SEF Working paper: 19/2011

December 2011

A note resolving the debate on

"The weighted average cost of capital is not quite right"

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A note resolving the debate on

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Abstract

Miller (2009a) derives a weighted average cost of capital for the special case where the cash flows to equity and the cashflows to debt are annuities. The paper attracts debate. We show that the weighted average cost of capital is redundant in a world where interest paid is not tax deductible. The required rate of return on unlevered equity will consistently and reliably estimate the net present value of any project no matter the idiocyncratic beliefs of the analyst as to the year-by-year leverage of the project, or of the firm. We recommend that the weighted average cost of capital method is discarded. Our recommendation also applies to a world where interest paid is tax deductible.

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1. Introduction

Miller (2009a) suggests there are projects that cannot be correctly valued by the textbook weighted average cost of capital. To prove his point, he derives a nonlinear weighted average cost of capital NLWACC model. He examines a project with a finite life where the cash flows to equity and the cash flows to debt are annuities. Bade (2009) and Pierru (2009a) take issue with Miller (2009a). Finally, Miller (2009b) offers a reply to Pierru (2009a) which is further debated by Pierru (2009b). Although there is random and sporadic reference to tax relief on interest paid in the papers cited above, it is clear that the main focus of these papers is on the weighted average cost of capital in a Modigliani & Miller (1958) world where interest paid is not tax deductible – that is, where the value of the levered firm is equal to the value of the unlevered firm.

The cash flows of Miller's project are shown in Table 1, Panel A. Miller uses this project to illustrate his NLWACC model. This model, derived from first principles in Miller (2009a, equations (13) to (23), pp. 134-135), is

$$IA_{NLWACC} = IA_e^L \times \frac{S^L}{S^L + B} + IA_b \times \frac{B}{S^L + B} \quad , \tag{1}$$

where $IA_r = \dfrac{r}{\left(1 - (1+r)^{-N}\right)}$ is the inverse of the annuity operator $A_r = \dfrac{\left(1 - (1+r)^{-N}\right)}{r}$ with

IA_{NLWACC}, IA_e^L and IA_b representing the operators for the NLWACC, the required rate of return on levered equity and the cost of debt, respectively; S^L is the market value of levered equity and B is the market value of debt. The corresponding textbook weighted average cost of capital is written as

$$r_{WACC} = r_e^L \times \frac{S^L}{S^L + B} + r_b \times \frac{B}{S^L + B} \quad , \tag{2}$$

where r_b is the cost of debt. A point to appreciate, as we show later, is that the required rate

of return on levered equity r_e^L is a function of leverage.

An important aspect of Miller's NLWACC model is that it generates a different estimate of the weighted average cost of capital to that generated by the textbook WACC. The economic truth is that the NLWACC is the appropriate discount rate for Miller's project. In this case the textbook WACC is incorrect. The implication is that the textbook WACC only applies to special cirsumstances. These cirumstances are a single period world, an infinite world and a world where leverage is constant over the finite life of the project (Miles & Ezzell, 1980).

Bade (2009) and Pierru (2009a, b) are not fully convinced by Miller's arguments. Indeed, Pierru (2009a, Abstract, p. 1219) believes that Miller's NLWACC model "*is not relevant*". Bade (2009, Table 2, p. 1479) and Pierru (2009a, Table 1, p. 1221) offer an alternative project to illustrate the well-known fact that the textbook WACC is correct when the leverage the project is constant. Bade (2009, Table 1, p. 1478) and Pierru (2009a, Table 2, p. 1222) show that the textbook WACC of Miller's project increases over time.

Our goal is to critically synthesise these differing points of view. The issue is important in a theoretical domain and is critical in a practical world. In a world where interest paid is not tax deductible, we show that the WACC, as a method and as a notion, is redundant. Indeed, it is an intrinsically risky procedure -- in unskilled hands, its application can easily lead to incorrect conclusions. Our solution is to use the required rate of return on unlevered equity to assess the net present value of the project – this is a foolproof method.

There are three points to appreciate. First, the series of cash flows (i.e., projects) used in Miller, Bade and Pierru, and in this current paper, have the special property that their net present value is zero. The express goal is to mimic the pricing of an asset in an efficient market. The WACC is defined as the discount rate r_{WACC} which converts the unlevered cash flows CF_t^U of the firm (Miller, 2009a, footnote 4, p. 130) to the combined market value of debt and equity, i.e.,

$$S^L + B = \sum_{t=1}^{N} \frac{CF_t^U}{(1 + r_{WACC})^t} \quad . \tag{3}$$

Alternatively, the WACC method determines the net present value of a project, i.e.,

$$NPV_0 = \sum_{t=1}^{N} \frac{CF_t^U}{(1 + r_{WACC})^t} - (S_0 + B) \quad , \tag{4}$$

where S_0 is the contribution of equity at time $= 0$ and B is the cash contribution by debt at time $= 0$. Noting that the net present value accrues solely to equity, we get $S^L = S_0 + NPV_0$. Simple rearrangement gives equation (3). The purpose of these $NPV_0 = 0$ projects is to confirm or deny the validity of estimates of the weighted average cost of capital arising from a variety of specifications. Second, in this paper, all discount (interest) rates are represented as a percentage per annum (or period). Third, our solution is predicated on the availability of a reliable estimate of the project's unlevered required rate of return on equity r_e^U. The vital aspect here is that this is the starting point for the estimation of the required rate of return on levered equity r_e^L (see equation 7) which is used in the textbook WACC (see equation 2).

For convenience our examples are based on a two-period world -- this is the simplest form of the general N-period case. The tenor of the results is not affected by increasing the maturity of the world, i.e., there is nothing to be gained in the examination of unecessarily complex scenarios.

2. Special cases

Our analysis starts with the consideration of two special cases. These are the cases where the IRR of the cash flows to equity, i.e., the required rate of return on levered equity, generates the correct weighted average cost of capital when applied to the appropriate WACC model. The cash flows of the project are -$260, +$160 and +$160 in successive years. Equity contributes $180 to the cost of the project – the remaining $80 is provided by debt. Debt requires a rate of return of 6%. By design, the project's expected cash flows are an annuity. The IRR of this project's unlevered cash flows is 15.03% -- as we show, this is the WACC of the project. Table 2 presents two examples. In our opinion, these are very special cases. The

only differences between these two examples are found in the debt repayment schedules. The common characteristic is that the project is the firm.

The first example is a Miller project where the future cash flows to debt are an annuity and the future levered cash flows to equity are also an annuity (Table 2, Panel A). Application of the Miller NLWACC model gives

$$
\begin{aligned}
IA_{NLWACC} &= IA_e^L \times \frac{S^L}{S^L + B} + IA_b \times \frac{B}{S^L + B} \\
&= 0.6465 \times \frac{180}{260} + 0.5454 \times \frac{80}{260} \\
&= 0.6154
\end{aligned}
\qquad (5)
$$

Using an iterative method to solve for the annual NLWACC gives $r_{NLWACC} = 15.03\%$. This is the IRR of the unlevered cash flows. This confirms the validity of the Miller NLWACC model. Supporting evidence is found in the fact that the NLWACC model becomes the textbook WACC in the special cases when $N = 1$ and $N =$ infinity. The conclusion is that the Bade (2009) and Pirerru (2009a) criticisms are moot.

The second special case is the Bade/Pierru approach of structuring the cash flows to debt so that the leverage ratio is a constant (Table 2, Panel B). We use the IRR of 15.03% to determine the value of the project each year. The value of debt each year is defined as 80/260 of the value of the project – this defines the annual cash flows to debt (see Bade, 2009 or Pierru, 2009a). Equity receives the residual cash flows – these have an IRR of 19.05%. Using the textbook WACC we get

$$
\begin{aligned}
r_{WACC} &= r_e^L \times \frac{S^L}{S^L + B} + r_b \times \frac{B}{S^L + B} \\
&= 19.05 \times \frac{180}{260} + 6 \times \frac{80}{260} \\
&= 15.03\%
\end{aligned}
\qquad (6)
$$

The pragmatic criticism of this approach is the neccessity to restructure the debt repayment schedule of the firm (project) in order to use the textbook WACC. Such a process is inefficient.

Pierru (2009a, p. 1220) recommends that "*A constant WACC implicitly requires that the debt ratio also remains constant ... Therefore, Miller should determine the debt repayment schedule so as to satisfy this debt ratio*". Pierru (2009a) does not restructure Miller's debt repayment schedule so as to achieve a constant leverage ratio. Rather, another project is used as an example (see Table 1). The reasons for these decisions are not obvious to us. The restructuring of the debt repayment schedule is not an onerous task -- it took us a few minutes to determine the resulting cash flows to debt and equity which are presented in Table 3. The IRR of these cash flows to equity is 12.071%. As we show below (see equation 8), the application of the textbook WACC results in Miller's estimated WACC of 10.5533%.

3. General case

We now examine the general case. The previous analysis has essentially treated the firm as being the project. The focus now will be on a firm which owns a number of different projects. The firm plans to maintain a constant leverage over the finite life of the project under evaluation. Thus the exact specification of the year-by-year cash flows to debt is not necessary – it is sufficient that they earn 6% from the cash flows generated by the project. Miles & Ezzell (1980) show that the textbook WACC model applies to a project with a finite life if the leverage ratio is a constant. The scenario is where interest paid is tax deductible, i.e., a Modigliani & Miller (1963) world where $V^L = V^U + B \times T_C$ where V^L and V^U are the values of the levered firm and the unlevered firm, respectively, and T_C is the corporate tax rate. The Miles & Ezzell (1980, Section III, pp. 126-127) argument naturally translates to a world where interest paid is not tax deductible.

To illustrate the method, we apply the approach to Miller's project (Table 1, Panel A). The firm's debt to equity ratio, by market value, is 1/3, that is, 25% debt and 75% equity. As before, the required rate of return on debt is 6%. We assume that the unlevered required rate of return r_e^U is reliably estimated at 10.5533% -- this is the IRR of Miller's project. The required rate of return on levered equity r_e^L is estimated, using the textbook model, as

$$r_e^L = r_e^U + \left(r_e^U - r_b\right) \times \frac{B}{S^L}$$
$$= 10.5533 + \left(10.5533 - 6\right) \times \frac{1}{3} \tag{7}$$
$$= 12.071\%$$

and the textbook WACC is estimated as

$$r_{WACC} = r_e^L \times \frac{S^L}{S^L + B} + r_b \times \frac{B}{S^L + B}$$
$$= 12.071 \times 0.75 + 6 \times 0.25 \qquad . \tag{8}$$
$$= 10.5533\%$$

This confirms, as one might reasonably expect, that the Miles & Ezzell (1980) logic is correct for the world where interest paid is not tax deductible. If Bade (2009) and Pierru (2009a) had addressed Miller's project in this classical way, they would have confirmed the validity of the Miller's NLWACC. As we show earlier (see Table 3), identical results are achieved if the cash flows to equity are restructured to achieve a constant leverage ratio.

The textbook r_e^L model (equation 7), like the textbook WACC model (equation 8), only applies in specific circumstances. Our best guess is that these circumstances are the same as those required for the textbook WACC. However, these two equations are 'cointegrated'. When combined, their end result, the estimated WACC, is applicable to any scenario -- two wrongs make a right. To prove the point, assume that the leverage ratio of the firm is 1, that is, $S^L = B$. Recalculations give $r_e^L = 15.1066\%$ and $r_{WACC} = 10.5533\%$. As if by magic, the same WACC number is obtained from any chosen leverage ratio. This insight leads to our recommendation.

4. Recommendation

Our recommendation is to discount the unlevered cash flows of the project by the project's required rate of return on unlevered equity. This approach applies to every conceiveable project no matter the leverage of the firm in a Modigliani & Miller (1958) world. The textbook WACC is written as

$$r_{WACC} = r_e^L \times \frac{S^L}{S^L + B} + r_b \times \frac{B}{S^L + B} \quad . \tag{9}$$

The corresponding textbook required rate of return on levered equity is written as

$$r_e^L = r_e^U + \left(r_e^U - r_b\right) \times \frac{B}{S^L} \quad . \tag{10}$$

Substitution of r_e^L into equation (9), followed by simple reorganisation gives

$$r_{WACC} = r_e^U \quad , \tag{11}$$

for any N. The conclusion is clear. There is no need to calculate the WACC in a world where interest paid is not tax deductible.

The Appendix addresses the issue of whether equation (11) also applies to Miller's NLWACC model. The process is to derive the appropriate required rate of return on levered equity, and then to substitute it into the NLWACC model. As might be reasonably expected, our recommendation applies to Miller's model.

5. Concluding remark

The WACC method is superfluous in a world where interest is not tax deductible. The net present value of any project can be obtained by discounting the unlevered cash flows by the required rate of return on unlevered equity. Our contribution is to recommend that the WACC method (and mentality) is discarded. Our logic naturally extends to a world where interest paid is tax deductible – a Modigliani & Miller (1963) world. In this case, Myer's (1974) APV method is recommended. Alternatively put, our suggestion is to use the modified APV method in a Modigliani & Miller (1958) world where interest paid is not tax deductible.

Appendix

The required rate of return on levered equity to match Miller's NLWACC is derived as follows. The unlevered cash flows of the project CF^U, the cash flows to debt CF^{Debt} and, as a direct consequence, the cash flows to equity CF^{Equity} are annuities with a life of N years. The market value of levered equity S^L is the present value of the cash flows to levered equity discounted by the required rate of return on levered equity r_e^L, that is,

$$S^L = \sum_{t=1}^{N} \frac{CF^{Equity}}{\left(1+r_e^L\right)^t} = \sum_{t=1}^{N} \frac{CF^U - CF^{Debt}}{\left(1+r_e^L\right)^t} \quad . \tag{A.1}$$

Apply the annuity operator A_e^L to get

$$S^L = A_e^L \times \left(CF^U - CF^{Debt}\right) . \tag{A.2}$$

Noting the present value of the annuity unlevered cash flows is given by $S^U = A_e^U \times CF^U$ and the corresponding present value for debt is $B = A_b \times CF^{Debt}$, we get

$$S^L = A_e^L \times \left(IA_e^U \times S^U - IA_b \times B\right) , \tag{A.3}$$

where IA_r is the inverse of thr annuity operator A_r. Noting, from Modigliani & Miller (1958) that $S^U \equiv V^U = V^L \equiv S^L + B$, we get

$$\begin{aligned} S^L &= A_e^L \times \left(IA_e^U \times S^L + IA_e^U \times B - IA_b \times B\right) \\ &= A_e^L \times \left(IA_e^U \times S^L + \left(IA_e^U - IA_b\right) \times B\right) \end{aligned} . \tag{A.4}$$

Simple rearrangement gives

$$IA_e^L = IA_e^U + \left(IA_e^U - IA_b\right) \times \frac{B}{S^L} \quad . \tag{A.5}$$

This is the required rate of return of levered equity to complement the Miller (2009a) NLWACC.

Substitution of IA_e^L into the Miller NLWACC, namely,

$$IA_{NLWACC} = IA_e^L \times \frac{S^L}{S^L + B} + IA_b \times \frac{B}{S^L + B} \quad , \tag{A.6}$$

gives

$$IA_{NLWACC} = \left(AI_e^U + \left(AI_e^U - AI_b \right) \times \frac{B}{S^L} \right) \times \frac{S^L}{S^L + B} + IA_b \times \frac{B}{S^L + B} \quad . \tag{A.7}$$

Simple rearrangement easily shows

$$AI_{NLWACC} = AI_e^U \quad , \tag{A.8}$$

which gives

$$r_{NLWACC} = r_e^U \quad , \tag{A.9}$$

for each year of any *N*. Our recommendation works for Miller's project.

References

Bade, B. (2009). Comment on "The weighted average cost of capital is not quite right". *Quarterly Review of Economics and Finance*, 49, 1476–1480.

Miles, J. and Ezzell, J. (1980). The weighted average cost of capital, perfect capital markets, and project life: A clarification. *Journal of Financial and Quantitative Analysis*, 15, 719-730.

Miller, R. A. (2009a). The weighted average cost of capital is not quite right. *Quarterly Review of Economics and Finance*, 49, 128–138.

Miller, R. A. (2009b). The weighted average cost of capital is not quite right: Reply to M. Pierru. *Quarterly Review of Economics and Finance*, 49, 1213-1218.

Modigliani, F. and Miller, M. (1958). The cost of capital, corporate finance and the theory of investment. *American Economic Review*, 48, 261-297.

Modigliani, F. and Miller, M. (1963). Corporate income taxes and the cost of capital: A correction. *American Economic Review,* 53, 433-443.

Myers, S. (1974). Interactions of corporate financing and investment decision implications for capital budgeting. *Journal of Finance*, 29, 1-25.

Pierru, A. (2009a). The weighted average cost of capital is not quite right: A comment. *Quarterly Review of Economics and Finance*, 49, 1219–1223.

Pierru, A. (2009b). The weighted average cost of capital is not quite right: A rejoinder. *Quarterly Review of Economics and Finance*, 49, 1481–1484.

Table 1
The projects

	Cash flows		

Period	Project	Debt	Equity

Panel A: Miller's project

0	-200,000	-50,000	-150,000
1 to 8	38,247.23	8,051.80	30,195.43
IRR	10.5533%	6.00%	12.00%

Panel B: Bade/Pierru project

0	-200,000	-50,000	-150,000
1 to 8	38,173.86	varies	varies
IRR	10.50%	6.00%	12.00%

Table 2
Special cases

	Cash flows		
Period	Project	Debt	Equity

Panel A: Miller model

Period	Project	Debt	Equity
0	-260	-80	-180
1	160	43.635	116.365
2	160	43.635	116.365
IRR	15.03%	6.00%	18.98%
IA_r	$IA_{WACC} = 0.6154$	$IA_b = 0.5454$	$IA_e^L = 0.6465$
	$r_{WACC} = 15.03\%$		

Panel B: Bade/Pierru model – constant leverage

Period	Project	Debt	Equity
0	-260	-80	-180
1	160	42.00	118.00
2	160	45.36	114.64
IRR	15.03%	6.00%	19.05%
	$r_{WACC} = 15.03\%$		

Table 3
Miller's (2009a) project with constant leverage

	Cash flows		
Period	Project	Debt	Equity
0	-200,000	-50,000	-150,000
1	38,247.23	7,285.15	30,962.08
2	38,247.23	7,480.26	30,766.97
3	38,247.23	7,695.97	30,551.26
4	38,247.23	7,934.44	30,312.79
5	38,247.23	8,198.08	30,049.15
6	38,247.23	8,489.54	29,757.69
7	38,247.23	8,811.76	29,435.47
8	38,247.23	9,167.99	29,079.24
IRR	10.5533%	6.0%	12.071%