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# The Role of Market Frictions on the Price Differential: A Search-Theoretical Approach

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Abstract: To shed light on how market frictions and the waiting time of imitators affect prices and how effective research subsidy and patent protection affect the price differential, this paper adopts a direct search-theoretical approach to capture the searching behaviors of consumers and producers in the innovative and imitative markets. As a result, this model shows that the price differential with endogenous market frictions would react to the change of quality the least. A shorter durability would result in a wider price differential in the model without the extra state for imitators than in the model with the extra state. While a research subsidy shrinks the price differential, and improve consumers' flow values, the patent protection widens the price differential, hurts imitators' profits and may not improve the consumers' flow values. The innovators could take the advantage of the effects of durability on price differential by inventing products which might influence the durability of the products currently hold by the consumers. This finding might provide an explanation on why the latest versions of computer products are invented less likely convertible to older versions of windows or associated software.

JEL Classification: O31, O33.

Keywords: market frictions, price differential, direct search, innovation, imitation

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# 1 Introduction

A high price of an invention is often observed, especially for the invention newly introduced to the market. Then the price of this invention reduces gradually after a period of time. One reason that causes the price to reduce is imitation activities, which result from technology diffusion. A consumer who is attracted to this invention has two options. One is to purchase the product now at a high price, and another is to delay the purchase and pay a lower price. The discussions related to this issue, whether technology diffusion, the market competitiveness, or innovation activities, are all connected to market frictions at some degree. The market frictions here are referred to the ratio of produces to consumers. For example, the technology diffusion may result in imitative activities and generate a more competitive environment for producers. An invention might speed up the retirement of the current product, and hence, affects the market frictions and the price level. By looking insight the price pattern of an invention in dynamics, one can observe the market frictions play a significant role, which motivates this exercise. Moreover, there exists a price differential between the innovative and imitative products, which might also be affected by the market frictions.

The following two examples may support how the price differential caused by technology diffusion products may depend not only on the quality of products but also the market frictions. The first example is DVD players. By classifying the electronic devices into two groups in terms of the quality, and the ease of use, the products produced by Sony, Panasonic, Hitachi and Toshiba may be grouped together, called “innovative products”, and the products produced by other companies, such as Oritron, may be grouped together, called “imitative products”. Soon after Panasonic introduced its single-disc DVD players, the companies in the same group, such as Sony, Toshiba and Hitachi, have their DVD players on the shelves. This could be because all these companies have their own research teams, which have similar facilities, technology and information of the next possible inventions. It is worth mentioning that it took years for Oritron to shelve its DVD players on the market, priced USD \$130, compared to Toshiba at USD\$160 and Panasonic at USD\$180 at the time. Note that Toshiba and Panasonic had reduced their prices after Oritron introduced its DVD players.

The second example is laptops. Again, by classifying the laptops into two groups in terms of the design, functions and the quality of screens, we may have Sony's laptops in one group and Compaq in another. In 2005, Sony had more than *1200* different types of laptops featured 40GB hard drive, 1.0GHz and 256MB RAM at price USD\$880 or more. Meanwhile, Compaq had *166* types of laptops with the same features at price USD\$724 or more. The price differential between Sony laptops and Compaq laptops with similar features was approximately USD\$156. For laptops with more advanced features, such as 3.0 GHz, 512MB RAM, integrated wireless lan, and DVD±RW, there are 7 types of laptops made by Sony at price USD\$1880 or more, but *none* made by Compaq. After having the DVD±RW feature replaced by CD-RW/DVD+RW, there are 2 types of laptops made by Compaq at price USD\$1499 or more. The price differential between Sony and Compaq laptops with these more advanced feature had become approximately USD\$381, which is more than double the price differential with common features. Note that the innovators are more likely to have products with the more advanced features for consumers to choose than the imitators. For laptops, whether with advanced or common features, the innovators provide more types than the imitators. Moreover, it does takes time for imitators to figure out how to imitate an advanced feature, which is DVD±RW in this example.

So where do market frictions take place? Clearly, the introduction of an advanced innovative product and/or an imitative product would affect the competitiveness of producers and the price levels, which may or may not have impacts on the price differential between innovative and imitative products. This price differential, however, might affect a consumer's decision on purchasing. The price a consumer would pay depends not only on the features of the laptop, but also on the group in which she purchases. If it is a laptop with common features, Sony produces more varieties at a higher price than Compaq does. If it is a laptop with the advanced features, such as DVD±RW, Sony provides several types at a very high price, but Compaq provides none. The amount of consumers shops in a group would certainly affect the market frictions of that particular group, and may affect the price level of that group, which might have impacts on the price differential.

In addition to the market frictions, we cannot ignore the production costs, which are also important in determining the price levels. The production costs for innovators include the research

costs invested in innovations, and the costs for imitators would include the waiting time cost. To shed light on how the market frictions and production costs, such as research costs and waiting time, influence both innovative and imitative activities and the price differential, and on how the innovation encouraging policies affect market frictions and the price differential, I adopt a search theoretical framework to address these questions.

The technology diffusion makes it possible for imitation and generate a more competitive market. In the literature of technology diffusion, the endogenous growth framework is often adopted to analyze how innovation and imitation might affect economic growth. Under this framework, Jovanovic and MacDonald (1994) find insufficient innovative and imitative efforts. Aghion, Harris, Howitt, and Vickers (2001) find that a little imitation might enhance growth, but too much imitation may do the opposite. Rustichini and Schmitz (1991) discover that subsidies might cause under-investment in both innovation and imitation sectors. Focusing on the interactions between R&D activities and price patterns, Laing, Palivos, Wang (2002) find that market frictions is crucial for the price structure and product diffusion pattern and that a more competitive market for producers will enhance more R&D activities. While market frictions are not counted in Jovanovic and MacDonald (1994) and Rustichini and Schmitz (1991), imitation activities is not included in the analysis of Laing, Palivos, Wang (2002).

In order to look insights the price patterns in the presence of market frictions, I extend Moen's (1997) direct search-theoretic framework to analyze the innovative and imitative activities as well as consumers' shopping behaviors. Since the producers with similar production activities would form their own submarket, there will be one innovative submarket and one imitative submarket. The quality levels of products are assumed the same within a submarket, and may be different across sub-markets. Consumers are free to travel to either submarket to purchase. The market frictions, defined as a ratio of producers to consumers, are crucial in determining the matching rates of consumers and producers, and can be solved endogenously.

As a result, this model confirms that the difference of marginal costs of innovative and imitative firms generates the price differential between the innovative and imitative submarkets. This is consistent with the findings of Reinganum (1979). I also show that the price differential with

endogenous market frictions would react to the change of quality the least, and the price differential without market frictions would react to such quality improvement the most, compared to the price differential with exogenous market frictions. The equilibrium market frictions and price differential in the models with and without the extra production state for imitators have similar characteristics, except the effects of durability. A shorter durability would result in a wider price differential in the model without the extra production state for imitators than in the model with the extra state.

Although both research subsidy and patent protection induce more research activities, their effects on the price differential and consumers' and producers' welfare are different. In this decentralised economy without central planner and tax, the welfare is implied in the flow values of consumers and producers. A research subsidy would generate more research activities and a more competitive innovative submarket for innovators. Consequently, the price differential shrinks, and consumers' flow value are improved. This result with regard to the link of R&D activities and market competitiveness is consistent with that of Laing, Palivos, Wang (2002). The patent protection, however, would widen the price differential, but it would hurt the imitators' profits and flow values, and may not improve the consumers' flow values. Alternatively, the innovators could take the advantage of the effects of durability on the price differential by inventing products which might influence the durability of the products currently hold by the consumers. The innovation which reduces the durability of the old product would enlarge the price differential by benefitting innovators more than imitators. This result provides an explanation on why the latest versions of computer products are invented less convertible to older versions of windows or associated software and hardware.

The rest of the paper is organized as follows. The environment of a general model with endogenous market frictions is described in Section 2, followed by the equilibrium in Section 3. In Section 4, the cases without market frictions and with exogenous market frictions are discussed and compared to the general model. In Section 5, I introduce an extra production state for imitators to examine whether the main results of the general model would sustain. Finally, conclusion and possible extensions are provided in Section 6.

## 2 Environment (general model: with endogenous market frictions)

Following the direct search-theoretic framework of Moen (1997), I assume an economy with only one type of good, which is indivisible. The economy is populated with a continuum of consumers and a continuum of producers. Consumers are searching for producers to purchase products, and producers are searching for consumers to sell products. Producers are distinguished by their production activities. Innovators would invest in research activities to invent more advanced products, while imitators would wait to imitate the most advanced products produced by innovators. Since producers who conduct similar production activities would form their own submarket, all innovators will form an innovative submarket ( $n$ ) and all imitators will form an imitative submarket ( $m$ ).

The advancements of products are in terms of qualities. More advanced products are with higher quality levels than less advanced ones. The concept of quality ladders, pioneered by Grossman and Helpman (1991), is introduced to model the different quality levels achieved by innovators and imitators [Figure 2]. The quality level of products determines the utility received by consumers. It is expected that the more complicated technology involved to produce an advanced product by innovators, the longer waiting time it will be required to imitate such products by imitators. To circumvent technological difficulties, the matching rate is assumed to depend on the market frictions, which is defined as a ratio of producers to consumers.

### 2.1 Market frictions and matching technology

Let  $b_i$  and  $s_i$  denote the mass of consumers (buyers) and producers (sellers) in sub-market  $i$ , respectively. The flow of the matching functions between producers and consumers in submarket  $i$  is  $x(s_i, b_i)$ , which is assumed concave ( $x' > 0$ ,  $x'' < 0$ ) and constant return to scale. Let  $\theta_i$  denote the market frictions of the submarket  $i$ ,  $\theta_i \equiv s_i/b_i$ . A higher  $\theta_i$  indicates a tighter submarket for producers. Then the consumers' matching rate in submarket  $i$  is  $d(\theta_i) = x(s_i, b_i)/b_i = x(\theta_i, 1)$ , and  $d'(\theta_i) > 0$ , and the producers' matching rate in submarket  $i$  is  $e(\theta_i) = x(s_i, b_i)/s_i = x(1, 1/\theta_i)$  and  $e'(\theta_i) < 0$ . Therefore, a higher  $\theta_i$  gives a lower matching rate for producers and a higher matching



rate for consumers.

$$\begin{aligned} \lim_{\theta \rightarrow 0} d(\theta) &= \lim_{\theta \rightarrow \infty} e(\theta) = 0 \\ \lim_{\theta \rightarrow \infty} d(\theta) &= \lim_{\theta \rightarrow 0} e(\theta) = \infty. \end{aligned}$$

## 2.2 Consumers

Consumers are assumed homogenous and can enter either submarket without costs. Each consumer has space to hold at most one unit of product, and free disposal applies. The consumers whose space is empty are in the unmatched state ( $U$ ) and the consumers whose space is filled are in the matched state ( $M$ ). The value function of a consumer in the  $U$  state is  $J^U$ , and the value function of a consumer in the  $M$  state is  $J^M$ . Since consumers could match in either sub-market,  $J_i^M (i = n, m)$  depends on the sub-market  $i$  from which the product is purchased. Let  $r$  denote the discounting factor, the flow value of  $J^U$  is the maximization of the expected flow value of changing to a matched state:

$$rJ^U = d(\theta_i) \text{Max}[J_i^M - J^U]. \quad (1)$$

After purchasing from a producer, the consumer will enjoy utility  $u_i$  from consuming the product by paying price  $P_i$ <sup>1</sup>. Both  $u_i$  and  $P_i$  are submarket specific. The price  $P_i$  of submarket  $i$  is decided by the producers of that sub-market. For simplicity, the average durability ( $\sigma$ ) of the product is assumed the same across submarkets. This durability could be interpreted as the period of time when consumers are satisfied with the product. This limited capacity of an individual means that consumers do not return to the market when they are satisfied with the product. Therefore, the arrival rate of consumers' entry to the market would be  $1/\sigma$ . The flow value of  $J_i^M$  is the net utility from consuming the product and the expected flow value of changing the state back to  $U$ :

$$rJ_i^M = u_i - rP_i + \frac{1}{\sigma}(J^U - J_i^M) \quad (2)$$

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<sup>1</sup>One could also assume that consumers receive different utility levels for the same product. In this model, the homogenous consumers would receive the same utility by consuming the products purchased in the same submarket and pay the same price.

Note that  $P_i$  is paid in full when the trade occurs rather than in several consecutive periodical payments to the producer. However, the utility received by consumers is in every period. So the discounting factor  $r$  appears in the price only.

### 2.3 Producers

In the retailer shops, the space to display one particular style of electronic devices is limited<sup>2</sup> and electronic devices are durable goods. Therefore, the shops often hold limited stock for each style. For simplicity, I assume that each space displayed in the retailer shop represents a producer. Each producer is assumed has space to hold at most one unit of product, so it will produce one unit per time period and will not hold inventory over the period. Entering the market costs each producer  $v_i^0 (i = n, m)$ , which varies across submarkets. The producers are distinguished by production activities. An innovator always invests sunk cost  $k$  in research, and a imitator has a delay process  $\tau$  to imitate the most advanced product produced by innovators. This delay process becomes the main cause of the quality differential between innovators and imitators, according to the quality ladder concept [Grossman and Helpman (1991)]. Let  $\lambda$  denote the exogenously determined economic growth rate. Then the growth factor at period  $t$  is  $e^{\lambda t}$ . Without losing generality, when the effective value of quality level is  $q$ , then the delayed process would result in a lower quality level<sup>3</sup>:  $qe^{-\lambda t}$ . This quality gap only exists across sub-markets, not within a sub-market. Both the research investment and the waiting time for imitation are publicly observable, and no hidden information exists with regard to the types of producers.

Following Moen (1997)<sup>4</sup>, all innovators in the model form an innovative sub-market and all imitators form an imitative sub-market. These are the only two sub-markets in the economy

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<sup>2</sup>For example, in a retailer shop, at most one laptop of each style is displayed on the shelves. Even for the electronic devices in a small size, such as cameras and mobile phones, the space used to display is usually limited to at most one for each style. Note that different colors can be considered different styles.

<sup>3</sup>Given  $\lambda$  and the growth factor  $e^{\lambda t}$ , by spending the whole period  $t$  on producing the product, a firm can achieve the real value of quality level  $qe^{\lambda t}$ . Define the effective value as the real value divided by the growth factor, then the effective value of quality level is  $q$ , and the effective value of quality achieved involving the delay process  $\tau$  is  $qe^{-\lambda \tau}$ , and  $qe^{-\lambda \tau} < q$ .

<sup>4</sup>Producers with the same production activities are assumed to gather, share ideas with each other, and form a sub-market of their own.

[Figure 1]. Moreover, it is assumed that only products produced by the producers of that sub-market will circulate in that sub-market, and there is no mix-up. Therefore, consumers who want to shop innovative products would have to enter the innovative sub-market, for example.

The instantaneous production process is assumed for all producers, both innovators and imitators. That is, no delay between obtaining an idea and producing the product and no extra production costs<sup>5</sup>. This assumption of instantaneous production leaves no production state. After production, the producer is in an unmatched state ( $U$ ), in which he holds this product and waits for a consumer to trade. After trading with the consumer, the producer will conduct the production activity and the instantaneous production allows the producer to return to the unmatched state ( $U$ ) right away. In this setup, there is no matching state for producers. One may argue that this may not be true for imitators because of their delayed process. This argument will be addressed in Section 5, in which the production state is restored for imitators, and it is shown that this assumption is not crucial for the main results.

Having a product at hand, an innovator is now waiting for a consumer to trade with. With the matching rate  $e(\theta_n)$ , the innovator will match with a consumer and receive the payment  $P_n$ , and then return to the next research project. By investing  $k$  on research, the innovator will obtain a new idea, has the product produced instantaneously, and returns to the unmatched state ( $U$ ). Let  $\Pi$  denote producers' value function. The superscript of  $\Pi$  shows the state of producers, and the subscript shows the sub-market where the producer locates. The flow value of an innovator  $r\Pi_n^U$  is the net profit of selling the product in the innovative sub-market  $n$ :

$$r\Pi_n^U = -k + re(\theta_n)P_n. \quad (3)$$

An imitator, however, will wait for the innovative product to imitate. After seeing the innovative products on the market, the imitator would spend time  $\tau$  to figure out the idea to imitate. Once the idea of imitation obtained, production is completed instantaneously. This unavoidable time cost lowers the quality level ( $qe^{-\lambda\tau}$ ) achieved by the imitator. With matching rate  $e(\theta_m)$ , the imitator would trade with a consumer and receive  $P_m$ . Then he will return to the unmatched

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<sup>5</sup>This assumption is for simplicity and wouldn't change the main results since the costs of materials to produce the products are similar for all producers.

state ( $U$ ) by spending time  $\tau$  to figure out the next imitation idea and completing the production instantaneously. Therefore, the flow value of an unmatched imitator is the expected payment of matching with a consumer:

$$r\Pi_m^U = re(\theta_m)P_m. \quad (4)$$

## 2.4 Price announcement and market frictions

In each submarket, producers would decide a price  $P_i$  to announce, which maximizes their profits subject to consumers' flow value  $J^U$ , which can be obtained by combining equations (1) and (2):

$$\frac{\sigma d(\theta_i)(u_i - rP_i)}{r[\sigma(r + d(\theta_i)) + 1]} = J^U, \text{ where } i = n, m. \quad (5)$$

Note that the entry condition, which is introduced in the next section, helps determining  $\theta_i$  endogenously. The costless entry to either submarket for consumers implies that the unmatched consumers would feel indifferent in entering either submarket. After trade, the combination of  $P_i$  and  $\theta_i$  would generate consumers' flow value  $rJ_i^M$ . Both consumer's indifference curve and producers' iso-profit curves are depicted in Figure 3, which shows that innovators have a higher marginal rate of substitution between  $\theta_n$  and  $P_n$  than imitators. It could be the research cost  $k$  that discourages the innovators to exchange a lower  $P_n$  for a lower  $\theta_n$  than imitators. For a given  $\theta_i$  and  $J^U$ , a producer  $i$  chooses  $P_i$  to maximize profit  $\Pi_i^U$  [equations (3) and (4)] and subject to  $J^U$  [equation (5)]. Let  $\eta_i$  denote the elasticity of producers' matching rate  $e$  with respect to the submarket frictions  $\theta_i$ :  $\eta_i = \eta(\theta_i) \equiv -\theta_i e'(\theta_i)/e(\theta_i)$ . The elasticity  $\eta_i$  ( $i = n, m$ ) is assumed constant for all producers. Then  $P_i$  ( $i = n, m$ ) can be solved as a function of  $J^U$ ,  $\eta_i$ , and  $u_i$  [see (A4) and (A5) in appendix].

## 3 Equilibrium (general model)

In equilibrium, without hidden information, none of the producers would deviate from its own type. Meanwhile, consumers are indifferent in purchasing products in either sub-market. Since the durability is assumed to be the same across submarkets, the utility level in this model is generated from the quality level of the product. This gives  $u_n = q$  and  $u_m = qe^{-\lambda\tau}$ . As a result, this quality differential ( $q - qe^{-\lambda\tau}$ ) together with various submarket frictions ( $\theta_i$ ) are one of the main causes for the price differential across sub-markets.

Substituting equation (5) into the reduced form of  $P_i$  [equations (A4) and (A5)] gives:

$$P_n(\theta_n) = \frac{(1 + \sigma r)(1 - \eta_n)q}{r[1 + \sigma r + \sigma \eta_n d(\theta_n)]}, \quad (6)$$

$$P_m(\theta_m) = \frac{(\sigma r + 1)(1 - \eta_m)qe^{-\lambda \tau}}{r[1 + \sigma r + \sigma \eta_m d(\theta_m)]}. \quad (7)$$

The entry cost of this model is exogenous and can be regarded as the sum of all types of transaction costs involved to learn the knowledge and/or technology, which is required to become a producer, and this cost is not the same for becoming an innovator and for becoming an imitator. Let  $v_i^0 (i = n, m)$  denote the entry cost of being a producer. In equilibrium, the entry condition requires:

$$v_i^0 = \Pi_i^U, (i = n, m). \quad (8)$$

By combining equation (8) with equations (3)-(4) and (6)-(7), we can get the equilibrium  $\theta_i^* (i = n, m)$ :

$$e(\theta_n^*) = \frac{r(\sigma r + 1)(v_n^0 + k/r)}{(\sigma r + 1)(1 - \eta_n)q - r\sigma \eta_n (v_n^0 + k/r)\theta_n^*}, \quad (9)$$

$$e(\theta_m^*) = \frac{r(\sigma r + 1)v_m^0}{(\sigma r + 1)(1 - \eta_m)qe^{-\lambda \tau} - r\sigma \eta_m v_m^0 \theta_m^*}, \quad (10)$$

where the possibilities of multiple equilibria cannot be ruled out, and every equilibrium satisfies equations (5) and (8), and  $P_i = \arg \max_{P_i} \Pi_i^U$ ,  $i = n, m$ . Substituting equation (9) and (10) into equations (6) and (7), the equilibrium price differential of this general case can be derived [see equation (13) in Section 4].

## 4 Results

To shed light on the role of market frictions on the price differential, I will discuss the case without market frictions and the case with exogenous market frictions in this section and compare these two cases to the general model, which is with endogenous market frictions and is derived in the previous section. This is followed by the analysis on the effects the innovation encouraging policies on the equilibrium price differential and the flow values of producers and consumers.

#### 4.1 Case 1: without market frictions

The case without market frictions means instantaneous matching between producers and consumers.

The price differential between submarkets ( $\Delta P^1$ ) becomes:

$$\Delta P^1 = \frac{q[1 - e^{-\lambda\tau}]}{r}. \quad (11)$$

Equation (11) implies that without market frictions,  $\Delta P^1$  is mainly affected by both  $q$  and  $\tau$ , and that an increase in either  $q$  or  $\tau$  would enlarge  $\Delta P^1$ . One explanation is that a higher  $q$  stimulates consumers' reservation price, which allows producers to charge more. This effect would benefit innovators more than imitators, and hence, enlarges  $\Delta P^1$ . The longer the amount of time taken to imitate, the lower the quality level that can be achieved by imitators. Thus, imitators have to lower  $P_m$  to attract consumers, and causes a wider  $\Delta P^1$ .

#### 4.2 Case 2: with exogenous market frictions

With exogenous market frictions for both submarkets, the price differential of this case ( $\Delta P^2$ ) becomes:

$$\Delta P^2 = \frac{(\sigma r + 1)(1 - \eta_n)q}{r[\sigma(r + \eta_n d(\theta_n)) + 1]} - \frac{(\sigma r + 1)(1 - \eta_m)qe^{-\lambda\tau}}{r[\sigma(r + \eta_m d(\theta_m)) + 1]}. \quad (12)$$

Equation (12) shows that the market frictions do affect the price levels and  $\Delta P^2$ , and allows the durability of the product ( $\sigma$ ) to play a role on  $\Delta P^2$ .

The properties of  $\Delta P^2$  shows that both  $\tau$  and  $q$  affect  $\Delta P^2$  in the same direction as case 1, but in a smaller size<sup>6</sup>, and that the effects of  $q$  on  $P_i$  are also smaller in case 2 than in case 1. This implies that the existence of market frictions restricts the producers to adjust  $P_i$ , and this restriction is stronger on innovators than on imitators. Therefore, in response to an increase in  $q$ , the size of an increase in  $P_n$  is smaller than in  $P_m$  in the presence of market frictions. As a result,  $\Delta P^2$  is smaller than  $\Delta P^1$ .

Additionally, the existence of market frictions ( $\theta_i$ ) allows the durability  $\sigma$  as well as  $\theta_i$  related variables to play a role on  $\Delta P^2$ . The  $\theta_i$  related effects are: the elasticity of producers' matching

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<sup>6</sup>The properties of  $\Delta P^2$  are:  $\partial\Delta P^2/\partial q > 0$ ,  $\partial\Delta P^2/\partial\tau > 0$ ,  $\partial\Delta P^2/\partial\eta_n < 0$ ,  $\partial\Delta P^2/\partial\eta_m > 0$ ,  $\partial\Delta P^2/\partial\theta_n < 0$ ,  $\partial\Delta P^2/\partial\theta_m > 0$ .

rate  $\eta_i$ , and the consumer's matching rate,  $d(\theta_i)$ . All three variables,  $\sigma$ ,  $\eta_i$  and  $d(\theta_i)$ , have negative impacts on  $P_i$ . The higher the  $\eta_i$ , the lower the  $P_i$  will be posted by the producers. A tighter market for producers (a higher  $\theta_i$ ) would increase  $d(\theta_i)$  and reduce consumers' reservation price for the product. Thus, producers have to lower  $P_i$  to attract consumers. Consequently,  $\Delta P^2$  would shrink (enlarge), if the submarket  $n$  ( $m$ ) gets tighter for innovators (imitators).

### 4.3 General model: with endogenous market frictions

Following what has been left in the general model (see Section 3), when the market frictions are endogenously determined, the equilibrium price differential can be derived as:

$$\Delta P^3 = \frac{(\sigma r + 1)(1 - \eta_n)q}{r[1 + \sigma r + \sigma \eta_n d(\theta_n^*)]} - \frac{(\sigma r + 1)(1 - \eta_m)q e^{-\lambda \tau}}{r[1 + \sigma r + \sigma \eta_m d(\theta_m^*)]}. \quad (13)$$

When the direct effects dominate, the properties of the equilibrium market frictions<sup>7</sup>  $\theta_i^*$  ( $i = n, m$ ) allow us to provide insight on how the market frictions and the price differential are affected by the variables.

Equation (13) shows that  $q$  affects both  $P_i$  and  $\theta_i$  positively, and  $\sigma$  and  $\eta_i$  affect both  $P_i$  and  $\theta_i$  negatively. The negative impacts of  $\theta_i$  on  $P_i$  offset the direct effects of these three variables ( $q, \sigma, \eta_i$ ) on  $P_i$ . Therefore, the overall effects of ( $q, \sigma, \eta_i$ ) on  $P_i$  in the case with endogenous market frictions is smaller than the case with exogenous market frictions.

The entry cost  $v_i^0$  and the production costs,  $k$  for innovators and  $\tau$  for imitators, would affect  $P_i$  indirectly via  $\theta_i$ . An increase in  $v_i^0$  discourages producers to enter the market and causes a lower  $\theta_i$ . This smaller  $\theta_i$  would drive up  $P_i$ . While an increase in  $k$  would increase  $P_n$  through lowering  $\theta_n$ , an increase in  $\tau$  may increase  $P_m$  through lowering  $\theta_m$ . However, different from  $v_i^0$  and  $k$ ,  $\tau$  has a direct negative effect on  $P_m$ . When the direct effect dominates, the lower quality level of imitative products caused by a longer waiting time ( $\tau$ ) is sufficiently strong to reduce  $P_m$ .

Comparing the price differential in all three cases, one can find that a change in quality differential  $q$  would affect  $\Delta P^1$  the most,  $\Delta P^2$  the second, and  $\Delta P^3$  the least:  $\partial \Delta P^1 / \partial q > \partial \Delta P^2 / \partial q > \partial \Delta P^3 / \partial q$ . This shows that the market frictions would reduce the effects of  $q$  on the price dif-

<sup>7</sup>The equilibrium market frictions  $\theta_i^*$  ( $i = n, m$ ) has the following properties:  $\partial \theta_i^* / \partial q > 0$ ,  $\partial \theta_i^* / \partial v_i^0 < 0$ ,  $\partial \theta_i^* / \partial \eta_i < 0$ ,  $\partial \theta_i^* / \partial \sigma < 0$ ,  $\partial \theta_n^* / \partial k < 0$ ,  $\partial \theta_m^* / \partial \tau < 0$ .

ferential. The endogenous market frictions would shrink the effects even further. One finding to emphasize is the negative effect of durability ( $\sigma$ ) on the price differential. An decrease in the length of durability of the product  $\sigma$  would drive up both  $P_n$  and  $P_m$ . Interestingly,  $P_n$  would respond more strongly than  $P_m$ . Consequently, the price differential is enlarged. In other words, the innovators could take advantage of this negative effects of  $\sigma$  on  $P_i$  by inventing products which would reduce the durability of the products currently held by the consumers.

## 5 Discussion

### 5.1 Alternative model

In this section, I allow for an extra production state ( $P$ ) for imitators, additional to the unmatched state ( $U$ ), to examine whether the main results in the general model still hold. The main reason for this extra state for imitators is due to the period of time that imitators have to wait for the ideas to imitate. It is now assumed that after completing the production and having the product at hand to sell, an imitator would change to the production state ( $P$ ) from the unmatched state ( $U$ ) [see Figure 4 for the environment]. Accordingly, the value functions of these two states are :

$$r\Pi_m^P = \frac{1}{\tau}(\Pi_m^U - \Pi_m^P), \quad (14)$$

$$r\Pi_m^U = e(\theta_m)(rP_m + \Pi_m^P - \Pi_m^U). \quad (15)$$

Similarly, combining with the entry condition [equation (8)] to solve the equilibrium market frictions  $(\theta_n^{A*}, \theta_m^{A*})$ , the equilibrium price differential ( $\Delta P^A$ ) can be solved:

$$\Delta P^A = \frac{(1 - \eta_n)(1 + \sigma r)u_n}{r[1 + \sigma r + \eta_n \sigma d(\theta_n^{A*})]} - \frac{[1 + \tau(r + e(\theta_m^{A*}))](1 - \eta_m)(1 + \sigma r)u_m}{r\{2[(1 + \tau e(\theta_m^{A*}))](1 - \eta_m) + \tau r\} + \eta_m \tau d(\theta_m^{A*})}. \quad (16)$$

The first and the second term of equation (16) shows  $P_n^{A*}$  and  $P_m^{A*}$ , respectively. The properties of market frictions  $\theta_i^{A*}$  ( $i = n, m$ ) are similar to those in the general case, except for the effects of  $\sigma$  and  $\eta_m$  on  $\theta_m^{A*}$ . The overall effect of  $\sigma$  on  $\theta_m^{A*}$  becomes positive, and the overall effect of  $\eta_m$  on  $\theta_m^{A*}$  becomes ambiguous. In turn, the overall effect of  $\sigma$  on  $P_m^{A*}$  also becomes positive. Both the general model and alternative model show the negative effects of  $\sigma$  on the price differential, and this negative effects of  $\sigma$  on the price differential is stronger in the alternative model than in



the general model. This is because an increase in durability this alternative model would increase  $P_m^{A*}$ , and shrink the price differential  $\Delta P^{A*}$  further than the price differential in the general model,  $\partial \Delta P^{A*} / \partial \sigma < \partial \Delta P^3 / \partial \sigma$ . One explanation is that in the general model without the extra production state for imitators, the durability has two opposite effects on  $P_i^*$ . On one hand, a higher  $\sigma$  delays consumers' next trip to the market, and causes the producers to reduce  $P_i^*$  to attract consumers. On the other hand, a higher  $\sigma$  increases consumers' reservation price, which allows the producers to charge more. In the general model, the former effect dominates, but in this alternative model, the latter effect dominates in the imitative submarket. This could be because the explicitly production state for imitators implies that a higher  $\sigma$  buys imitators more time to develop imitative ideas while consumers are still consuming their previous product.

## 5.2 The impact of innovation encouraging policies

The policies, such as research subsidy and patent protection, are often considered to encourage innovation activities. In this section, I will focus on these two policies, and examine how such policies might affect the price levels, and whether such policies might improve the welfare of consumers and producers. In this decentralized economy without a central planner, the flow values can be viewed as the welfare of an individual.

One interesting finding is that although both research subsidy and patent protection achieve the goal to encourage innovation well, they affect the price differential in the opposite directions. Moreover, these two policies have different impact on the welfare of consumers and producers. A research subsidy (a lower  $k$ ) would lower  $P_n$  while leaving  $P_m$  unchanged, and shrink the price differential. Therefore, a consumer could enjoy the innovative product at a lower price, and improves her flow matched value ( $J_i^M$ ). Meanwhile, innovators flow value are improved as well. The patent protection (a higher  $\tau$ ), however, would reduce an imitator's flow value by lowering  $P_m$ , and enlarge the price differential. Although a lower  $P_m$  might improve consumers' matched value ( $J_m^M$ ), the lower quality of imitative products would decrease  $J_m^M$  by reducing consumers' utility level. Overall, the patent policy may not improve consumers' flow matched value  $J_m^M$ , but it certainly decreases the imitators' flow value while leaving innovators' flow value unchanged.

Another interesting finding is the negative effect of durability  $\sigma$  on the price levels and on the

price differential. In order to charge a higher price and to enlarge the price differential, innovators might want to invent products, which could retire the old products earlier. Taking computer software as an example, a version of "windows" can last very long, and has different software, such as word and explorer, associated to it. From time to time, we are provided upgraded versions of software, which are invented to be adapted to the newest version of "windows". Although resisting upgrading the old windows, which is still workable, consumers are aware of the fact that the old windows is becoming less likely to accommodate the latest versions of software and has to be replaced or upgraded. So the invention of the latest versions of software, which are less likely convertible to the old windows, have retired the old "windows" earlier. This might speed up consumers' next trip returning to the market, and allow the producers to charge more. Similarly, the old computers without DVD drive, wireless len or USB drive might become not- upgradable at some stage, and have to be retired earlier than expected. So the inventions of DVD drive, wireless len and USB drive have shorten the period of time when consumers are satisfied with the product. This might allow the producers of computers with DVD drive, wireless len, or USB drive to charge more and enlarge the price differential. Therefore, the inventions that reduce the durability of the old product would enlarge the price differential, and benefit the innovators more than imitators. This might encourage innovative activities in return.

## 6 Conclusion and Extension

The search-theoretical framework allows us to shed light on the role of market frictions on the price differential between innovative and imitative products and to examine the impact of innovation encouraging policies on the welfare of producers and consumers. Interestingly, I find that the market frictions would shrink the price differential, and endogenous market frictions would shrink the price differential further than exogenous market frictions. The equilibrium market frictions and the price differential in the models with (*alternative model*) and without (*general model*) the extra production state for imitators have similar characteristics. Both the general model and alternative model show the negative effects of the durability on the price differential, and this negative effects is stronger in the alternative model than in the general model. The only exception is the effect of

durability, which becomes positive on the price of imitative goods in the alternative model. Thus, a shorter durability would result in a wider price differential in the general model than in the alternative model.

The innovation encouraging policies, research subsidy and patent protection affect the price differential and consumers' and producers' welfare, differently. . A research subsidy would shrink the price differential and improve consumers' welfare. The patent protection would widen the price differential, but it would hurt the imitators' flow values and may not improve the consumers' flow values. Alternatively, the effects of durability could motivate the innovators to invent products which might influence the durability of the existing products currently hold by the consumers. The innovations which reduce the durability of old products could enlarge the price differential and benefit innovators more than imitators. This result provides an explanation on the observations in the electronic devices.

This model may be extended in several ways to analyze the innovative and imitative activities in different aspects. One is to have the price level determined by bargaining process. This may allow the model to discuss how the price patterns matter to the innovative and imitative activities in the presence of market frictions. Another possible extension is to allow for multi-unit capacity for producers and consumers. This may enable the discussion on how innovators and imitators would adjust the quantity to produce in response to the change of market conditions with and without market frictions.

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## Appendix

1. Figure 3: Derivation of the iso-profit curves and the indifference curves.

From value functions of innovators, imitators and consumers, the following equations can be derived and signed:

$$\begin{aligned} \frac{\partial \theta}{\partial P} \Big|_{\Pi_n^U} &= -\frac{e(\theta_n)}{eI(\theta_n)P} > 0; & \frac{\partial^2 \theta}{\partial P^2} \Big|_{\Pi_n^U} &= \frac{eI(\theta_n)e(\theta_n)}{[eI(\theta_n)P]^2} < 0, \\ \frac{\partial \theta}{\partial P} \Big|_{\Pi_m^U} &= \frac{(\tau r + 1)e(\theta_m)}{-[P(\tau r + 1) - \tau]eI(\theta_m)} > 0; & \frac{\partial^2 \theta}{\partial P^2} \Big|_{\Pi_m^U} &= \frac{(\tau r + 1)^2 eI(\theta_m)e(\theta_m)}{\{eI(\theta_m)[\tau - P(\tau r + 1)]\}^2} < 0, \\ \frac{\partial \theta}{\partial P} \Big|_{J^U} &= \frac{\sigma d(\theta)}{\sigma dI(\theta)(U - P - r)} > 0; & \frac{\partial^2 \theta}{\partial P^2} \Big|_{J^U} &= \frac{\sigma^2 dI(\theta)d(\theta)}{[\sigma dI(\theta)(U - P - r)]^2} > 0. \end{aligned}$$

Thus, Figure 3 can be depicted.

2. Solve  $P_i$  as a function of  $J^U$ ,  $\eta_i$  and  $u_i$ :

Totally differentiate equations (3) and (4) with respect to  $P_i$ :

$$e(\theta_i) \left[ 1 - \eta_i \left( \frac{P_i}{\theta_i} \right) \left( \frac{\partial \theta_i}{\partial P_i} \right) \right] = 0, \text{ where } i = n, m \quad (\text{A.1})$$

Consider the interior solution only, then the condition satisfying (A.1) is:

$$\eta_i \left( \frac{P_i}{\theta_i} \right) \left( \frac{\partial \theta_i}{\partial P_i} \right) = 1. \quad (\text{A.2})$$

Rearranging equation (5):  $d(\theta_i) = \theta_i e(\theta_i) = \frac{r(\sigma r + 1)J^U}{\sigma(u_i - rP_i - rJ^U)}$  and totally differentiating (A.3) with respect to  $\theta_i$  and  $P_i$  give:

$$\frac{\partial \theta_i}{\partial P_i} = \frac{r\theta_i}{(1 - \eta_i)(u_i - rP_i - rJ^U)} \quad (\text{A.3})$$

Plugging (A.3) into (A.2), then  $P_i$  can be solved as a function of  $J^U$ ,  $\eta_i$  and  $u_i$ :

$$P_n(\theta_n) = (1 - \eta_n) \left( \frac{u_n}{r} - J^U \right), \quad (\text{A.4})$$

$$P_m(\theta_m) = (1 - \eta_m) \left( \frac{u_m}{r} - J^U \right). \quad (\text{A.5})$$

3. Section 5: alternative model

Combining equations (11) and (17)-(18), the equilibrium market tightness  $(\theta_n^{A*}, \theta_m^{A*})$  can be solved:

$$e(\theta_n^{A*}) = \frac{(1 + \sigma r)(v_0^n + k)}{(1 - \eta_n)(1 + \sigma r)u_n - \sigma\eta_n(v_0^n + k)\theta_n^{A*}}, \quad (\text{A.6})$$

$$e^2(\theta_m^{A*}) = \frac{\left. \begin{aligned} &2(1 + \tau r)v_0^m(1 - \eta_m + \tau r) \\ &+ e(\theta_m^{A*})v_0^m\tau \left\{ \begin{aligned} &(1 + \tau r)[2(1 - \eta_m) + \eta_m\theta_m^{A*}] \\ &+ [2(1 - \eta_m) + \tau r] \end{aligned} \right\} \\ &- e(\theta_m^{A*})u_m(1 + \tau r)^2(1 - \eta_m)(1 + \sigma r) \end{aligned} \right\}}{\tau \left\{ \begin{aligned} &(1 + \tau r)(1 - \eta_m)(1 + \sigma r)u_m \\ &- \tau v_0^m[2(1 - \eta_m) + \eta_m\theta_m^{A*}] \end{aligned} \right\}}. \quad (\text{A.7})$$

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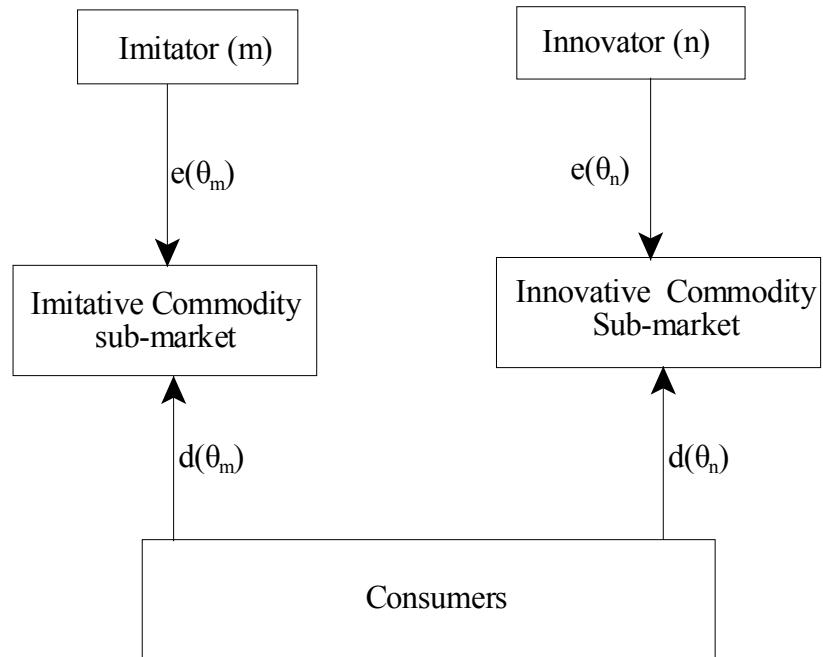


Figure 1: Environment (without production state)

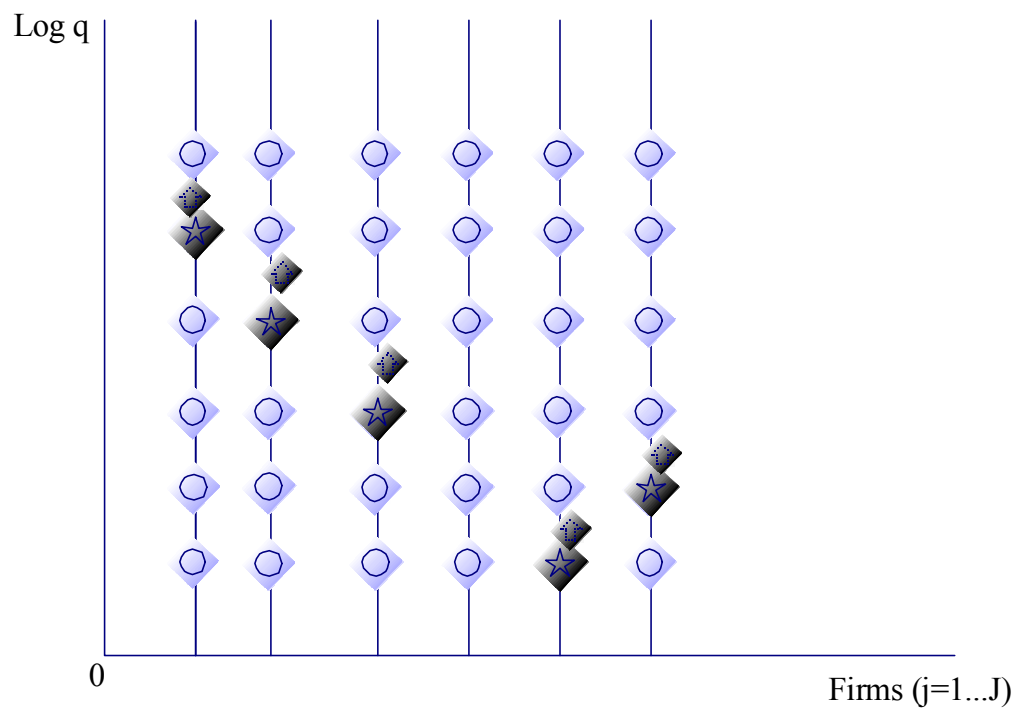


Figure 2: Quality ladders (Grossman and Helpman, 1991)



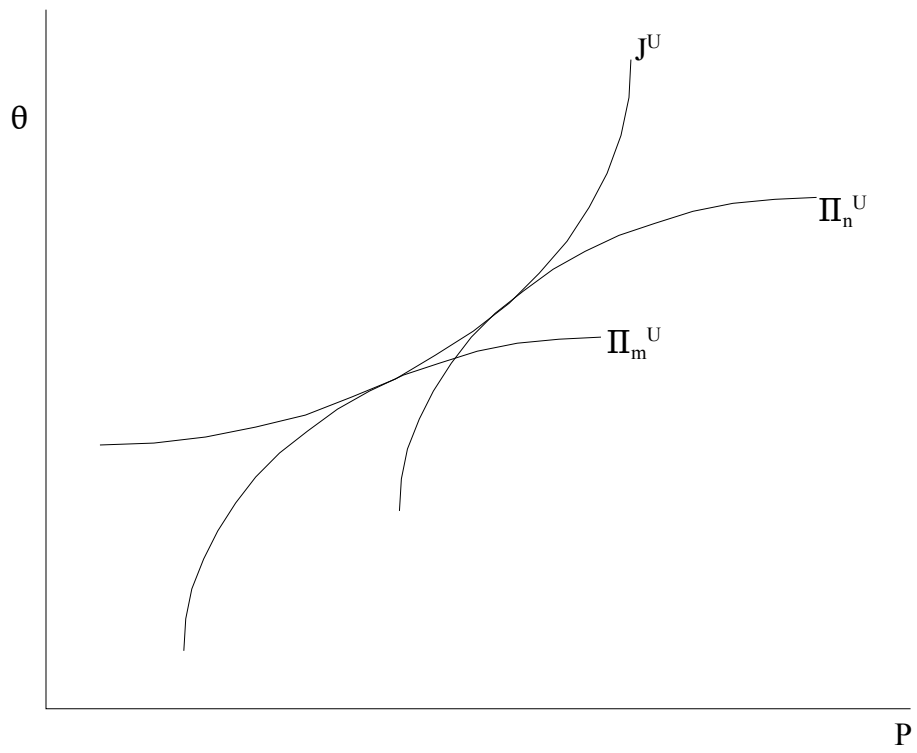


Figure 3: Interaction between firms and consumers

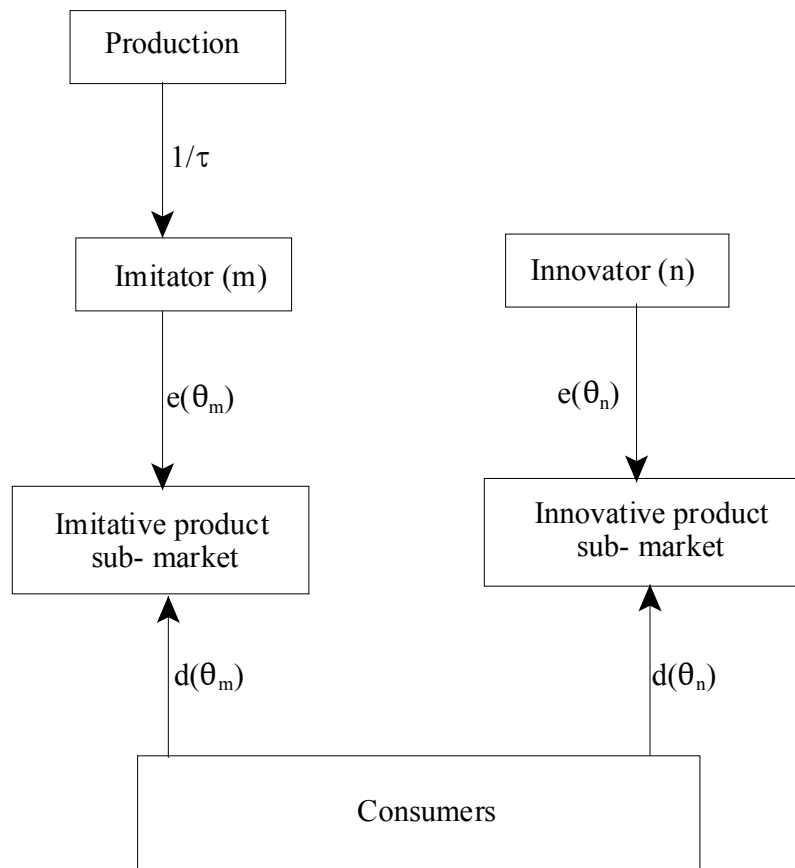


Figure 4: Environment (alternative model-with production state for imitators)