will depend on the initial values of  $\kappa^A$  and  $\kappa^L$ . If the economy begins with  $\kappa^i < (\kappa^i)^* (i = A, L)$ , the rental rate of physical capital  $r^i$ , will decline monotonically toward  $(r^i)^*$ . This fall in  $r^i$  implies a decline in  $\dot{C}^i / C^i$  [equations (13a) and (13b)]. The reverse is true. This relationship is shown in figure 12.

The growth rates of physical capital  $K^i$  (i = A, L) can be obtained by combining equations (13a), (13b), (A28a), (A28b), (A30a), and (A30b):

$$\frac{K^{i}}{K^{i}} = (\gamma^{i})^{*} + \left[z^{i} - (z^{i})^{*}\right] - [\chi^{i} - (\chi^{i})^{*}], \quad i = A, L$$
(20)

where  $(\gamma^A)^* \equiv \pi^A [\beta z^A - \delta - \rho]$  and  $(\gamma^L)^* \equiv \pi^L [\theta z^L - \delta - \rho]$ .

Equation (20) shows that the growth rate of the physical capital behave differently in case one and in case two when  $z^i > (z^i)^* (i = A, L)$ . In case one,  $\dot{K}^i/K^i$  (i = A, L) move ambiguously with respect to  $\kappa^i$ , since the second term is positive and the last term is negative in both equations. In case two,  $\dot{K}^i/K^i$  (i = A, L) is increasing in  $\kappa^i$  since both the second term and the last term are positive in both equations.

Similarly, we could obtain the growth rates of human capital H by combining equations (20a), (20b), (A32a), (A32b), (A28a), and (A28b):

<sup>&</sup>lt;sup>6</sup>When  $z^i < (z^i)^*$  (i = A, L), the growth rate of  $z^i$  is positive, and  $z^i$  increases toward  $(z^i)^*$  (i = A, L). When  $z^i > (z^i)^*$  (i = A, L), the growth rate of  $z^i$  is negative, and  $z^i$  decreases toward  $(z^i)^*$  (i = A, L). As  $t \to \infty$ ,  $z^A \to (z^A)^*$  and  $z^L \to (z^L)^*$  if  $(z^i)^*$  (i = A, L) is stable. <sup>7</sup>For the advanced sector, the rental rate is  $r^A = \beta z^A - \delta$  and the wage rate is  $w^A = (1 - \beta) B^A u^{-\beta} (w^A)^{\beta} = z^A - \delta$ .

<sup>&</sup>lt;sup>*i*</sup> For the advanced sector, the rental rate is  $r^A = \beta z^A - \delta$  and the wage rate is  $w^A = (1 - \beta) B^A u^{-\beta} (w^A)^{\beta} = (B^A)^{\frac{1}{1-\beta}} (1 - \beta) (z^A)^{\frac{-\beta}{1-\beta}}$ . For the laggard sector, the rental rate is  $r^L = \theta z^L - \delta$ , and the wage rate is  $w^L = (1 - \theta) B^L v^{-\theta} (w^L)^{\theta} = (B^L)^{\frac{1}{1-\theta}} (1 - \theta) (z^L)^{\frac{-\theta}{1-\theta}}$ .

$$\frac{\dot{H}^A}{H^A} = \left(\gamma^A\right)^* - \left(u - u^*\right) + \left(1 - \alpha\xi^A N\right)$$
(21a)

$$\frac{\dot{H}^{L}}{H^{L}} = (\gamma^{L})^{*} - (v - v^{*}) - \left[1 - (1 - \alpha)\xi^{L}N\right]$$
(21b)

Equations (21a) and (21b) show that in case one,  $\dot{H}^i/H^i(i = A, L)$  is increasing in  $\kappa^i$  since  $(u - u^*)$  and  $(v - v^*)$  are declining in  $\kappa^i$ . In case two, however,  $\dot{H}^i/H^i(i = A, L)$  is decreasing in  $\kappa^i$  because  $(u - u^*)$  and  $(v - v^*)$  are increasing in  $\kappa^i$ .

#### 7. SIMULATION

The more general case is discussed in section 3. In the special case is if there is no new entries (i.e. N = 0), then our model will be similar to that of the generalized Uzawa-Lucas model. Barro(2004) carried out some simulations for the generalized Uzawa-Lucas model with two sectors. We adopt his parameter values  $\beta = \theta = 0.4$ , while the parameters  $\phi$  and  $\mu$  are varied between 0 and 0.4. As a representative case;  $\delta = 0.05$ ,  $\rho = 0.02$ , and  $\sigma = 3$ . For  $\phi = \mu = 0$  and  $B^i$  normalized to one, Barro found that the steady state interest rate is 0.08 and the steady-state growth rate is 0.02.

In our simulation with new entries, we use Barro's parameter values. The number of new entries is normalized to one: N = 1. The human capital contributions in each sector is set equal:  $\xi^A = \xi^L = 0.9$ . Half of the new entries go to the advanced sector:  $\alpha = 0.5$ . This is a realistic representation of the television industry, as there is a similar number of new entries in both plasma and LCD markets. Finally,  $C^L = 0.8$  and  $C^A = 0.2$ . This also reflects the television market, as the sale of plasmas are significantly greater than the sale of LCDs in the larger screen sizes. We obtained the following results. The steady state interest rates are  $r^A = r^L = 0.12711902$ . The steady state growth rate for the advanced sector is  $\gamma^A = 0.0856$  and for the laggard sector is  $\gamma^L = 0.04686$ . These steady state growth rates are consistent with the empirical evidence that growth rates increase with new entries, as established in our model.

If there are no new entries (N = 0) then the results are the same as in Lucas (1993). If N > 0, the fraction of human capital devoted to human capital accumulation is increasing with new entries. That is, (1 - u) and (1 - v) are positively related to new entries.

We assume that both  $\xi^A$  and  $\xi^L$  are less than one. When  $\alpha$  is set to 0.5, both markets have the same number of new entries. The contribution of new entries into the advanced market  $(\xi^A)$  is assumed to be the same as that in the laggard market  $(\xi^L)$ . Also, the parameters  $\theta$  and  $\beta$  are set equal, so production in the advanced and laggard sectors is equally intensive in physical and human capital. Then, for the fraction of human capital devoted to human capital accumulation in the advanced good, (1 - u), to be greater than that of the laggard good, (1 - v),  $\pi^A$  must be greater than  $\pi^L$ .

Because  $\lambda$  is less than one, the value of  $C^L$  needs to be sufficiently larger than CA. For the case of the flat screen televisions; plasma and LCDs are relatively close substitutes, so  $\lambda$  will be closer to one. Currently, plasma televisions dominate the market of the larger screens because of their lower prices, so  $C^L > C^A$ . This implies that u < v, which is consistent with the empirical evidence of heterogeneity between laggard and advanced firms presented in Aghion(2006).

The intuition for this result is that the advanced sector will invest more into education and knowledge when the threat of new entries is increased. It is similar to the escape entry effect in Aghion et al, where the advanced incumbents invest to protect their market share. The effect of increased entry on the laggard sectors will still be positive, but much weaker than that in the advanced sector.

If  $\pi^A$  is larger than  $\pi^L$  and the other parameters are as described above,  $\omega^A$  will be larger than  $\omega^L$ . This means that the proportion of physical to human capital used to produce advanced goods is larger than that for the laggard good.  $\pi^A > \pi^L$  also implies that  $\chi^A$  will be greater than  $\chi^L$ . This means that the growth rate in the advanced sector will be higher than that in the laggard sector.

#### 8. CONCLUSION

This paper investigates the effects of new entry on growth in an economy with advanced and laggard sectors. The theoretical model has four sectors with four Cobb-Douglas production functions. The main result of the model is that new entries have a positive effect on the fraction of human capital devoted to the accumulation of human capital in both the advanced and laggard sectors. However, under certain conditions, this effect is much stronger in the advanced sector than in the laggard sector. Thus, new entries have a larger impact on growth in the advanced sector than in the laggard sector.

However, one weakness of this model that is shared by the bulk of the existing literature is that we study a closed economy. This is not a accurate reflection of the real world. One possible direction for future research would be to have an open economy allowing for imports and exports. Another possible extension is to include leisure in the utility function, similar to the extension in Lucas(1988).

#### APPENDIX A: APPENDIX

Section 1. The first order conditions of Hamiltonian with respect to the choice variables,  $C^A,\,C^L,\,s,\,q$  ,  $u,\,{\rm and}\,\,v.$ 

$$\frac{dU\left(C^A + \lambda C^L\right)}{dC^A} = e^{\rho t} \eta^A \tag{A1a}$$

$$\frac{dU\left(C^A + \lambda C^L\right)}{dC^L} = e^{\rho t} \eta^L \tag{A1b}$$

$$K^{A} \left\{ \eta^{A} \beta B^{A} \left( sK^{A} \right)^{\beta-1} \left( uH^{A} \right)^{1-\beta} - \varphi^{A} \phi [(1-s)K^{A}]^{\phi-1} [(1-u)H^{A}]^{1-\phi} \right\} = (\mathbf{A}2a)$$
$$K^{L} \left\{ \eta^{L} \theta B^{L} \left( qK^{L} \right)^{\theta-1} (vH^{A})^{1-\theta} - \varphi^{L} [(1-q)K^{L}]^{\mu-1} [(1-v)H^{L}]^{1-\mu} \right\} = (\mathbf{A}2b)$$

$$H^{A}\left\{\eta^{A}(1-\beta)B^{A}(sK^{A})^{\beta}(uH^{A})^{-\beta} - \varphi^{A}(1-\phi)\left[(1-s)K^{A}\right]^{\phi}\left[(1-u)H^{A}\right]^{-\phi}\right\} = (A3a)$$
$$H^{L}\left\{\pi^{L}(1-\theta)B^{L}(aK^{L})^{\theta}(a)H^{L}\right\}^{-\theta} - \varphi^{L}(1-u)\left[(1-u)K^{L}\right]^{\mu}\left[(1-u)H^{L}\right]^{-\mu}\right\} = (A3b)$$

$$H^{L}\left\{\eta^{L}(1-\theta)B^{L}\left(qK^{L}\right)^{\nu}\left(\upsilon H^{L}\right)^{-\nu} - \varphi^{L}(1-\mu)\left[(1-\upsilon)K^{L}\right]^{\mu}\left[(1-\upsilon)H^{L}\right]^{-\mu}\right\} = (\mathbf{A}3\mathbf{b})$$

**Section 2.** The conditions  $\dot{\varphi}^A = -\frac{\partial J}{\partial H^A}$ ,  $\dot{\varphi}^L = -\frac{\partial J}{\partial H^L}$ ,  $\dot{\eta}^A = -\frac{\partial J}{\partial K^A}$ , and  $\dot{\eta}^L = -\frac{\partial J}{\partial K^L}$  imply:

$$\frac{\dot{\varphi}^{A}}{\varphi^{A}} = -\frac{\eta^{A}}{\varphi^{A}}B^{A}(1-\beta)(u)\left(sK^{A}\right)^{\beta}\left(uH^{A}\right)^{-\beta} -(1-\phi)(1-u)\left[(1-s)K^{A}\right]^{\phi}\left[(1-u)H^{A}\right]^{-\phi} -\alpha\xi^{A}N + \delta$$

$$\dot{\varphi}^{L} = \eta^{L} - \xi \left(s-x\right)\left(s-x\right)\left(s-x\right)^{-\theta} - \alpha\xi^{A}N + \delta$$
(A4a)

$$\frac{\varphi^{2}}{\varphi^{L}} = -\frac{\eta^{L}}{\varphi^{L}}B^{L}(1-\theta)(v)\left(qK^{L}\right)^{\theta}\left(vH^{L}\right)^{-\theta} -(1-\mu)(1-v)\left[(1-q)K^{L}\right]^{\mu}\left[(1-v)H^{L}\right]^{-\mu} - (1-\alpha)\xi^{L}N + \delta$$
(A4b)

$$\frac{\dot{\eta}^{A}}{\eta^{A}} = -B^{A}\beta s \left(sK^{A}\right)^{\beta-1} \left(uH^{A}\right)^{1-\beta} + \delta - \frac{\varphi^{A}}{\eta^{A}}\phi(1-s) \left[(1-s)K^{A}\right]^{\phi-1} \left[(1-u)H^{A}\right]^{1} \left[A^{5}_{5a}\right]^{\phi-1} \left[(1-u)H^{A}\right]^{1} \left[A^{5}_{5a}\right]^{\phi-1} \left[(1-u)H^{A}\right]^{1} \left[A^{5}_{5a}\right]^{\phi-1} \left[A^$$

$$\frac{\dot{\eta}^{L}}{\eta^{L}} = -B^{L}\theta q \left(qK^{L}\right)^{\theta-1} \left(vH^{L}\right)^{1-\theta} + \delta - \frac{\varphi^{L}}{\eta^{L}} \mu(1-q) \left[(1-q)K^{L}\right]^{\mu-1} \left[(1-v)H^{L}\right]^{1-\mu} (A5b)$$

The above first order conditions determine the growth rates of consumption of the two goods. Differentiating (5a) and (5b) with respect to time gives expressions for  $\dot{\eta}^A$  and  $\dot{\eta}^L$ . Substituting these expressions into (9a) and (9b) and rearranging yields  $\dot{C}^A/C^A$  and  $\dot{C}^L/C^L$ .

Section 3. The first order conditions of J with respect to s, u, q and v can be re-written as functions of  $P^i$  (i = A, L) [equations (A6a)-(A7b)]. Then eliminating  $P^i$  (i = A, L) from these equations yields the requirement of efficiency in production.

$$B^{A}\beta(sK^{A})^{\beta-1}(uH^{A})^{1-\beta} = P^{A}\phi[(1-s)K^{A}]^{\phi-1}[(1-u)H^{A}]^{1-\phi}$$
(A6a)

$$B^{A}(1-\beta)(sK^{A})^{\beta}(uH^{A})^{-\beta} = P^{A}(1-\phi)[(1-s)K^{A}]^{\phi}[(1-u)H^{A}]^{-\phi}$$
(A6b)  
$$B^{L}\theta(qK^{L})^{\theta-1}(vH^{L})^{1-\theta} = P^{L}\mu[(1-q)K^{L}]^{\mu-1}[(1-v)H^{L}]^{1-\mu}$$
(A7a)

$$B^{L}\theta(qK^{L})^{\theta-1}(vH^{L})^{1-\theta} = P^{L}\mu[(1-q)K^{L}]^{\mu-1}[(1-v)H^{L}]^{1-\mu}$$
(A7a)

$$B^{L}(1-\theta)(qK^{L})^{\theta}(\upsilon H^{L})^{1-\theta} = P^{L}(1-\mu)[(1-q)K^{L}]^{\mu}[(1-\upsilon)H^{L}]^{-\mu}$$
(A7b)

Then eliminating  $P^{i}$  (i = A, L) from these equations yields the requirement of efficiency in production.

Section 4. The second efficiency condition.

The net return of physical capital investment is its net marginal product in goods production for both the advanced and laggard sectors:  $r^A = B^A \beta (sK^A)^{\beta-1} (uH^A)^{1-\beta} - \delta$  and  $r^L = B^L \theta (qK^L)^{\theta-1} (vH^L)^{1-\theta} - \delta$ . The net return of  $1/P^i$  units of human capital investment in terms of physical capital is:  $r^{A^*} = (1-\phi)[(1-s)K^A]^{\phi}[(1-u)H^A]^{-\phi} + \alpha\xi^A N - \delta + (\dot{P}^A/P^A)$  and  $r^{L^*} = (1-\mu)[(1-q)K^L]^{\mu}[(1-v)H^L]^{-\mu} + (1-\alpha)\xi^L N - \delta + (\dot{P}^L/P^L)$  for the advanced and laggard sectors, respectively. In equilibrium,  $r^i = r^{i^*}$  gives

$$\frac{\dot{P}^{A}}{P^{A}} = \left(B^{A}\right)^{\frac{1-\phi}{\beta-\phi}} \beta^{\frac{\beta(1-\phi)}{\beta-\phi}} \phi^{\frac{\phi(\beta-1)}{\beta-\phi}} \left(\frac{1-\phi}{1-\beta}\right)^{\frac{(1-\phi)(\beta-1)}{\beta-\phi}} \left(P^{A}\right)^{\frac{\beta-1}{\beta-\phi}} \\
- \left(B^{A}\right)^{\frac{-\phi}{\beta-\phi}} \left(\frac{\phi}{\beta}\right)^{\frac{\phi(\phi-1)+\beta}{\beta-\phi}} (1-\phi) \left(\frac{1-\phi}{1-\beta}\right)^{\frac{\phi(1-\beta)}{\beta-\phi}} \left(P^{A}\right)^{\frac{\phi}{\beta-\phi}} - \alpha\xi^{A}N, \quad (A8a)$$

$$\frac{\dot{P}^{L}}{P^{L}} = \left(B^{L}\right)^{\frac{1-\mu}{\theta-\mu}} \theta^{\frac{\theta(1-\mu)}{\theta-\mu}} \mu^{\frac{\mu(\theta-1)}{\theta-\mu}} \left(\frac{1-\mu}{1-\theta}\right)^{\frac{(1-\mu)(\theta-1)}{\theta-\mu}} \left(P^{L}\right)^{\frac{\theta-1}{\theta-\mu}} \\
- \left(B^{L}\right)^{\frac{-\mu}{\theta-\mu}} \left(\frac{\mu}{\theta}\right)^{\frac{\mu(\mu-1)+\theta}{\theta-\mu}} (1-\mu) \left(\frac{1-\mu}{1-\theta}\right)^{\frac{\mu(1-\theta)}{\theta-\mu}} \left(P^{L}\right)^{\frac{\mu}{\theta-\mu}} - (1-\alpha)\xi^{L}N. \quad (A8b)$$

Equations (17a) and (17b)] requires that the interior solutions for  $P^A$  (or  $P^L$ ) requires the slope of  $(r^A)^*$  or  $(r^L)^*$ flatter than that of  $(r^A)$  or  $(r^L)$ . Section 5. By differentiating  $k_y^i$ ,  $k_x^i$ ,  $r^i$ , and  $w^i$  (i = A, L), applying Cremer's rule, and

**Section 5.** By differentiating  $k_y^i$ ,  $k_x^i$ ,  $r^i$ , and  $w^i$  (i = A, L), applying Cremer's rule, and combining with the full employment constraint, one can obtain the allocation of human capital between sectors and the scaled goods production in each sector.

$$\frac{dk_y^A}{dP^A} = \left(\frac{k_y^A}{k_x^A}\right) \left[\frac{k_x^A}{1-\beta} + \frac{k_y^A}{\beta}\right], \frac{dk_y^L}{dP^L} = \left(\frac{k_y^L}{k_x^L}\right) \left[\frac{k_x^L}{1-\theta} + \frac{k_y^L}{\theta}\right]$$
(A9)

$$\frac{dk_x^A}{dP^A} = \frac{B^A \left(k_y^A\right)^{\beta}}{\left(\phi - 1\right) \phi \left(k_x^A\right)^{\phi - 2} \left(k_x^A - k_y^A\right)}, \frac{dk_x^L}{dP^L} = \frac{B^L \left(k_y^L\right)^{\theta}}{\left(\mu - 1\right) \mu \left(k_x^L\right)^{\mu - 2} \left(k_x^L - k_y^L\right)}$$
(A10)

$$\frac{dr^A}{dP^A} = \beta \left(\beta - 1\right) B^A \left(k_y^A\right)^{\beta - 1}, \frac{dr^L}{dP^L} = \theta \left(\theta - 1\right) B^L \left(k_y^L\right)^{\theta - 2} B^L k_y^L \left(P^L\right)$$
(A11)

$$\frac{dw^{A}}{dP^{A}} = \phi \left(1 - \phi\right) \left(k_{y}^{A}\right)^{2} \left(k_{x}^{A}\right)^{\phi-2}, \frac{dw^{L}}{dP^{L}} = \mu \left(1 - \mu\right) \left(k_{y}^{L}\right)^{2} \left(k_{x}^{L}\right)^{\mu-2} k_{y}^{L} \left(P^{L}\right)$$
(A12)

These derivatives can be combined with the full employment condition to derive the allocation of human capital between sectors and the scaled output of each sector.

Section 6. Derivation of the growth rates of consumption on both advanced and laggard goods for the special case.

Reset the current-value Hamiltonian accordingly:

$$J = U(C^{A}, C^{L})e^{-\rho t} + \eta^{A} \left\{ B^{A}(K^{A})^{\beta}(uH^{A})^{1-\beta} - C^{A} - \delta K^{A} \right\} + \eta^{L} \left\{ B^{L}(K^{L})^{\theta}(vH^{L})^{1-\theta} - C^{L} - \delta K^{L} \right\} + \varphi^{A} \left\{ [(1-u) + \alpha \xi^{A}N]H^{A} - \delta H^{A} \right\} (A13) + \varphi^{L} \left\{ [(1-v) + (1-\alpha)\xi^{L}N]H^{L} - \delta H^{L} \right\},$$

the growth rates of  $K^i$  and  $H^i$  (i = A, L)can be derived:

$$\frac{\dot{K}^{A}}{K^{A}} = B^{A} u^{1-\beta} (\kappa^{A})^{\beta-1} - \chi^{A} - \delta, \\ \frac{\dot{K}^{L}}{K^{L}} = B^{L} v^{1-\theta} (\kappa^{L})^{\theta-1} - \chi^{L} - \delta$$
(A14)

$$\frac{H^{A}}{H^{A}} = (1-u) + \alpha \xi^{A} N - \delta, \\ \frac{H^{L}}{H^{L}} = (1-v) + (1-\alpha) \xi^{L} N - \delta$$
(A15)

where  $\kappa^i \equiv K^i/H^i$ ,  $\chi^i \equiv C^i/K^i$ , (i = A, L). Hence, the growth rates of  $\kappa^i$  (i = A, L) are simply the difference between equations (A14) and (A15):

$$\frac{\dot{\kappa}^{A}}{\kappa_{A}^{A}} = \frac{\dot{K}^{A}}{\kappa_{A}^{A}} - \frac{\dot{H}^{A}}{H^{A}} = B^{A} u^{1-\beta} (\kappa^{A})^{\beta-1} - \chi^{A} - (1-u) - \alpha \xi^{A} N$$
(A16a)

$$\frac{\dot{\kappa}^{L}}{\kappa^{L}} = \frac{\dot{K}^{L}}{K^{L}} - \frac{\dot{H}^{L}}{H^{L}} = B^{L} \upsilon^{1-\theta} (\kappa^{L})^{\theta-1} - \chi^{L} - (1-\upsilon) - (1-\alpha)\xi^{L} N$$
(A16b)

Combining the first order conditions of Hamiltonian with respect to  $C^A$  ,  $C^L$  , v, and u:

$$\eta^i e^{\rho t} = \frac{\partial U}{\partial C^i}, \quad i = A, L$$
 (A17)

$$\frac{\eta^{A}}{\varphi^{A}} = \frac{u^{\beta} \left(\kappa^{A}\right)^{-\beta}}{\left(1-\beta\right) B^{A}}, \frac{\eta^{L}}{\varphi^{L}} = \frac{v^{\theta} \left(\kappa^{L}\right)^{-\theta}}{\left(1-\theta\right) B^{L}}.$$
(A18)

and the relevant conditions:  $\dot{\varphi}^i = -\partial J/\partial H^i$  and  $\dot{\eta}^i = -\partial J/\partial K^i$ , (i = A, L) gives:

$$\frac{\dot{\varphi}^{A}}{\varphi^{A}} = -B^{A}u^{1-\beta}(\kappa^{A})^{\beta}(1-\beta)\left(\frac{\eta^{A}}{\varphi^{A}}\right) - \left[1-u+\alpha\xi^{A}N\right] + \delta,$$
(A19a)

$$\frac{\dot{\varphi}^L}{\varphi^L} = -B^L \upsilon^{1-\theta}(\kappa^L)^{\theta} \left(1-\theta\right) \left(\frac{\eta^L}{\varphi^L}\right) - \left[1-\upsilon + (1-\alpha)\xi^L N\right] + \delta, \qquad (A19b)$$

$$\frac{\dot{\eta}^A}{\eta^A} = -\beta B^A u^{1-\beta} (\kappa^A)^{\beta-1} + \delta, \\ \frac{\dot{\eta}^L}{\eta^L} = -\theta B^L v^{1-\theta} (\omega^L)^{\theta-1} + \delta.$$
(A20)

The above conditions can be used to obtain the growth rates of consumption for both the advanced and the laggard sectors. Differentiating equation (A17) with respect to time gives  $\dot{\eta}^A$  and  $\dot{\eta}^L$ , respectively, which can be imposed into equation (A20) to determine the consumption growth rates. By substituting equation (A14) into equations (13a), and (13b), one could derive  $\dot{\chi}^i/\chi^i \equiv \dot{C}^i/C^i - \dot{K}^i/K^i$  (i = A, L):

$$\frac{\dot{\chi}^{A}}{\chi^{A}} = \left(\frac{C^{A} + \lambda C^{L}}{\sigma C^{A}}\right) \left[\beta B^{A} u^{1-\beta} (\kappa^{A})^{\beta-1} - \delta - \rho\right] - B^{A} u^{1-\beta} (\kappa^{A})^{\beta-1} + \chi^{A} + \delta (A21a)$$

$$\frac{\dot{\chi}^{L}}{\chi^{L}} = \left(\frac{C^{A} + \lambda C^{L}}{\sigma C^{L}}\right) \left[\theta B^{L} v^{1-\theta} (\kappa^{L})^{\theta-1} - \delta - \rho\right] - B^{L} v^{1-\theta} (\kappa^{L})^{\theta-1} + \chi^{L} + \delta \quad (A21b)$$

Combining equations (A16a), (A16b), and (A18)-(A20) gives:

$$\frac{\dot{u}}{u} = \left(\frac{1+\alpha\xi^A N}{\beta}\right) - \chi^A - \left[1-u+\alpha\xi^A N\right], \qquad (A22a)$$

$$\frac{\dot{\upsilon}}{\upsilon} = \left(\frac{1+(1-\alpha)\xi^L N}{\theta}\right) - \chi^L - \left[1-\upsilon+(1-\alpha)\xi^L N\right].$$
 (A22b)

#### Section 7. Steady State Analysis

The steady state of this system can be characterized by setting the six time derivatives to zero. This yields

$$\left(\chi^{A}\right)^{*} = \frac{\left(1 + \alpha\xi^{A}N\right)\left(\pi^{A} - 1\right)}{\beta} + \pi^{A}\left(\delta + \rho\right) - \delta, \qquad (A23a)$$

$$\left(\chi^{L}\right)^{*} = \frac{\left(1 + (1 - \alpha)\xi^{L}N\right)\left(\pi^{L} - 1\right)}{\theta} + \pi^{L}\left(\delta + \rho\right) - \delta, \qquad (A23b)$$

$$u^* = -\left(\frac{1+\alpha\xi^A N}{\beta}\right)\left(\beta+\pi^A\right) + \pi^A\left(\delta+\rho\right) - \delta, \qquad (A24a)$$

$$v^* = -\left(\frac{1+(1-\alpha)\xi^L N}{\theta}\right)\left(\theta+\pi^L\right) + \pi^L\left(\delta+\rho\right) - \delta, \qquad (A24b)$$

$$\left(\kappa^{A}\right)^{*} = \left[-\left(\frac{1+\alpha\xi^{A}N}{\beta B^{A}}\right)\right]^{\frac{1}{\beta-1}} \left[\left(\frac{1+\alpha\xi^{A}N}{\beta}\right)\left(\beta+\pi^{A}\right)+\pi^{A}\left(\delta+\rho\right)-\delta\right], \quad (A25a)$$

$$\left(\kappa^{L}\right)^{*} = \left[-\left(\frac{1+(1-\alpha)\xi^{L}N}{\theta B^{L}}\right)\right]^{\frac{1}{\theta-1}} \left[\left(\frac{1+(1-\alpha)\xi^{L}N}{\theta}\right)\left(\theta+\pi^{L}\right)+\pi^{L}\left(\delta+\rho\right)-(\mathbf{A})^{2}\mathbf{5}\mathbf{b}\right)^{\frac{1}{\theta-1}}\right]$$

where  $\pi^{A} \equiv \frac{C^{A} + \lambda C^{L}}{\sigma C^{A}}$  and  $\pi^{L} \equiv \frac{C^{A} + \lambda C^{L}}{\sigma C^{L}}$ . Section 8: Transitional Dynamics

Rewrite a system of equations [equations (A16a), (A16b), (A21a), (A21b), (A22a), and (A22b)] with  $\kappa^i(i=A,L)$  in terms of  $z^i(i=A,L)$  for the variables  $\{\kappa^A, \kappa^L, \chi^A, \chi^L, u, v\}$ .

$$\frac{\dot{z}^{A}}{z^{A}} = -(1-\beta) \left[ z^{A} - (z^{A})^{*} \right]$$
(A26a)
$$\frac{\dot{z}^{L}}{z^{L}} = -(1-\theta) \left[ z^{L} - (z^{L})^{*} \right]$$
(A26b)

$$\frac{z^L}{z_L} = -(1-\theta) \left[ z^L - \left( z^L \right)^* \right]$$
(A26b)

$$\frac{\dot{\chi}^{A}}{\chi^{A}} = \left[z^{A} - \left(z^{A}\right)^{*}\right] \left(\pi^{A}\beta - 1\right) + \left[\chi^{A} - \left(\chi^{A}\right)^{*}\right]$$
(A27a)

$$\frac{\dot{\chi}^{L}}{\chi^{L}} = \left[z^{L} - \left(z^{L}\right)^{*}\right] \left(\pi^{L}\theta - 1\right) + \left[\chi^{L} - \left(\chi^{L}\right)^{*}\right]$$
(A27b)

$$\frac{\dot{u}}{u} = (u - u^*) - \left[\chi^A - \left(\chi^A\right)^*\right]$$
(A28a)

$$\frac{\dot{v}}{v} = (v - v^*) - \left[\chi^L - \left(\chi^L\right)^*\right]$$
(A28b)

where  $z^{i*}(i = A, L)$  is the steady-state value of  $z^i(i = A, L)$ . Using  $\kappa^{A*}$ ,  $\kappa^{L*}$ ,  $\chi^{A*}$ ,  $\chi^{L*}$ ,  $u^*$ , and  $v^*$  as found earlier, along with the definitions of  $z^A$  and  $z^L$ , we obtain the steady state values of  $z^A$  and  $z^L$ .

#### Section 9. Transitional dynamics for the two cases

Case one:  $\pi^A \beta < 1$  for the advanced sector, and  $\pi^L \theta < 1$  for the laggard sector

As depicted in Figure 4, the stable path has  $\chi^i > (\chi^i)^*(i = A, L)$ . For  $\chi^i \le (\chi^i)^*(i = A, L)$ ,  $\chi^i(0) < 0(i = A, L)$ . Thus,  $\chi^i(i = A, L)$  would diverge from  $(\chi^i)^*$  (i = A, L) and reach zero. If  $\dot{\chi}^i \ge 0(i = A, L)$  for some t, then  $\dot{\chi}^i > 0$  (i = A, L) for all subsequent t. The negative term  $(\pi^A\beta - 1)\left[z^A - (z^A)^*\right]$  for the advanced sector and  $(\pi^L\theta - 1)\left[z^L - (z^L)^*\right]$  for the laggard sector decreases in size over time. The variable  $\chi^i(i = A, L)$  would diverge from  $(\chi^i)^*$  (i = A, L) and approach  $\infty$ . The stable path is therefore  $\dot{\chi}^i < 0$  for all t.

Case two:  $\pi^A \beta > 1$  for the advanced sector, and  $\pi^L \theta > 1$  for the laggard sector

As shown in Figure 5, The first terms in both equation (A27a) and (A27b) become positive. If  $\chi^i = (\chi^i)^*(i = A, L)$  then  $\dot{\chi}^i > 0(i = A, L)$ . Thus,  $\chi^i(i = A, L)$  would diverge from  $(\chi^i)^*(i = A, L)$  and approach  $\infty$ . The stable path therefore has  $\chi^i < (\chi^i)^*(i = A, L)$ . If  $\dot{\chi}^i \leq 0(i = A, L)$  for some t, then  $\dot{\chi}^i < 0$  for all subsequent t. The positive term  $(\pi^A \beta - 1) \left[ z^A - (z^A)^* \right]$ for the advanced sector and  $(\pi^L \theta - 1) \left[ z^L - (z^L)^* \right]$  for the laggard sector decrease in size over

time. The variable  $\chi^i(i = A, L)$  would diverge from  $(\chi^i)^*(i = A, L)$  and approach zero. The stable path is therefore  $\dot{\chi}^i > 0$  for all t.

The dynamics of u and v, the fractions of human capital used in production in the advanced and laggard sectors respectively are given by equations (A28a) and (A28b). The  $\dot{u} = 0$  and  $\dot{v} = 0$ loci are given by;

$$u = u^* + [\chi^A - (\chi^A)^*]$$
 (A29a)

$$v = v^* + [\chi^L - (\chi^L)^*]$$
 (A29b)

Case one:

The locus is linear and upward sloping in  $(u, \chi^A)$  space for the advanced sector [or  $(v, \chi^L)$  space for the laggard sector] as shown below. In figure 5,  $u(\chi^A)$ [or  $v(\chi^L)$ ] shows the stable saddle paths. If  $z^i(0) > (z^i)^*(i = A, L)$ , then  $\chi^i > (\chi^i)^*(i = A, L)$ , and  $\dot{\chi}^i < 0(i = A, L)$  as determined earlier. If for some  $t, u \leq u^*$  (or  $v = v^*$  for the laggard sector) then  $\dot{u} < 0$ (or  $\dot{v} < 0$ ) for all subsequent t. Therefore, u(or v) moves from  $u^*$  (or  $v^*$ ) and approaches zero. Thus the stable path is  $u > u^*$  (or  $v > v^*$ ). If  $\dot{u} \geq 0$ (or  $\dot{v} \geq 0$  for the laggard sector) for some t, then  $\dot{u} > 0$ (or  $\dot{v} > 0$ ) for all subsequent t, because  $\left\{-\left[\chi^i - (\chi^i)^*\right]\right\}$  (i = A, L) is negative and decreasing in size. Therefore  $\dot{u} < 0$ (or  $\dot{v} < 0$ ) and  $u > u^*$  (or  $v > v^*$ ) holds for all t.

#### Case two:

If  $z^i(0)(i = A, L)$ , then  $\chi^i < (\chi^i)^*$ ,  $\dot{\chi}^i > 0$ . If for some  $t, u \ge u^*$  (or  $v \ge v^*$  for the laggard sector) then  $\dot{u} > 0$  (or  $\dot{v} > 0$ ) for all subsequent t. Therefore, u(or v) moves from  $u^*$  (or  $v^*$ ) for all subsequent t and approaches  $\infty$ . Thus the stable path is  $u < u^*$  (or  $v < v^*$ ).

If  $\dot{u} \leq 0$  (or  $\dot{v} \leq 0$  for the laggard sector) for some t, then  $\dot{u} < 0$  (or  $\dot{v} < 0$ ) for all subsequent t, because  $-\left[\chi^i - (\chi^i)^*\right]$  (i = A, L) is positive and decreasing in size. Therefore  $\dot{u} > 0$  (or  $\dot{v} > 0$ ) and  $u < u^*$  (or  $v < v^*$ ) holds for all t.

It is also important to consider the relationship between  $z^i (i = A, L)$ , the gross average product of physical capital, and the state variable  $\kappa^i (i = A, L)$ . The dynamics of  $\kappa^A$  and  $\kappa^L$  can be described by the following equations written in terms of  $z^i (i = A, L)$ :

$$\frac{\dot{\kappa}^{A}}{\kappa^{A}} = z^{A} - \chi^{A} - (1 - u) - \alpha \xi^{A} N$$

$$= \left[ z^{A} - \left( z^{A} \right)^{*} \right] - \left[ \chi^{A} - \left( \chi^{A} \right)^{*} \right] + (u - u^{*}) + 1 - \alpha \xi^{A} N$$
(A30a)

$$\frac{\dot{\kappa}^{L}}{\kappa^{L}} = z^{L} - \chi^{L} - (1 - u) - (1 - \alpha) \xi^{l} N$$
  
=  $\left[ z^{L} - (z^{L})^{*} \right] - \left[ \chi^{L} - (\chi^{L})^{*} \right] + (v - v^{*}) - \alpha \xi^{L} N$  (A30b)

If we then use equations (A28a) and (A28b) to substitute in for  $[\chi^i - (\chi^i)^*]$  (i = A, L), we get

$$\frac{\dot{\kappa}^A}{\kappa^A} = \pi^A \beta \left[ z^A - \left( z^A \right)^* \right] - \gamma_{\chi^A} + \left( u - u^* \right) + 1 - \alpha \xi^A N \tag{A31a}$$

$$\frac{\dot{\kappa}^{L}}{\kappa^{L}} = \pi^{L}\theta \left[ z^{L} - \left( z^{L} \right)^{*} \right] - \gamma_{\chi^{L}} + \left( v - v^{*} \right) - \alpha \xi^{L} N$$
(A31b)

If for case one  $z^i(0) > (z^i)^*(i = A, L)$  then the conditions;  $[z^i - (z^i)^*] > 0(i = A, L)$ ,  $\dot{\chi}^i \leq 0(i = A, L)$ , and  $(u - u^*) \geq 0$  (or  $(v - v^*) \geq 0$  for laggard sector) imply that  $\dot{\kappa}^A/\kappa^A > 0$  (or  $\dot{\kappa}^L/\kappa^L > 0$ ). Hence, the system can only be on a stable path if  $\kappa^i(0) < (\kappa^A)^*$ . Then  $\kappa^i$  will rise monotonically from  $\kappa^i(0)$  towards  $(\kappa^i)^*$ .

If for case one  $z^i(0) < (z^i)^*(i = A, L)$  then the conditions;  $[z^i - (z^i)^*] < 0(i = A, L)$ ,  $\dot{\chi}^i \ge 0(i = A, L)$ , and  $(u - u^*) \le 0$  (or  $(v - v^*) \le 0$  for laggard sector) imply that  $\dot{\kappa}^A/\kappa^A < 0$ . Hence, the system can only be on a stable path if  $\kappa^i(0) > (\kappa^A)^*$ . Then  $\kappa^i$  will fall monotonically from  $\kappa^i(0)$  towards  $(\kappa^i)^*$ .

Thus,  $z^i$  and  $\kappa^i(i = A, L)$  are inversely related, with  $z^i(0) \ge (z^i)^*(i = A, L)$  as  $\kappa^i(0) \ge (\kappa^i)^*(i = A, L)$ . A lower starting value of the state variable  $\kappa^i(i = A, L)$  is associated with a higher initial value of  $z^i(0)$ .

If we use equations (A30a) and (A30b) again, and use (A27a) and (A27b) to plug in an expression for  $(u - u^*)$ . we get;

$$\frac{\dot{\kappa}^{A}}{\kappa^{A}} = \left[z^{A} - \left(z^{A}\right)^{*}\right] + \gamma_{u} + 1 - \alpha \xi^{A} N$$
(A32a)

$$\frac{k^L}{k^L} = \left[ z^L - \left( z^L \right)^* \right] + \gamma_v - \alpha \xi^l N$$
(A32b)

For case two, if  $z^i(0) > (z^i)^*(i = A, L)$  then:  $\dot{u} > 0$  and  $\dot{z}^A < 0$ . This implies that  $\dot{\kappa}^A/\kappa^A > 0$ . However, if  $z^i(0) < (z^i)^*(i = A, L)$ , then  $\dot{u} < 0$  and  $\dot{z}^A < 0$ . This implies  $\dot{\kappa}^A/\kappa^A < 0$ . Therefore  $\kappa^i$  (i = A, L) is always inversely related to  $z^i$ .

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Figure 1



Figure 2



Figure 3



Figure 4



Figure 5



Figure 6



Figure 7



Figure 8



Figure 9



Figure 10



Figure 11







Figure 13



Figure 14



Figure 15